An interview of Alain Connes
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I'm here with the Professor Alain Connes at ICTP ; he's visiting in conjunction with the workshop on Non-commutative geometry.
So the first question would be a sort of an ICTP specific question in this : if you had advice to give young students who want to study math, in particular, students from developing countries, what advice would you give to them ?

It's difficult : you know, to study maths, of course, they are challenges and somehow, I mean, I have always thought that the key step in studying mathematics is to understand that, you know, you don't learn mathematics. You make it, you do it and until you are really able to take a problem and solve it by yourself or try to solve it by yourself, you are not doing mathematics. Because learning maths, they are not topics that you can learn, there are some scientific topics that you can learn, but this is not the case for mathematics. For mathematics, you have to do it yourself. So that would be the best I could say, in a very short time. So it's really like ..., for instance, you know, to give a comparison, if you try to become a pianist by reading books, it's the same story, you have to practise. The practise is far more important than whatever, reading books, and all that. In that way, it's a very democratic subject and there is a key step also, the magnificience of this key step is that when a student finds a mistake of the teacher, because he is able to think by himself, and find out that he is right, and the teacher is wrong, this is something which is very important in mathematics, and which is different from other topics. Because other topics require so much knowledge that, somehow this will not be possible to do it for a beginner, but in mathematics, it's possible.

So next question is thinking about this quest to find a unified theory for the universe, I think it's interesting the impact that maybe this has had on the interaction between maths and physics. And so maybe some have prospective that at one time one field set another, but it seems that now there's sort of a symbiotic relationship and I wanted your perspective on the evolution of relationship between maths and physics.

Yeah, I think it is a very delicate issue in the sense that there is one Graal, one problem which people are trying to solve which is the Quantum Gravity, so we know that quantum gravity (?) exists, we know it's quite difficult. But in a way, for mathematicians, at least as far as they are concerned, the issue is even more important for mathematics in the following sense : for instance when Riemann gave his inaugural

Transcript of a video that can be watched here : https://www.youtube.com/watch?v=XxtnTtvlhMw talk, he was very clear on the fact that the hypothesis that he had for Riemannian geometry, the hypothesis for geometry, would not hold at the very very small scale. And he was so lucid and precise, that he had already foreseen developments that would come much later, and in particular in non-commutative geometry, because of the fact he wanted the notion of (...) or the notion of light-ray no longer make sense in the very very small whereas there were these notions that were crucial in his definition of geometry. So there is a symbiosis, but there are also, I would say, deviations, and what I hear for instance in some talks, I hear deviations because some people just want to change the rules of physics (to do what they want ?...).

So I think we must be very careful. And at the same time, what I would say, is that there is an intermediate goal to complete geometry, and that goal is very precise, it's to understand the effect, the impact on the notion of geometry that the experimental physics has provided for us in one century, where the inward-band trip, that began at the end of the XIX$^{\text{th}}$, with the discovery of the electron and of radioactivity. And there has increased our perception of the small structure of space-time by a factor of ten to the power eight in the century and that has implications on the geometric model we have of space-time and that application will be fully understood in non-commutative geometry and what happens is that space-time is no longer a purely continuum but it's a mixture of the continuum and the discrete. And so this is a lesson that was understood, it's a lesson which is very strange, that forced to change the riemannian paradigm. But this change in riemannian paradigm, of course, Riemann couldn't foresee it because it involves the quantum mechanics. So the new paradigm on geometry is very close to the riemannian one but there are nuances, and those nuances come from the quantum, they come from the formalism of quantum mechanics that has been discovered by von Neumann in 1960s. And it turned out then that the idea or the notion we want for geometric space becomes more natural, and is more easy to understand in the quantum formalism.

So you describe that there is not just the immensity of the universe but there are also these very small scales. How would you define a point ?

Okay, that's an interesting question because we can ask within a primary approach to that how do we define a point, and that question that is simple is how do we communicate with extraterrestrians or possible other civilizations the place where we are. Well, if I tell you that we are in Trieste, and so on, well, that don't tell because first of all, these people don't understand what Trieste, and then, there will have people who know general relativity, to whom we just have to give our coordinates in a coordinates system but that's also foolish because which coordinates system do we take, which invariant way do we have, and it turns out that what I was talking about before, this reunderstanding of duality has its answer that is exactly provided by the formalism. So the first question becomes how do we communicate this space in which we are, just global, not by giving a picture, how do we communicate the space in which we are, and second question, how do we define where we are in it. For communicating the space in which we are, it turns out that the best way is to give the music of the space. So if you take a shape, this is a well known metaphor if you want, which goes back to Mark Kac, so if you give a shape, like a drum for instance, of drums of various shapes and so on, it turns out that each drum has its shape as a special scale, a musical scale, and from which frequencies are depending of the shape.

And it turns out that if you want to give invariantly the space, you have to give a list of quantities which are assigned to this space in an invariant manner. Now the scale of the space is invariantly defined ; you can rotate the space, you can do whatever you want, and you will not change its scale. So this an invariant of the space. And it turns out that Helmholtz found in which is called Helmholtz's equation because he use the scale of the space, it turns out that there is a small refinement in this equation, Helmholtz was taking for line element the squareroot of the Laplacian, and you have to divide this basis by the Dirac operator, but this is small nuance. And when you know this small nuance, then you can actually reconstruct the space but you need to know a little more. You need a little more than the scale of the space, you need to know precisely what are the points. And what are the points ? Each point is defined by a chord on the scale. A point in a space, technically speaking, how do you specify the point ? So technically speaking, what you do, you take what are called the eigen vectors for the Dirac operator, their inceptions are bounded in the space (...) and you evaluate them at the point. When you evaluate them, you cannot just give a number, so you get a number in terms of the metrics, which are scalar products of those various cells, at the point, you use the metrics and it turns out that modulo the invariance, this metrics is exactly what you need to know the point.

So the picture, the mental picture is that by a misunderstanding, the space is understood by a musical scale and possible chords. And the possible chords are the points. So in a way, what happens is that you reconstitute the space by a kind of Fourier transform. And I believe that this is exactly what the brain does when you see, because when you see, there are photons which are coming in a moment because when you see, you have the photons at space eigen scale and the brain reconstitutes space like we are used to see it but what is even more important is that this is exactly the way we perceive the distant universe. Because we perceive the distant universe by looking at spectra of galaxies, spectra of stars, spectra of nebulae, and it is in this sense that the spectra can calibrate information that we receive from far stars.

In this formalism, we find out that not only, it's useful for microscopic distances but it is also reshuffles and changes the point of view on the large distances, but in a way which is perfectly coherent with our perceptions of the universe. For instance, typically, what happens is that we know that things are very very distant, you have to remember that there was some time where people didn't even know that there were things outside our galaxy, ok, it took very brave astronomers to find that. But now, we know that things are very very very distant, just because of the redshift. And this is again the spectral prover.

And here there's a concept of distance or unit of length in term of the wavelength

Sure. That's also a very important step, which is so much fun to explain because it relates to very concrete stuff. So the story starts in France more or less, during the French Revolution. You see, there were, more or less, there was a unit of length per city. There were on thousand of unit of length. That mean that where people were selling for instance tissu, travelling from one place to another, they had to measure with respect to the unit that was at the entrance of the village, of course *(laughs from Lucia-Dora Simonelli)*. Revolution was an idea of course to unify things, and they had grand purposes and all that, so they decided to... they had very good scientists, they decided to try to unify the system, by defining a unit of length. So what did they do. They took the largest available object, which is the Earth, and they defined the unit of length so that, when you multiply this unit of length by 40,000,000, you obtain the circumference of the Earth. So this is what they tried.

And in order to... Of course, they couldn't go to the pole, to measure the entire meridian, they measure angles between the stars they pointed with their instruments, so they only necessitated to measure some angular portion of the meridian. And they choose the angular portion which was between Dunkirk that is at the North of France and Barcelone which is in Spain. And in 1792, so this was during the fool period of Revolution, they sent two people, Delambre and Méchain, were sent out, to do the following; the idea was that they would first of all have a base, what we call a base. So they had lined down on a sufficiently long distance some bars, if you want, and they had taken that as a base. They were only measuring angles, which is a very smart idea. They were putting telescopes on top of hills and measuring angles, and by doing triangulations, they were comparing the base with the distance between Barcelone and Dunkirk. And out of that was defined the unit of length, which was actually a metal bar. It was a very interesting story because there were all sources of development behind this story because one of the guys, I think it was Méchain, had to make measurements in Spain and of course, so he was measuring angles, by putting the telescope on top of the hill, and of course, he had a lot of troubles because there was a war, between France and Spain, at that time, and he had to explain to the Spanish army that, by putting his telescope on top of the hill and looking in his telescope, he was not a spy, he was trying to define the unit of length *(laughs from both)*.

So there were a lot of anecdotes interesting to develop, I love the details of this stories, I don't know why. And then what happened was the following. This unit of length was actually deposed near Paris, and when I was kid, I learned that "The unit of length is the meter which is deposit in Pavillon de Breteuil near Paris.", and so on.

So I was thinking and I'm sure many people were thinking "This is not very practical" because if you want to measure your bed, of course, they made duplicates of this. So that was the situation at the time. But then, some very interesting event happened. So there were periodic meetings of the metric system people. And this meeting has been carried on very periodically for years. I'm not sure the period was one year. But around the 1930s, they noticed that actually, the platinium bar, defining the unit of length was changing length. And how do they notice that. They noticed that wy actually measuring its length very precisely, and by comparing it with the krypton wavelength for a specific transition. So that was very bad, and gradually, they took the right step. And the right step of course was to take this wavelength, as the unit of length. That took some time. But what is very interesting to know is that now, there are instruments which are so common, you can buy them in a shop, and these instruments are based again on the wavelength. The element that is used is no longer krypton. It is cesium because cesium is very easily available and sold, and moreover, the wavelength of cesium which is used is a microwave. So it's like when you put something in the microwave oven, it's a wavelenth that is of the order of 3 centimeters. And it's an instrument which allows you to measure length up to 12 decimals, so I mean, it's abolutely incredible. And this is now what is taken as the unit of length. Of course, people will tell you it's not a unit of length it'a a unit of time but because of the constancy of speed of light, speed of light has been fixed to a very specific number. So things have evolved and now, what you see from that is that there was a complete change in the paradigm because the unit of length is no longer a localized object, which is somewhere, but it's a spectral data. And it turns out that the new paradigm which comes from quantum mechanics, which is the paradigm of non-commutative geometry, which is called spectral geometry if you want, is exactly parallel to this change of paradigm in physics. So it's very concrete. It's something which is very very concrete and the enormous advantage is that if we had, for instance, to unify the metric system, not on Earth, but in the galaxy, for instance, if you tell to people, "ok, come to Paris and compare your unit of length to the unit of length we have defined there..." *(laughs)*, they would laugh at you, they would roar, because they would say "we have our unit of length", whereas if you tell to people "take a chemical element". Of course, cesium is a little bit complicated because...

For your business, it... maybe, you need something very common...

Yeah, exactly, like helium or hydrogen. I would vote for hydrogen, because hydrogen is essentially present anywhere, whereas cesium or heavy elements of that kind, in fact, one has to know that they only come from, not only from supernovae, but from very, very exceptional supernovae. So their abondance in the universe is not so clear, but if you take hydrogen for instance, there are spectral rays of hydrogen which are very precisely defined, there would be very specific patterns, but then one would have to find hyperfine splitting, because the advantage of hyperfine splitting which is used for cesium is that an hyperfine splitting is a difference of energy which is very,

very small, and that would in the inverse law, when you pass to the wavelength, it will generate microwaves, which is much more practical, whereas if you take a huge difference of frequency, like for a transition, you will get a very, very tiny unit of length and that would not (?) be good. What I am saying is that when you communicate with people, by sending a probe, and if you are able to tell them what is your unit of length, this is marvellous. And you just send a copy of the spectral rays of hydrogen and you explain which one you want to find out. I mean this is very simple, and if they are smart, they will understand, whereas if you do otherwise, it would never work.

In this description of the fine structure of space-time, you describe it in term of the spectrum of an operator, which allows...

It's a little bit more complicated, as I said, you know, of course, the spectrum of the operator gives you the unit of length,...

Does this allow you, in a way, to combine a discrete concept with a continuous concept ?

Well, what allows to combine the discrete and the continuum is the fact that, essentially, it's a mixture of the discrete and the continuum and what the discovery of the experimental physics have unvailed, over the century, is exactly what is the structure of the discrete space. So at first, the discrete space, with my collaborators, Chamseddine, and Walter Van Suijlekom and Mukhanov, what we found, at first, we were proceeding with a bottom-up approach, namely, we are taking from experiments, and trying to fit with what was going on, etc., and gradually we found what the finite space should be, but in a recent work about 2 or 3 years ago, with Chamseddine and Mukhanov, we were very amazed because we were asking a purely geometric problem, which was motivated of course by non-commutative geometry, but which was totally disjoint from the physics and the Standard model, and so on, and by developing this problem in dimension 4, we found exactly the same finite space in the same algebra, that was put in by hand before. So, we believe that we have a piece of the truth.

But naively, why is it important to include this discrete concept ?

Why naively, is it important, this is easy to explain somehow, but I need a piece of paper, *(He takes one)*. It's very easy to understand. You see, why is it important to have this discrete piece. It is that the most obvious problem you have, if you don't have this discrete piece, is that the Higgs boson, the Brout-Englert-Higgs, I knew Brout very much, I mean, he died just one year before the particle was discovered. The particle was discovered, we know it's there, but it doesn't fit with standard geometry, why ? Because in standard geometry, if you take a function on a space, you will differentiate it, and you will get what is called a gauge potential, a one-form, okay.

Why ? Because the differentiation depends on the direction in which you differentiate, so this is why you get something which is called spin 1 if you want, which depends of the direction. But the Brout-Englert-Higgs particle is a particle which is spin 0. So it doesn't depend of direction. So you wonder how you can obtain geometrically a particle of spin 0. Now imagine that instead of having just this manifold, okay, there is a discrete element, the discrete element is just an element that tells "am I on the top or am I on the bottom ?", so now I have more information, I know if I am on the top or on the bottom. And I take a function. This function will have a development here, and it will have a development under *(showing with his hand the two faces of the sheet of paper).* They don't have to be the same, so I can differentiate my function up *(AC makes his hand turning against the top of the sheet)*, and I can differentiate it down *(AC makes his hand turning against the bottom of the sheet)*, but I can also take the finite difference across *(showing the difference on the slice of the sheet)*. And the finite difference across, it does not depend on which direction I am taking. That's the boson of spin 0, and that corresponds to the Brout-Englert-Higgs boson. So the Brout-Englert-Higgs boson was a completely clear unmistakable sign, on a discrete structure, which was present.

And I knew Robert Brout very much and he was very interested, of course, by this understanding, which is the understanding of why, if you want, the experimental fights that physicists had, because the Brout-Englert-Higgs mechanism, it was obtain after years and years and years of thoughts, of how to give masses to particles. So all the masses of the particles actually come from this mechanism and what you find out in this model we have developed is that in fact, the main ingredient which is the metrics of masses and mixing angles, and so on, of the particles, is in fact exactly the line element for the finite structure. So the line element for the finite structure contains exactly this information, which means if you want that in this model, you have a mixture of the continuum and the discrete. But the discrete contains the information about the mass and the mixing angles part.

Thank you very much for your time and the exposure.