Alain Connes : So at the start, we could approach mathematics by considering it as a part of physics, as a language that is developed to better understand the physical world around us ; and effectively, we realize that mathematics has this remarkable efficiency in this domain there. And very often, we have an understanding from the outside of these things that is in a way too superficial, and one example I wanted to take is the version which we now come to, with physical reality.

So physical reality is nothing other than the superimposition of possible imaginaries. And I will not give the formula but in this sentence, there is something extraordinary, it is that imaginary numbers, complex numbers are implicated, and these numbers, by the same name, as imaginary numbers, at the beginning, were mathematical fictions.

So there is this incredible efficiency of mathematics in the physical world which even upsets our philosophical conception of what reality is and which questions, of course, materialism as a somewhat naive idea because the materialism is a theory based on a partial understanding of things and which identifies the real with the material.

Now from what I said, precisely, the fact that the real is this superposition of imaginary possibilities shows how much more subtle reality is. But in fact, after a while, we realize that there is a clean journey, inside from the mathematical world, which becomes disjoint from the physical world, and the main tool that makes it possible to start this journey and begin, is the analogy ; it's the fact that the human mind is able to see between very, very different areas, certain reflections, certain correspondences, and from these reflections, these correspondences, to transpose, to transplant ideas that were valid in a domain, to transplant them into another domain ; this is how mathematicians have discovered whole swathes of mathematics that have nothing to do with the physical world, and that it would be illusory to want to find in the physical world.

This is called, for example, the p-adic world. The p-adic world is the world in which for example the integer two is very small, and when, by analogy, we transplant the concepts we have for real numbers, we notice, a little as Alice in Wonderland, we discover 36 things we wouldn't have never suspected.

So this is the first thing, the first thing is that there is a world of mathematics, which is absolutely not subject to the world of physics and when we explore this world much further, we realize in fact that in this mathematical world, there are for

Transcription of a video that can be played here https://www.youtube.com/watch?v=i08kF-NLVS4 example things that are true but not demonstrable.

So this is something that is quite difficult to explain, I tried to explain it in a book we wrote with Lichnerowicz and SchÃ¼tzenberger, by the fable of hare and turtle. So it would be technically complicated to explain but itâs a typical example of a mathematical statement, which we know is true, we know also that it is unprovable in what is called Peano arithmetic, it is said in simple ways, and the reason we know it to be true is what is called ordinals theory, set theory, Zermelo-Fraenkel' theory, etc. So in fact, we have since managed to see that if we take simple statements, arithmetic statements that can be formulated in a simple way, and in fact, we know that most of these statements are unprovable, that is to say that the proportion of statements that are true but not demonstrable tends to one, that is to say that most of the statements that are true are actually unprovable. And the image that must be kept in mind for the relationship between the mathematician and this mathematical reality that I call archaic mathematical reality, because rightly, there are things that are true but that we cannot perceive in a direct way, this is the same relationship as the one between external reality and a court : in the court, you have data, so in mathematics, these are called axioms, and from this data, you can make a number of deductions, this is what the mathematician does while working. But, it would be wrong to identify the deductions that are made inside the court with external reality and this for obvious reasons.