

Programmation d'une formule extraite du livre *Werke : Hoehere Arithmetik* de Gauss (Denise Vella-Chemla, février 2024)

On trouve à la page 22 du livre *Werke : Hoehere Arithmetik* les deux définitions suivantes :

$$\begin{aligned}
 T &= 1 + \cos k\omega + \cos 4k\omega + \cos 9k\omega + \dots + \cos(n-1)^2k\omega \\
 U &= \sin k\omega + \sin 4k\omega + \sin 9k\omega + \dots + \sin(n-1)^2k\omega
 \end{aligned}$$

et à la page 31 le tableau suivant :

T	U	selon que n est de la forme
$\pm\sqrt{n}$	$\pm\sqrt{n}$	4μ
$\pm\sqrt{n}$	0	$4\mu + 1$
0	0	$4\mu + 2$
0	$\pm\sqrt{n}$	$4\mu + 3$

On vérifie ces formules par le programme :

```

import math
from math import pi, sin, cos, sqrt

for n in range(1,101):
    print('',n, '---->')
    if n%4 == 0: print('4k')
    else:
        if n%4 == 1: print('4k+1')
        else:
            if n%4 == 2: print('4k+2')
            else: print('4k+3')

    T = 1
    U = 0
    for k in range(1, n):
        T = T+cos(k*k*2*pi/n)
        U = U+sin(k*k*2*pi/n)
    print('T = ', T)
    print('U = ', U)
    print('sqrt = ', sqrt(n))

```

dont le résultat montre que les formules sont correctes :

```

1 ---->
4k+1
T = 1
U = 0
sqrt = 1.0

```

2 --->
4k+2
T = 0.0
U = 1.2246467991473532e-16
sqrt = 1.4142135623730951

3 --->
4k+3
T = 9.992007221626409e-16
U = 1.7320508075688779
sqrt = 1.7320508075688772

4 --->
4k
T = 2.0000000000000004
U = 1.9999999999999998
sqrt = 2.0

5 --->
4k+1
T = 2.23606797749979
U = -5.551115123125783e-16
sqrt = 2.23606797749979

6 --->
4k+2
T = -3.164135620181696e-15
U = 1.9984014443252818e-15
sqrt = 2.449489742783178

7 --->
4k+3
T = 1.4432899320127035e-15
U = 2.6457513110645907
sqrt = 2.6457513110645907

8 --->
4k
T = 2.828427124746193
U = 2.828427124746187
sqrt = 2.8284271247461903

9 --->
4k+1
T = 3.0000000000000036

U = 1.2212453270876722e-15
sqrt = 3.0

10 --->

4k+2

T = 6.661338147750939e-16
U = -5.662137425588298e-15
sqrt = 3.1622776601683795

11 --->

4k+3

T = 3.219646771412954e-15
U = 3.316624790355405
sqrt = 3.3166247903554

12 --->

4k

T = 3.4641016151377526
U = 3.464101615137754
sqrt = 3.4641016151377544

13 --->

4k+1

T = 3.605551275463985
U = -8.215650382226158e-15
sqrt = 3.605551275463989

14 --->

4k+2

T = 1.7763568394002505e-15
U = -6.2727600891321345e-15
sqrt = 3.7416573867739413

15 --->

4k+3

T = 8.43769498715119e-15
U = 3.8729833462074135
sqrt = 3.872983346207417

16 --->

4k

T = 4.0000000000000115
U = 3.9999999999999876
sqrt = 4.0

17 --->

4k+1

T = 4.123105625617655

U = 1.2712053631958042e-14

sqrt = 4.123105625617661

18 --->

4k+2

T = 0.0

U = -2.2315482794965646e-14

sqrt = 4.242640687119285

19 --->

4k+3

T = 1.176836406102666e-14

U = 4.358898943540667

sqrt = 4.358898943540674

20 --->

4k

T = 4.472135954999589

U = 4.472135954999556

sqrt = 4.47213595499958

21 --->

4k+1

T = 4.582575694955837

U = 8.43769498715119e-15

sqrt = 4.58257569495584

22 --->

4k+2

T = 1.0658141036401503e-14

U = 6.938893903907228e-15

sqrt = 4.69041575982343

23 --->

4k+3

T = -1.1435297153639112e-14

U = 4.795831523312737

sqrt = 4.795831523312719

24 --->

4k

T = 4.898979485566374

U = 4.8989794855663416
sqrt = 4.898979485566356

25 --->

4k+1
T = 4.999999999999992
U = -1.3128387266192476e-14
sqrt = 5.0

26 --->

4k+2
T = 5.218048215738236e-15
U = -1.2656542480726785e-14
sqrt = 5.0990195135927845

27 --->

4k+3
T = 1.6209256159527285e-14
U = 5.1961524227066676
sqrt = 5.196152422706632

28 --->

4k
T = 5.291502622129176
U = 5.291502622129131
sqrt = 5.291502622129181

29 --->

4k+1
T = 5.385164807134534
U = -1.940114735532461e-14
sqrt = 5.385164807134504

30 --->

4k+2
T = 2.1316282072803006e-14
U = -9.048317650695026e-15
sqrt = 5.477225575051661

31 --->

4k+3
T = 3.774758283725532e-14
U = 5.567764362830035
sqrt = 5.5677643628300215

32 --->

4k

T = 5.656854249492386

U = 5.656854249492382

sqrt = 5.656854249492381

33 --->

4k+1

T = 5.744562646538029

U = -2.6256774532384952e-14

sqrt = 5.744562646538029

34 --->

4k+2

T = -9.43689570931383e-15

U = 4.218847493575595e-15

sqrt = 5.830951894845301

35 --->

4k+3

T = 3.186340080674199e-14

U = 5.9160797830996135

sqrt = 5.916079783099616

36 --->

4k

T = 6.00000000000000195

U = 5.999999999999989

sqrt = 6.0

37 --->

4k+1

T = 6.082762530298207

U = -4.41591208044656e-14

sqrt = 6.082762530298219

38 --->

4k+2

T = 8.548717289613705e-15

U = -1.3600232051658168e-14

sqrt = 6.164414002968976

39 --->

4k+3

T = 2.2648549702353193e-14

U = 6.2449979983983654
sqrt = 6.244997998398398

40 --->

4k
T = 6.324555320336819
U = 6.324555320336705
sqrt = 6.324555320336759

41 --->

4k+1
T = 6.403124237432841
U = -6.375455718909961e-14
sqrt = 6.4031242374328485

42 --->

4k+2
T = 3.6193270602780103e-14
U = 1.4377388168895777e-14
sqrt = 6.48074069840786

43 --->

4k+3
T = 1.3766765505351941e-14
U = 6.55743852430199
sqrt = 6.557438524302

44 --->

4k
T = 6.633249580710818
U = 6.63324958071077
sqrt = 6.6332495807108

45 --->

4k+1
T = 6.708203932499365
U = -3.574918139293004e-14
sqrt = 6.708203932499369

46 --->

4k+2
T = 7.37188088351104e-14
U = 6.13398221105399e-15
sqrt = 6.782329983125268

47 --->
4k+3
T = -6.417089082333405e-14
U = 6.855654600401059
sqrt = 6.855654600401044

48 --->
4k
T = 6.928203230275569
U = 6.928203230275426
sqrt = 6.928203230275509

49 --->
4k+1
T = 7.000000000000001
U = 7.382983113757291e-15
sqrt = 7.0

50 --->
4k+2
T = -2.2870594307278225e-14
U = 5.0959236830294685e-14
sqrt = 7.0710678118654755

51 --->
4k+3
T = -1.6986412276764895e-14
U = 7.141428428542828
sqrt = 7.14142842854285

52 --->
4k
T = 7.211102550927947
U = 7.211102550928012
sqrt = 7.211102550927978

53 --->
4k+1
T = 7.280109889280554
U = -1.7694179454963432e-14
sqrt = 7.280109889280518

54 --->
4k+2
T = 1.9984014443252818e-15

U = 2.416122857340497e-14
sqrt = 7.3484692283495345

55 --->

4k+3

T = 7.494005416219807e-14

U = 7.416198487095638

sqrt = 7.416198487095663

56 --->

4k

T = 7.4833147735479235

U = 7.483314773547801

sqrt = 7.483314773547883

57 --->

4k+1

T = 7.5498344352707445

U = -6.063205493234136e-14

sqrt = 7.54983443527075

58 --->

4k+2

T = -3.2307490016592055e-14

U = -1.1331213745080504e-13

sqrt = 7.615773105863909

59 --->

4k+3

T = 8.315570454442422e-14

U = 7.681145747868639

sqrt = 7.681145747868608

60 --->

4k

T = 7.745966692414893

U = 7.745966692414805

sqrt = 7.745966692414834

61 --->

4k+1

T = 7.810249675906595

U = 3.048949981376836e-14

sqrt = 7.810249675906654

62 --->
4k+2
T = 5.917488721252084e-14
U = -2.148281552649678e-14
sqrt = 7.874007874011811

63 --->
4k+3
T = 2.6645352591003757e-14
U = 7.93725393319381
sqrt = 7.937253933193772

64 --->
4k
T = 7.999999999999994
U = 7.999999999999949
sqrt = 8.0

65 --->
4k+1
T = 8.06225774829863
U = -1.1203538097248611e-13
sqrt = 8.06225774829855

66 --->
4k+2
T = -5.939693181744587e-14
U = -1.4585554986012994e-14
sqrt = 8.12403840463596

67 --->
4k+3
T = -5.995204332975845e-15
U = 8.185352771872607
sqrt = 8.18535277187245

68 --->
4k
T = 8.24621125123535
U = 8.246211251235247
sqrt = 8.246211251235321

69 --->
4k+1
T = 8.306623862918043

U = 3.176625629208729e-14
sqrt = 8.306623862918075

70 --->

4k+2

T = 9.00390872971002e-14
U = -6.517009154549669e-14
sqrt = 8.366600265340756

71 --->

4k+3

T = 1.0902390101819037e-13
U = 8.426149773176352
sqrt = 8.426149773176359

72 --->

4k

T = 8.485281374238665
U = 8.485281374238472
sqrt = 8.48528137423857

73 --->

4k+1

T = 8.544003745317587
U = -1.593447596093256e-13
sqrt = 8.54400374531753

74 --->

4k+2

T = 1.2312373343092986e-13
U = 1.0906553438161382e-13
sqrt = 8.602325267042627

75 --->

4k+3

T = -1.0247358517290195e-13
U = 8.660254037844435
sqrt = 8.660254037844387

76 --->

4k

T = 8.717797887081487
U = 8.717797887081339
sqrt = 8.717797887081348

77 --->

4k+1

T = 8.77496438739204

U = 2.167710455580618e-14

sqrt = 8.774964387392123

78 --->

4k+2

T = 2.1871393585115584e-14

U = -5.4969917506753063e-14

sqrt = 8.831760866327848

79 --->

4k+3

T = -1.1102230246251565e-16

U = 8.88819441731542

sqrt = 8.888194417315589

80 --->

4k

T = 8.944271909999317

U = 8.944271909999062

sqrt = 8.94427190999916

81 --->

4k+1

T = 8.99999999999982

U = -1.802724636235098e-13

sqrt = 9.0

82 --->

4k+2

T = 1.3244960683778118e-13

U = 1.6815715486728777e-13

sqrt = 9.055385138137417

83 --->

4k+3

T = 5.262457136723242e-14

U = 9.110433579144326

sqrt = 9.1104335791443

84 --->

4k

T = 9.165151389911593

U = 9.1651513899116
sqrt = 9.16515138991168

85 --->

4k+1
T = 9.219544457292818
U = -1.49366630175507e-13
sqrt = 9.219544457292887

86 --->

4k+2
T = 8.204548151979907e-14
U = -1.322830733840874e-13
sqrt = 9.273618495495704

87 --->

4k+3
T = 9.703349235223868e-14
U = 9.3273790530887
sqrt = 9.327379053088816

88 --->

4k
T = 9.380831519646996
U = 9.380831519646694
sqrt = 9.38083151964686

89 --->

4k+1
T = 9.433981132056614
U = 2.245426067304379e-14
sqrt = 9.433981132056603

90 --->

4k+2
T = -2.1649348980190553e-14
U = 4.621303340002214e-14
sqrt = 9.486832980505138

91 --->

4k+3
T = 8.737455203799982e-14
U = 9.539392014169453
sqrt = 9.539392014169456

92 --->
4k
T = 9.59166304662542
U = 9.59166304662546
sqrt = 9.591663046625438

93 --->
4k+1
T = 9.643650760993097
U = -1.6238399513923696e-13
sqrt = 9.643650760992955

94 --->
4k+2
T = 8.693046282814976e-14
U = -7.308043059595093e-14
sqrt = 9.695359714832659

95 --->
4k+3
T = 9.137135492665038e-14
U = 9.7467943448089
sqrt = 9.746794344808963

96 --->
4k
T = 9.797958971132939
U = 9.797958971132573
sqrt = 9.797958971132712

97 --->
4k+1
T = 9.84885780179616
U = 1.817296313433303e-13
sqrt = 9.848857801796104

98 --->
4k+2
T = -2.6978419498391304e-14
U = 2.222250161665329e-13
sqrt = 9.899494936611665

99 --->
4k+3
T = -5.873079800267078e-14

U = 9.949874371066427
sqrt = 9.9498743710662

100 --->

4k

T = 9.99999999999835

U = 9.99999999999675

sqrt = 10.0