Alain Connes on Mathematics

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Noncommutative geometry
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I believe that mathematics is one way the human mind can create concepts. In many ways mathematics plays a role that philosophy could have played in creating concepts that then can be used in the real world. It takes time for them to evolve and be used in the real world, but the real factory is mathematics. Its concepts have to do with shape and abstract things among others, and they are much more subtle and diverse than numbers. This is probably something that the general public does not realize. Mathematicians use numbers only when they are needed. One could say that the idea of energy comes from physics, but in fact, it originates in mathematics. Mathematics is the ultimate language in which there is a distillation of abstract ideas that become very precise and can then be used in different fields. At the same time, mathematics can be very tough because it's resistant. It's a very stubborn reality ; you cannot do whatever you want. It is scary. One shouldn't be scared. There's a beautiful saying by Alexander Grothendieck who said, "To be afraid of making a mistake is the same as to be afraid of the truth."

There's story of a friend's child that shows the essence of mathematics very clearly. At age five, he was on the beach with his father. He had been quite ill at age three and the father was, always a bit worried about his health. For one hour, the child sat on the beach quietly looking pale. The father was worried. Then the child came to his father and said, "Dad, there is no largest number." The father was amazed. The father, who was not a mathematician, asked, "How do you know ?". The child gave him a proof. We hear a lot of nonsense about children learning how to count on their fingers. Here you have a little child of five years old who found, on his own, a true mathematical fact and he found it in his brain, not in a book. He discovered it by pure thought and found the proof. This is the essence of mathematics. Of course there is a tradition. There are plenty of books, and things that we learn do not evaporate because they have a proof. On the other hand, mathematics is something with which you can have direct contact without any intermediate tool. This is the most swiking feature of mathematics. You can be completely by yourself and you can still think about mathematics. You wouldn't necessarily do the mathematics that is important now because in order to do that, you have to have read the latest things. I'm not saying you should work in isolation. You wouldn't go anywhere if you did that. But what I am saying is that when you get started, to really become a mathematician, the key step is to realize that at some point you have to stop reading books. You have to think on your own. You have to become your own authority. There is not an authority to which you have to refer At some point, you have to realize that whether something is written in a book or not doesn't matter. What matters is whether you have a proof and whether you're sure of it. The rest doesn't matter. This can happen in a child very early.

With regard to my own work, my thesis, you have the Cartesian point of view, which is ordinary geometry. Here you have coordinates and so on. But, there are spaces that are more complicated

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because they are spaces where you not only look at the points of sets but you also look at the relations between the points. These new sets, sets with relations, can be described by algebras, but these algebras are noncommutative. This was at first found by physicists and it can be explained extremely simply. When you write a word on a piece of paper, you have to pay attention to the order of the letters. A friend wrote me an email at one point and for some time I couldn't make sense of it because in four places I couldn't identify the meaning. It took me some time to realize it was my own name but written with a different order of the letters. When one does ordinary number calculus or ordinary algebra, as it's called, then you can permute the letters. The order of the letters doesn't matter : if you write 3×5, it's the same as 5×3. In physics, it's been found that this is not the case when you look at microscopic systems. You have to be more careful. You have to pay more attention. What I found in my thesis was that if you look at algebras where you pay attention to the order, time emerges. Time emerges from this noncommutativity : the fact that you pay attention to the order of the letters. This led me to my work on the classification of factors. After working on this for ten years, I developed in full a new geometry called "noncommutative geometry", in which one refines all the usual geometrical ideas and applies them to the new spaces. These spaces have amazing properties that generate their own time. Not only do they generate their own time, but they have features which enable you to cool them down or warm them up. You can do thermodynamics with them. There is an entirely new part of geometry and algebra that is related to these new spaces, called noncommutative geometry, on which I have been working essentially all my life.