Résumé au sujet des sommes quadratiques de Gauss, Denise Vella-Chemla, septembre 2025

On synthétise dans un tableau ce qui est connu sur les sommes quadratiques de Gauss, et on le vérifie par programme.

On appelle :

$$S = \sum_{x=0}^{n-1} \cos \frac{2\pi x^2}{n} + i \sin \frac{2\pi x^2}{n}$$

n	S
$4k$	$\sqrt{n} + i\sqrt{n}$
$4k+2$	0
$4k+1$	$\sqrt{n}$
$4k+3$	$i\sqrt{n}$

Dans le programme et son résultat ci-après, on rappelle qu'en python le nombre complexe $i = \sqrt{-1}$ s'écrit $1j$.

Voici le programme python :

```
import cmath
from cmath import pi,cos,sin
import sympy
from sympy.ntheory.residue_ntheory import is_quad_residue

def premier(atester):
     k = 2
     if atester in [0, 1]:  return False
     if atester in [2, 3, 5, 7]:  return True
     while True:
          if k * k > atester:  return True
          else:
               if atester % k == 0:  return False
               else:  k = k + 1

for n in range(1,104):
     print('',n,':::::::::',end='')
     somme = 0
     for x in range(0,n):
          somme = somme+cos(2*pi*x*x/n)+1j*sin(2*pi*x*x/n)
     print(somme)
```

Voici son résultat :

1 ::::::::(1+0j)

2 ::::::::1.2246467991473532e-16j

3 ::::::::(9.992007221626409e-16+1.7320508075688779j)

4 ::::::::(2.0000000000000004+1.9999999999999998j)

5 ::::::::(2.23606797749979-5.551115123125783e-16j)

6 ::::::::(-3.164135620181696e-15+1.9984014443252818e-15j)

7 ::::::::(1.4432899320127035e-15+2.6457513110645907j)

8 ::::::::(2.828427124746193+2.828427124746187j)

9 ::::::::(3.0000000000000036+1.2212453270876722e-15j)

10 ::::::::(6.661338147750939e-16-5.662137425588298e-15j)

11 ::::::::(3.219646771412954e-15+3.316624790355405j)

12 ::::::::(3.464101615137756+3.4641016151377477j)

13 ::::::::(3.6055512754639985-3.219646771412954e-15j)

14 ::::::::(-3.774758283725532e-15-1.8318679906315083e-15j)

15 ::::::::(1.1324274851176597e-14+3.872983346207407j)

16 ::::::::(4.00000000000001+3.9999999999999942j)

17 ::::::::(4.123105625617659+7.382983113757291e-15j)

18 ::::::::(-1.199040866595169e-14-7.66053886991358e-15j)

19 ::::::::(2.9976021664879227e-15+4.358898943540675j)

20 ::::::::(4.472135954999587+4.472135954999535j)

21 ::::::::(4.582575694955844+9.325873406851315e-15j)

22 ::::::::(-4.329869760381105e-15+3.652633751016765e-14j)

23 ::::::::(-3.774758283725532e-15+4.795831523312704j)

24 ::::::::(4.898979485566365+4.898979485566325j)

25 ::::::::(5.00000000000001-1.4904744105592727e-14j)

26 ::::::::(-2.2315482794965646e-14+5.084821452783217e-14j)

27 ::::::::(1.176836406102666e-14+5.196152422706646j)

28 ::::::::(5.2915026221292205+5.291502622129135j)

29 ::::::::(5.385164807134513-5.412337245047638e-15j)

30 ::::::::(5.1514348342607263e-14-3.169686735304822e-14j)

31 ::::::::(5.773159728050814e-15+5.567764362830042j)

32 ::::::::(5.6568542494924055+5.6568542494923815j)

33 ::::::::(5.744562646538009-2.6256774532384952e-14j)

34 ::::::::(6.106226635438361e-15+1.354472090042691e-14j)

35 ::::::::(1.609823385706477e-14+5.916079783099626j)

36 ::::::::(5.999999999999995+6.000000000000034j)

37 ::::::::(6.082762530298203-3.938516179857743e-14j)

38 ::::::::(-2.9976021664879227e-15-5.512257317263902e-14j)

39 ::::::::(1.9095836023552692e-14+6.244997998398367j)

40 ::::::::(6.324555320336834+6.324555320336728j)

41 ::::::::(6.403124237432813-4.6879167214797235e-14j)

42 ::::::::(4.063416270128073e-14+3.7192471324942744e-15j)

43 ::::::::(2.8976820942716586e-14+6.557438524302063j)

44 ::::::::(6.633249580710773+6.633249580710777j)

45 ::::::::(6.708203932499357-9.747758156208874e-14j)

46 ::::::::(5.129230373768223e-14-3.039235529911366e-14j)

47 ::::::::(-1.7430501486614958e-14+6.855654600401043j)

48 ::::::::(6.928203230275573+6.928203230275494j)

49 ::::::::(7.000000000000065+2.3814283878209608e-14j)

50 ::::::::(-7.083222897108499e-14-2.5868196473766147e-14j)

51 ::::::::(5.206945985491984e-14+7.141428428542721j)

52 ::::::::(7.211102550928081+7.211102550928045j)

53 ::::::::(7.280109889280493+3.622102617839573e-15j)

54 ::::::::(-2.3314683517128287e-14-6.732114865570793e-14j)

55 ::::::::(6.339373470609644e-14+7.416198487095701j)

56 ::::::::(7.483314773547854+7.483314773547758j)

57 ::::::::(7.5498344352707445-8.372469384454462e-14j)

58 ::::::::(3.9190872769268026e-14-3.4264258097493894e-14j)

59 ::::::::(2.7644553313166398e-14+7.681145747868653j)

60 ::::::::(7.745966692414922+7.745966692414744j)

61 ::::::::(7.810249675906573+9.132972156322694e-14j)

62 ::::::::(7.693845560652335e-14-7.299716386910404e-14j)

63 ::::::::(-2.3092638912203256e-14+7.937253933193833j)

64 ::::::::(8.000000000000048+7.999999999999954j)

65 ::::::::(8.062257748298565-1.0315359677548486e-13j)

66 ::::::::(5.4289905904170155e-14-6.894484982922222e-14j)

67 ::::::::(-8.903988657493755e-14+8.185352771872498j)

68 ::::::::(8.246211251235465+8.246211251235222j)

69 ::::::::(8.30662386291809-5.911937606128959e-14j)

70 ::::::::(8.626432901337466e-14-1.2156942119645464e-14j)

71 ::::::::(1.254552017826427e-13+8.42614977317636j)

72 ::::::::(8.485281374238564+8.485281374238609j)

73 ::::::::(8.54400374531749-1.3225531780847177e-13j)

74 ::::::::(1.1834977442504169e-13-2.085831507514513e-14j)

75 ::::::::(-9.037215420448774e-14+8.660254037844293j)

76 ::::::::(8.717797887081511+8.717797887081327j)

77 ::::::::(8.77496438739216-6.078471059822732e-15j)

78 ::::::::(1.3911094498553211e-13+3.649858193455202e-15j)

79 ::::::::(8.515410598874951e-14+8.888194417315523j)

80 ::::::::(8.944271909999362+8.944271909999088j)

81 ::::::::(8.999999999999947-1.1543543898540065e-13j)

82 ::::::::(6.838973831690964e-14+1.644517855226013e-14j)

83 ::::::::(1.5265566588595902e-13+9.110433579144157j)

84 ::::::::(9.165151389911708+9.165151389911731j)

85 ::::::::(9.219544457292667-1.0637324354689781e-13j)

86 ::::::::(3.006483950684924e-13-2.565170298396424e-13j)

87 ::::::::(6.727951529228449e-14+9.327379053088999j)

88 ::::::::(9.38083151964684+9.380831519646819j)

89 ::::::::(9.433981132056495-4.232725281383409e-14j)

90 ::::::::(-2.930988785010413e-14-1.6861512186494565e-13j)

91 ::::::::(2.565725409908737e-13+9.539392014169414j)

92 ::::::::(9.591663046625557+9.591663046625241j)

93 ::::::::(9.643650760993168+1.437044927499187e-13j)

94 ::::::::(-3.610445276081009e-13+6.800116025829084e-16j)

95 ::::::::(-7.427392034742297e-14+9.746794344808677j)

96 ::::::::(9.797958971132733+9.79795897113241j)

97 ::::::::(9.848857801796308+8.125444761475364e-14j)

98 ::::::::(1.9373391779708982e-13+1.1053657988924215e-13j)

99 ::::::::(6.7390537594747e-14+9.94987437106646j)

100 ::::::::(9.999999999999929+9.999999999999881j)

101 ::::::::(10.04987562112098+2.883804306463844e-14j)

102 ::::::::(2.4236168627567167e-13-1.7703200017038512e-13j)

103 ::::::::(7.505107646466058e-14+10.148891565092185j)