

Werk

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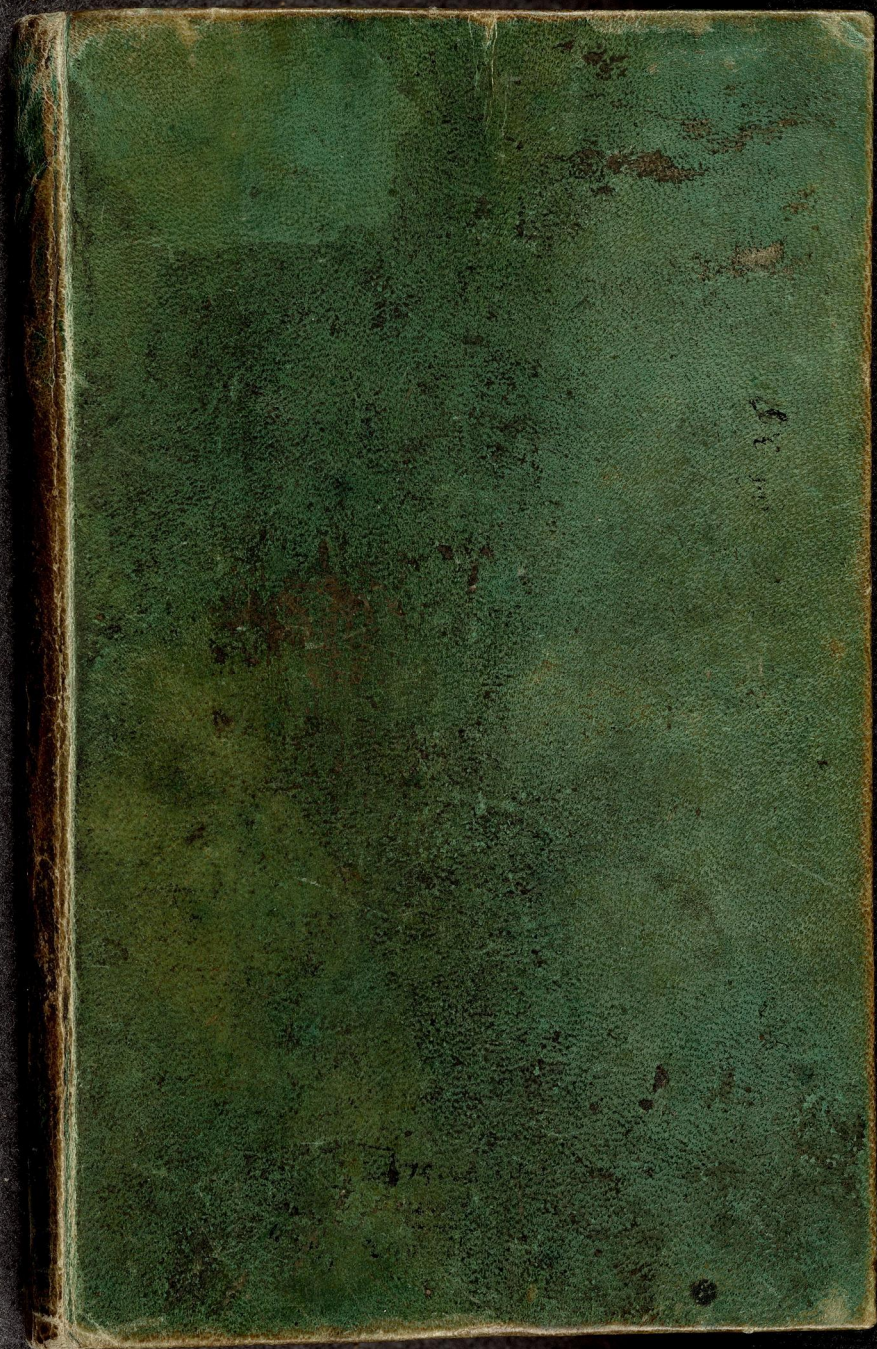
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Cod. Ms. Gants, Blatt 48

GEGAN
WAEQEGAN

29 Bl. 15.4.76. m

(Bl. 29 = Deckblatt d. Fache)

1796.

* Principia quibus innititur sectio circuli,
ac divisibilitas eiusdem geometrica in
septemdecim partes &c. Mart. 30. Brunsv.

* Numerorum primorum non omnes
numeros infra ipsos residua quadratica
esse posse demonstratione munitum.

Apr. 8. Ibid.

Formula pro cosinibus angulorum periphe-
rie submultiplicum expressionem gene-
raliorem ^{novis} admittentem nisi in duab. periodis

Apr. 12. Ibid.

* Amplificatio normae residuorum ad residua
et mensuras non indivisibiles.

Apr. 29. Gotsing.

Numeri cuiusvis divisibilitas varia in binos primos

Mai. 14. Golt.

* Coefficientes aequationum per radicum potestates
additas facile dantur. Mai. 23. Golt.

Transformatio seriei $1 - 2 + 8 - 64 \dots$ in fractionem
continua $\frac{1}{1+2}$

$\frac{1+2}{1+2}$

$\frac{1+8}{1+2}$

$\frac{1+32}{1+2}$

$\frac{1+56}{1+2}$

$\frac{1+128}{1+2}$

$\frac{1+256}{1+2}$

$\frac{1+512}{1+2}$

$\frac{1+1024}{1+2}$

$$1 - 1 + 1 \cdot 2 - 1 \cdot 3 \cdot 2 + 1 \cdot 3 \cdot 2 \cdot 4 = \frac{1+56}{1+2}$$

$\frac{1}{1+1}$

$\frac{1+2}{1+2}$

$\frac{1+8}{1+2}$

$\frac{1+32}{1+2}$

$\frac{1+56}{1+2}$

$\frac{1+128}{1+2}$

$\frac{1+256}{1+2}$

$\frac{1+512}{1+2}$

$\frac{1+1024}{1+2}$

et alia

$\frac{1+12}{1+2}$

$\frac{1+28}{1+2}$

$\frac{1+64}{1+2}$

$\frac{1+144}{1+2}$

$\frac{1+320}{1+2}$

$\frac{1+704}{1+2}$

Scalam simplicem in ^{seriebus} fractionibus variatim recurrentibus
esse functionem simplicem secundi ordinis scalarum
componentium

Comparationes infinitorum in numeris primis & factoribus. ^{26 Mai.} ~~com. eod.~~ 31 M. G.

Scala utriusque termini sunt producta vel adeo functionis
quacunque terminorum quibuscunque seriem 3 Jun. G.

Formula pro summa factorum numeri cuiusvis
compositi $f. gen. \frac{a^{n+1}-1}{a-1}$ 5 Jun. G.

Periodorum minima omnibus infra modulum numeris
pro elementis sumtis fact. gen. $(n+1)a - na^{n-1}$ 5 Jun. G.

Leges distributionis 19 Jun. G.

Factorum summa in infinito = $\frac{\pi^2}{6}$ sum. Num. 20 Jun. G.

Conde multiplicatoribus in formis divisorum
formis. qu.) connexis cogitare 22 Jun. G.

* Nova theoremati aurei demonstratio a priori
toto cōlo diversa eaque hanc parum elegans 27 Jun.

Quaeque partitio numeri a in tria dat formam in
trina \square separabilibus. 3 Jul.

Summa trium quadratorum continue proportionalium non
quam primis esse potest & conspicimus exemplum in quo
et quae ratio in rebus ~~debetur~~

** E Y P H K A. num. = $\Delta + \Delta + \Delta$ 10 Jul. Göt
Determinatio Euleriana formarum in quibus numeri col. po
siti plus unâ vice continentur

Principia componendi scalas serierum variatim recurrentium 16 Jul. Göt.

Methodus Euleriana pro demonstranda relatione inter ~~producta~~
rectangula sub segmentis tectorum sese sequentium in sectione
conicis ad omnes curvas applicatam 21 Jul.

* $a \equiv 1 \pmod{2^{n+1}}$ Semper Soluere in potestate Aug. 3^o Gött.

Rationem theorematum aurei quomodo ~~atque~~ profun-
dus perquirari oporteat respexi et ad hoc accingor
supra ~~primam~~ quadraticas aequationes excedi con-
tus. Inventio formularum qui semper per ~~pr~~ primos:

$\sqrt[n]{1}$ (numerice) diuidi possunt. Aug. 13^o Ibid.

Obiter $(a + bv - 1)^{m + nv - 1}$ evolutam — 14
Kei summa iam iam intellecta. Restat ut singu — 16. G.
la maniantur

$(a^p) \equiv (a) \pmod{p}$, a radia aequationis cuiusvis
quomodocunque irrationalis. 18

Si P, Q functiones algebr. quantitatis indeterminatae fuerint inc.
Datur $TP + uQ = 1$ tum in algebra tum specia 19 G.
ta tum numerica.

Exprimuntur potestates radicum aequationis propositae
aggregatae per coefficientes aequationis lege perquam
simplici. (cum aliis quibusdam geometricis, in Exerce.) 21. G.
Summatio series infinitae $1 + \frac{1}{1 \dots n} + \frac{1}{1 \dots 2n}$ &c. eod.

* Minutis quibusdam exceptis felicitas coepum
attigi scilicet si $p^n \equiv 1 \pmod{\pi}$ fore $x^n - 1$ compositum
e factoribus gradum n non excedentibus ~~condi~~ & proin
aequationem conditionalem fore solubilem. Sept. 2 G.

vide duas theos: aurei de n on the. deduxi.
Numerus fractionum inequalium quarum denominatores certum limitem
non superat ad numerum fractionum omnium quarum num. aut
denom. sint diuersi infra eundem limitem in infinito ut $6: \pi$ Sept. 6.

Si $\int \frac{dx}{\sqrt{(1-x^2)}}$ stat. $\Pi: x = z^n$. $x = \Phi: z$ erit

$$\Phi: z = x - \frac{1}{8} z^4 + \frac{1}{112} z^7 - \frac{1}{1792} z^{10} + \frac{3}{1792 \cdot 52} z^{13} - \frac{3 \cdot 185}{1792 \cdot 52 \cdot 14 \cdot 15 \cdot 16} z^{16}$$

Si $\Phi \int \frac{dx}{\sqrt{(1-x^n)}} = x$ erit:

$$\Phi: z = z - \frac{1}{2 \cdot n + 1} z^n A + \frac{n-1}{4 \cdot 2n+1} z^n B - \frac{n(n-1)}{2 \cdot n+1 \cdot 3n+1} C \dots$$

Methodus facilis inveniendi aeq. in y ex x Sept. 14
aeq. in x si ponatur $x^n + ax^{n-1} + bx^{n-2} \dots = y$

fractiones quarum denominator continet quantitates irrationales (quomodocunque) in alias transmutare

et hoc incommoda liberatas. Sept. 16

Coefficientes aeq. auxiliariae eliminationi inferuentis ex radicibus aeq. datae determinati eod.

Novae methodus qua resolutionem aequationum univarsalem investigare forsitanque invenire licebit Sept. 17.

Scilicet transmutat aeq. in aliam cuius radices

$\alpha \zeta^n + \beta \zeta^{n-1} + \gamma \zeta^{n-2} + \dots$ ubi $\sqrt[n]{1} = \alpha, \zeta, \zeta^2, \dots$ & n gaudens aequationis gradum denotans

Si mentem mihi venit radices aeq. $x^n - 1 = x$ aeq., communi radices habentibus elicere et adeo plerumque tantum aequationes coefficientibus rationalibus gaudentes eas resolvi oporteat Sept. 29 Bruns.

Aequatio tertii gradus est haec: $x^3 + ax - nx + \frac{un - 3n - 1 - mp}{n} = 0$ ubi $3n+1 = p$ & m numerus resid. cubico. \mp finis sui excipitantes. Unde sequitur si $n = 3k$ fore $m+1 = 3l$ si $n = 3k \pm 1$ fore $m = 3l$. Octob. 1 Bruns.

$$\text{huc } z^3 - 3p z + (pp - 8p - 9pm) = 0$$

Not m pentus determinatum $m+1$ semper $\square + 9\square$

Equationis $x^p - 1 = 0$ radices per integro
 multiplicati aggregati agram producere
 non possunt. © Oct. 9. Brunsv.

Quaedam sese obtulerunt de multiplicatoribus
 equationum ut certi termini evadantur, quae
 praedicta pollicentur © Oct. 16. Brunsv.

Lex detecta: quando et demonta erit
 systema ad perfectionem evexerimus Oct. 18 Brunsv.

* Vicinus G, G, A, X Oct. 21. Brunsv.
 formula interpolationis elegans Nov. 25 G.

Incepi Expressionem $1 - \frac{1}{2}\omega + \frac{1}{3}\omega^2 \dots$ in seriem
 transmutare secundum potestates ipsius
 ω procedentem. Nov. 26. G.

Formulae trigonometricae per series expelle
 per Dec.
 Differentiationes generalissimae Dec. 23.

Curvam parabolicam quadrare suscepi.
 cuius puncta quovis dantur Oct. 26.

Demonstrationem genuinam theorematum Lagrangiani
 detexi Dec. 27.

$$\left. \begin{aligned} \int \sqrt{\sin x} \cdot dx &= 2 \int \frac{y y \partial y}{\sqrt{(1-y^2)}} \\ \int \sqrt{\tan x} \partial x &= 2 \int \frac{\partial y}{\sqrt{(1-y^4)}} \\ \int \sqrt{\frac{1}{\sin x}} \partial x &= 2 \int \frac{\partial y}{\sqrt{(1-y^4)}} \end{aligned} \right\} y = \frac{\sin x}{\cos x} \quad 1797 \text{ Jan. 7.}$$

Unifortum
Curvam elasticam a $\int \frac{dx}{\sqrt{1-x^2}}$ pendente

perscrutari copi Jan. 8
Criterii Euleriani rationem sponte detexi Jan. 10.

Integrale complet. $\int \frac{dx}{\sqrt{1-x^n}}$ ad circ. quad. reducy
commentus $\int \frac{dx}{\sqrt{1-x^n}}$ Jan. 12.

Methodus facilis $\int \frac{x^m dx}{1+x^n}$ Determinandi

Supplementum eximium ad polygonorum Descriptio-
nem inveni. sc. si a, b, c, d sint factores primi
numeri primi nichil truncati tunc ad polygoni p laterum
nichil aliud requiri quam ut 1^o arcus indefinitus in a, b, c
d. partes faceret 2. ut polygona a, b, c, d. laterum
describantur.

Theorema de Less. - 172 simili methodo partem d m
stratae ut cetera ... Golt. Jan. 19
Golt. Febr. 4.

Forma $\frac{aa+bb+cc}{bc-ac-ab}$ quod ad diuisores
attinet conuenit cum hac $aa+3bb$ Febr. 5

Amplificatio prop. penult. p. 1. scilicet
 $1-a+a^3-a^6+a^{10} \dots =$ Febr. 16

Unde facile
omnis series
ubi exp. p. p. c.
ordinis crescent
transformatur
 $\frac{1+a}{1+a^2-a}$
 $\frac{1+a^3}{1+a^2-a^2}$
 $\frac{1+a^5}{1+a^3}$
etc.

Formularum integralium formae

$$\int e^{-ta} dt \text{ et } \int \frac{du}{\sqrt{1+u^2}}$$

inter se comparationem institui. M^o 2.

Cur ~~potest~~ ad aequationem perveniat
gradus non $\frac{1}{2}$ dividendo curvam Lemni-
scatam in n partes M^o 4

A potestabilibus integ. $\int \frac{p dx}{\sqrt{(1-x^2)^{q-1}}}$ pendit
$$\sum \left(\frac{mm+6mn+nn}{(mm+nn)^2} \right)^k$$

Lemniscata geometricè in quinque
partes dividitur M^o 21

* Inter multa alia Curvam Lemniscatam
spectantia observavi Numeratorem sinus
decompositi, arcus duplicis esse =

2 Num. Denom. Sinus \times Num. Den. Cos. arcus simpl.
Denominatorem vero =

Num. sin. 4 + Denom. sin. 4 . Jam si Deno-
minatus pro arcu π^1 ponatur θ erit Denom.

sin arcus $k\pi^1$, = θ^{kk} . Jam $\theta = 4,810480$

cuius numeri logarithmus hyperbolicus est =
1,570796 i.e. = $\frac{1}{2}\pi$ quod maxime est memorabile
cuiusque proprietatis demonstratio gravissima
analysis incrementa ~~per~~ pollicetur. M^o 24.

Demonstrationes elegantiores pro nexu

divisorum formae $\square - x$, $+1$ cum -1 , $+2$
inveni Jun 17 Götting

Deductionem secundam theoriae polygonorum
excolui Jul. 17 Götting

Per utraque methodum monstrari potest
puras tantum aequationes solui oportere.

Quod Oct. 1. per ind. invenimus demonstratione
munivimus Jul. 20.

Casum singularem solutionis congruentiae $x^n - 1 \equiv 0$
(scilicet quando ~~quod~~ congr. aux. radices aequales habet) qui
tunc diu ~~no~~ vexavit felicissimo successu vicimus, ex
prop. congruentiarum solutione si modulus est numeri
primi potest. Jul. 21.

$$Ax^{m+n} + ax^{m+n-1} + bx^{m+n-2} \dots + n \quad (A)$$

$$per x^m + \alpha x^{m-1} + \beta x^{m-2} \dots + m \quad (B)$$

dividatur atque omnes coefficientes in (A), a, b, c
se sint numeri integri

~~nullus numerus~~ ~~est~~ ~~possit esse numerus rationalis~~
~~liber factus, si omnes rationales esse~~

coefficientes vero omnes in B, rationales etiam
 hi omnes erunt integri ultimique n ultimis
 in metibus. Jul. 23.

Forsan omnia Producta

ex $(a + bp + cp^2 + dp^3 \dots)$ exp.
 designante p omnes radices prim. ~~exp.~~ $x^n = 1$
 ad formam $(x - p^1 y)(x - p^2 y) \dots$
 reduci potest. Est enim

$$\begin{aligned} (a + bp + cp^2) \times (a + bp^2 + cp) &= (a-b)^2 + (a-b)(c-a) + (c-a)^2 \\ (a + bp + cp^2 + dp^3) \times (a + bp^3 + cp^2 + dp) & \\ &= (a-c)^2 + (b-d)^2 \\ (a + bp + cp^2 + dp^3 + ep^4 + fp^5) \times &= (a+b-d-e)^2 \\ &- (a+b-d-e)(a-c-d-f) \\ &+ (a-c-d-f)^2 \\ &= (a+b-d-e)^2 \\ &+ (a+b-d-e)(b+c-e-f) \\ &+ (b+c-e-f)^2 \end{aligned}$$

vid. Febr. 4.

Falsum est

hinc enim sequeretur bini numeres r forma

Me $(x - p^1 y)$ contentis productum in eadem forma Me
 quod facile requiratur

* Radicum exp. $x^n = 1$ period. pluris eadem summa labore
 non potest inveniri

Jul. 27. Feb.

Plani possibilitatem demonstravi. Jul. 28

Quod Jul. 27 inscrip. errorem involvit: sed eo Felicius ^{Gottin}
num rem catavimus, quoniam probari possumus nullum
periodum esse posse numerum ~~fact~~ ^{rationalem}. Aug. 1

Quomodo periodorum numerum duplicando
signa advenire oporteat

Functionum primarum multitudinem per analysis
simplicissimam erui. Aug. 26

Theorema. si $a + bx + b^2x^2 + bc \dots + mx^m$ est functio secundum
modulum p. prima erit

$$a + bx + a$$

$$d + x + x^p + x^{pp} + bc x^{p^{p-1}}$$

per hanc scdm Idem
hunc modulum divisibilis

Aug. 30

Ec. Ec

Demonstratum, utique ad multa maiora per introit.
Modulorum multiplicium proata Aug. 31.

Aug. 1. generalius ad quosvis modulus adaptati
Sept. 4.

Principia delixi, ad quae congruentiarum secundum
modulos multiplices resolutio ad congruentias secundum
modulum linearem reducitur. Sept. 9

Aequationes habere radices imaginarias
methodo genuina demonstrata. Nov. Oct.

Prin. in dissert. p. 100. Mense Aug. 1799.

Novi Theorematis Pythagorae Dem. Brunsv. Oct. 16

Seriem $x - \frac{1}{2}x^2 + \frac{1}{12}x^3 - \frac{1}{144}x^4 \dots$ summam consideramus
invenimusque eam $= 0$ si.

~~2~~ $2\sqrt{x} + \frac{3}{16} \frac{1}{\sqrt{x}} - \frac{21}{1024} \frac{1}{\sqrt[3]{x}}$ $= (k + \frac{1}{4})^2$

Brunsv. Oct. 16.

Positis $L(x) = \phi^x$; $L(1 + \phi^x) = \phi^x x$; $L(1 + \phi^x) = \phi^{xx}$

ec. erit $\phi^x = \sqrt[2]{\frac{1}{2}} +$ Brunsv. Apr.

Clases dari in quavis ordine: hincque
numerorum in ternis quadrata discrepabilitas
in theorema solidam reduita
Brunsv. Apr. 6

Demonstrationem genuinam compositionis virium
erimus Gotting. Mai.

Theoremata la Grange de transformatione
functionem ad functiones quotcumque varia
bitum extendi. Gotting. Mai.

Series $1 + \frac{1}{4} + \left(\frac{1.1}{2.4}\right)^2 + \left(\frac{1.1.3}{2.4.6}\right)^2 + 8c. = \frac{4}{\pi}$ Jun

et ~~et~~ ~~omnibus~~ ~~generibus~~ involventum
simul cum theoria generali serierum angulis sinus et
cosinus angulorum arithmetice crescentium

Calculus probabilitatis contra La Place defensus
Gott. Jun 17.

Problema eliminationis ita solutum ut nihil
 amplius desiderari possit. Gott. Fin.
 Varia elegantiuscula circa attractionem
 sphaerae

$$1 + \frac{1}{9} \frac{1.3}{4.4} + \frac{1}{81} \frac{1.3.5.7}{4.4.8.8} + \frac{1}{729} \frac{1.3.5.7.9.11}{4.4.8.8.12.12} \dots =$$

$$1,02220\dots = \frac{1,3110\dots}{3,1415\dots} \sqrt{6}$$

~~arc. sin lemn. sin φ = arc. sin lemn. cos φ~~

~~= $\frac{2\varphi}{\pi}$~~

sin. lemnisc. = 0,95500598 sin. 1,198
 = 0,0430495 sin 3, 1,57
 + 0,0018605 sin 5
 - 0,0000803 sin 7

\sin^2 lemn. = 0,4569472 = $\frac{1}{2} \frac{1}{\cos. 2}$

arc. sin. lemn. sin φ = $\frac{\varphi}{\pi} \varphi$

+ $\left(\frac{\varphi}{\pi} - \frac{2}{\pi}\right) \sin 2\varphi$

+ $\left(\frac{11}{2} \frac{\varphi}{\pi} - \frac{12}{\pi}\right) \sin 4\varphi$

sin φ = 0,4775031... sin -
 + 0,03

De Lemniscata, elegantissima omnes expectati-
ones superantia acquisitionibus et quidem
per methodos quae campum propus-
nonum nobis aperuunt. Gott. Jul.

Solutio problematis ballistici Gott. Jul.

Cometarum theoriam perfectiorem reddidi Gott. Jul.
Novus in analysi campus se nobis aperuit,
scilicet investigatio functionum etc.

Formas superiores considerare coepimus
Br. Febr. 17¹⁷⁷⁴

Formulas novas exactas pro parallaxi
eruenit ——— Br. Apr. 8.

Terminum medium arithmetico-geometricum
inter 1 et $\sqrt{2}$ esse = $\frac{\pi}{\sqrt{2}}$ usque
ad figuram vnderiman comprobavimus, quare
demonstrata proorsus novus campus in analysi
certo aperietur
Br. Mai. 30.

In principis Geometriae egregios progressus
fecimus
Br. Sept.

Circa terminos medios arithmetico-geometricos
multa nova deteximus.
Br. Novemb.

Quod Medium arithmetico-geometricum tanquam
quotientem duarum functionum transcendentalium
representabilem esse iam pridem inueneramus:
nunc alteram harum functionum ad quantitates
integrales reducibilem esse deteximus. Helmsl.

Medium Arithmetico-Geometricum ipsum est quanti-
tas integralis ———— Dem ———— Dec. 14
Dec. 23

In theoria formarum binariarum formas reductas
affigere contigit 1800. Febr. 13

Seriem $a \cos t + a' \cos(t + \varphi) + a'' \cos(t + 2\varphi) + \text{etc.}$

ad limitem conuenit, si a, a', a'' etc. constituant pro-
gressionem sine mutatione signi ad 0 continuam-
uentem. Demonstratum Brunov. Apr. 27.

Theoriam quantitatum transcendentalium.

$$\int dx$$
$$\frac{1}{(1-ax)(1-bx)}$$

ad summam vniuersalitatem perduximus

Brunov. Mai. 6.

Incrementum ingens huius theoriae Brunov. Mai. 22
inuenire contigit, per quod simul omnia praecedentia
nec non theoria mediorum arithmetico-geometricorum pulcher-
rime nectuntur infinitesque augetur.

Isdem diebus circa (Mai 16) problema chronologicum
de festo paschalis ita eleganter resoluiamus.

Lehman Curtius auf dem Haußberggraben bei Merseburg
 Rhodanus und dem Kogelmann Nr. 291.

33'

| | | | |
|--------|-------|-------|----|
| 24,45 | 28,8 | 29,43 | 28 |
| 25,4,5 | 28,35 | 30,36 | 23 |

| | |
|----------|---------|
| 26,26,50 | 31,35,5 |
| 26,49,75 | 31,59,5 |

295879
 213
 329

27
 29
 19

| | | | | |
|-------|---------|-------|-------|-----|
| 38 18 | 41 47 | 47,6 | 50 24 | 198 |
| 39 23 | 43 7 | 48 34 | 51 32 | 178 |
| 39 33 | 43 9,5 | 48 50 | 50 58 | 122 |
| 41 32 | 45 20 ♂ | 49 50 | 53 37 | 227 |

| |
|---------|
| 40,2,5 |
| 41,15 |
| 41,21,5 |
| 42,26 |
| 43,26 |

| | |
|----------|---------|
| 26,26,50 | 31,35,5 |
| 26,49,75 | 31,59,5 |

| | | |
|-------|------|-----|
| 51,15 | 53,1 | 106 |
| 48,45 | | |
| 49,57 | | |
| 50,3 | | |

Gras

3835
 125 - 12
 278 257
 91 - 147
 187 454

61 Wolfenb.
 68 Bettenberg
 93 f. Brözen
 107 Aegy.

57
 23° 215 185
 104 81 289 46
 356 57
 16

Numeratorem et denominatorem sonus lemi-
stabilis (vniuersalissime accepti) ad quantitates integra-
les reducere contigit; simul omnium functionum
lemnificarum quae excogitari possunt, euolutiones
in series infinitas per principia geminis huiusmodi
inuentum pulcherrimum sate nullique praecedentium inferius

Praeterea iisdem diebus principia deteximus secundum
quae series arithmetico-geometricae interpolari debent; ita
ut omnes termini in progressionem datam ad terminum indicem quem-
cumque rationalem pertinetes per aequationes algebraicas
exhibere iam in potestate sit. Maior. Jun 2. 3.

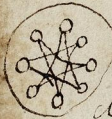
Inter duos numeros datos ^{int} duo semper dantur
infinita multi termini medii tam arithmetico geometrici huius
harmonico geometrici, quorum nexum mutuum ex asse
perspicendi felicitas adhuc est facta. Junio 3. Probar.

Theoriam nostram iam ad transcendentes
ellipticas immediate applicauimus Junio 5

Rectificatio Ellipseos tabes Newtoni Jun 10
resis absoluta.

Calculum Numerico-Exponentialem omnino co-
nam inuenimus Jun 12

Problema e calculo probabilitatis circa fractiones continuas obui
frustra tentatum solimus Oct. 25



Nov. 30, Felix fuit dies quo multitudinem
classium formarum binarum per triplicem methodum
assignare largitam est nobis puta 1) per prod. infin.
2) per aggregationem infinitam 3) per aggregationem fini-
tam cotangentium seu logarithm. sinuum. Bonn.

Dec. 3. Methodum quartam ^{ex} omnibus simplicissimam
deteximus pro dett. negativis ex sola multitudine nume-
rorum ξ, ξ' etc. petita si $Ax + \xi, Ax + \xi'$ etc. sunt
formae lineares diviformes in $\Pi + D$. Ibid.

* Impossibile esse ut sectio circuli ad aequationes inferi-
ores quam theoria nostra suggerit reducatui demon-
stratum Brunsv. Apr. 6

Isidem diebus Pascha Judaeorum per methodum novam
decominare docuimus (Apr. 1)

* Methodus quinta theorema fundamentale demonstrandi
se obtinet adimento theorematum elegantissimi theoriae sectionis circuli
puta

$$\sum \left\{ \begin{array}{l} \sin \\ \cos \end{array} \right\} \frac{nn}{a} P = \begin{array}{c|c|c|c} +\sqrt{a} & 0 & 0 & +\sqrt{a} \\ +\sqrt{a} & +\sqrt{a} & 0 & 0 \end{array} \left| \begin{array}{l} \text{substituendo pro } n \\ \text{omnes numeros} \\ a \text{ o usque ad } a-1 \end{array} \right.$$

puta $a \equiv 0 \quad 1 \quad 2 \quad 3 \pmod{4}$

Brunsv. Mai. medio

Methodus nova simplicissima expeditissi-
ma motu elementa orbitarum corporum colle-
gerim investigandi. Brunsv. Sept. m

Theoriam motus Lunae aggressi sumus Aug.

Formulas per multas novas in Astronomia theoretica
utilissimas eruiimus 1801 Mense Octobri

Annis insequentibus 1802. 1803. 1804 occupationes astronomicae
maximam otii partem abstulerunt, calculi imprimis, circa
planetarum novorum theoriam instituti. Unde evenit, quod
hisce annis catalogus huius neglectus est. Hae diebus Quare Dies
itaque, quibus aliquid ad matheseos incrementa conferre datum
est memoriae exciderunt. —

Demonstratio theorematis venustissimi supra 1801 Maii commemorati
quam per Haaras et ultra omni contentione quaesiveramus tandem
perfectimus Comment. vol. I 1805. Aug. 30.

Theoriam interpolationis ulterius excolimus 1805 Novbr 10

Methodum, ex duobus locis heliocentricis corporis circa solem motus
eiusdem orbitam elementa determinandi novam perfectissimam
deteximus. 1806. Januarius.

Methodum e tribus planetae locis geocentricis eius orbitam defecimus
nandi ad summum perfectionis gradum eveximus 1806. Maii.

Methodus nova ellipsin et hyperbolam ad parabolam reducendis
1806 April.

Eodem circiter tempore resolutionem functionis $\frac{x^2-1}{x-1}$ in factores
quatuor absolvimus.

Methodus nova e quatuor planetae locis geocentricis, quorum
duo externi sunt incompleti eius orbitam determinandi 1807 Jan 21

Theoria Residuorum cubicorum et biquadraticorum incepta 1807 febr. 15
ulterius excolta et completa reddita febr. 17. Demonstratione adhaeret:

Demonstratio huius theoriae per methodum decantissimam inventa
ita ut penitus perfecta sit nihilque amplius desideretur. 1807 febr. 22
Huiusmodi residua et non residua quadratica egregie illustratur.

Theoremata, quae theoriae praecedenti incrementum maximi
 pretii adiungunt, demonstratione eleganti munita { scilicet pro
 quibusdam radicibus primitivis habere oporteat ipsum b positivum
 pro quibusque regnum, $aa + 27bb = 4p$; $aa + 4bb = p$ } Febr. 24.

* Demonstratio omnino nova theorematis fundamentalis
principiis omnino elementaribus innixam deteximus
 Maii 6.

Theoria divisionis ⁱⁿ periodos tres (art. 338) ad
 principia longe simpliciora reducta 1808 May 10

Aequationem $X-1=0$, quae continet omnes radices
 primitivas aequationis $x^n-1=0$, in factores
 cum coefficientibus rationalibus dissecti non posse,
 demonstrat. pro valoribus compositis ipsius 1808 Jun. 12

* Theoriam formarum cubicarum, solutionem aequ.
 $x^3 + xy^3 + nz^3 - 3nxyz = 1$ aggressus sum Dec. 23

* Theorema de residuis cubicis 3 per methodum novam elegantem
 demonstratum per $x+x+1$ per considerant. valorum $\frac{x+1}{x}$ ubi
 termini semper habent $a, a\epsilon, a\epsilon\epsilon$ exceptis duobus quidant $\epsilon, \epsilon\epsilon$
 hi vero sunt $\frac{1}{\epsilon-1} = \frac{2\epsilon-1}{3}$ adeoque productum $\equiv \frac{1}{3}$ 1809 Jan 6
 $\frac{1}{\epsilon\epsilon-1} = \frac{\epsilon-1}{3}$

Series ad Media arithmetico geometrica pertinentia fusius excoluit
 1809 Jun 20

* Binomique sectionem pro mediis arithm: Geom. absol. 1809 Jun 29

42

Catalogum praecedentem per fata iniqua iterum interruptum
 initio anni 1812 resumimus. In mense Nov. 1811 contigerat
 demonstrationem theorematis fundamentalis in doctrina aequa-
 tionum pure analyticam completam reddere; sed quum nihil
 chartis servatum fuerit, pars quaedam ^{essentiales} memoriae penitus
 exiderat. Hanc per satis longum temporis intervallum
 frustra quaesitam tandem feliciter redinvenimus 1812 febr. 29

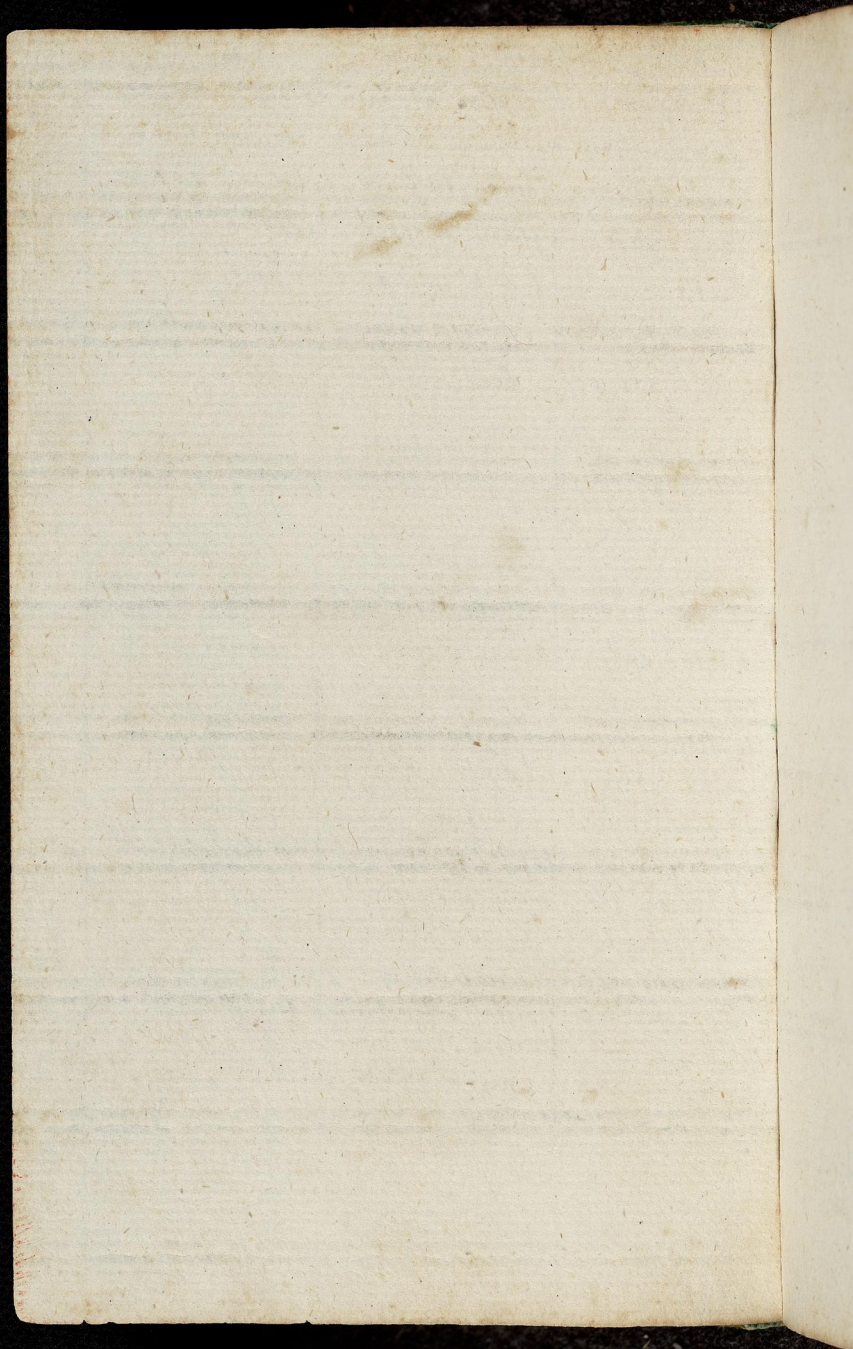
Theoriam Attractionis Sphaeroidis Elliptici in puncta
extra solidum sita prorsus novam invenimus
 Seeber. 1812. Sept. 26

Etam partes reliquas eiusdem theoriae per methodum
novam vires simplicitatis absolvi mus 1812 Oct. 15 Gott.

Fundamentum theoriae residuorum biquadraticorum et
~~per~~ generalis, per septem propemodum annos summa con-
 tentione sed semper frustra quaesitum tandem feliciter dete-
 ximus eodem die quo filius nobis natus est. 1813 Oct. 23 Gott.

Subtilissimum hoc est omnium eorum quae unquam
 perfecimus. Vix itaque operae pretium est, his in terminis
 mentionem quarundam simplificationum ad calculum
 orbitalium parabolicarum pertinentium.

Observatio per inductionem facta gravissima theoriam residuorum biquadra-
 ticorum cum functionibus lemniscaticis elegantissime redens. Puta si $a + bi$ est
 numerus primus $a - 1 + bi$ per $2 + i$ divisibilis, multitudine omnium solutionum
 congruentiae $1 = xx + yy + zzzz$ (quod $a + bi$), inclusis $x = \infty, y = \pm i,$
 $z = \pm i, y = \infty$ fit $= (a-1)^2 + 6b$ 1814 Jul 9.



20

Que le
Passé
Toute
N'est
Le p
Quand
N'est
Pour

Hela
Un p
Le ch
L'ann
Si je
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Mon
Est

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L'ann
Tout
Des
Ouvr
Les c
Si t
de m

Alors
Kremlin
Appare
Bourne
Kingsley
Suppl
Schmidt

2 Jan 70 C
 10 J
 10 G.
 30 $\frac{1}{2}$
15. 14
 B 135.14
 P. 1. 21. 3
 137. 11. 3

1. 0. -b -cm
 0. m. a. bm

 m. 0. -dm
 a. bm. cm
 am. bm. cm

Antgabe

Jan. 5. Brief - - 8.
 Assamblee - 1. 19. - R = bm
 Brief 4. - - S = bm
 Konferenz - 1. 5. -
 Brief - - 1. 4

R = bm
 S = bm
 Ro

$\pm 2m \quad \pm m \quad \frac{1}{2}$

1. 22. 485
 4.

1. 0. $\frac{1}{2}(1 \mp b)$ -cm
 0 $\pm 2m \quad \pm \frac{1}{2}a \quad (b \pm 1)m$

| |
|------------|
| 1. 0. -557 |
| 1. 23 -28 |
| -28. 5. 19 |
| 19. 14 -19 |

$\pm 2m \quad \pm m \quad \pm \frac{1}{2}m(1-d)$
 ~~$\pm \frac{1}{2}mb$~~
 $\pm \frac{1}{2}a \quad mb \quad \pm 2cm$

$\begin{matrix} 1-b & b+1 \\ 1+b & b-1 \end{matrix} S \quad \begin{matrix} \mp m & + mb \\ \pm m & + mb \end{matrix}$
 $\pm \frac{1}{2}(1-bb)m R$
 $\pm \frac{1}{2}acm$
 $am \quad bm \quad cm$
 $(b \pm 1)m$
 $-(b \mp 1)m$

$P_{90^\circ} = \sqrt[4]{2}$

$Q_{90^\circ} = \sqrt[4]{2}$

$p_{90^\circ} = 0$

$q_{90^\circ} = \sqrt{2}$

$p_{2x} = \frac{1}{2}p^4 + p^2q^2 - \frac{1}{2}q^4 = -P^4 - 2P^2Q^2 + Q^4$

$q_{2x} = -\frac{1}{2}p^4 + p^2q^2 + \frac{1}{2}q^4 = -P^4 + 2P^2Q^2 + Q^4$

$P_{2x} = 2P^2Q^2$

$Q_{2x} = P^4 + Q^4 = \frac{1}{2}(p^4 + q^4)$

$P^4 = \frac{1}{2}Q_{2x} - \frac{1}{4}p_{2x} - \frac{1}{4}q_{2x}$

$Q^4 = \frac{1}{2}Q_{2x} + \frac{1}{4}p_{2x} + \frac{1}{4}q_{2x}$

$p^4 = Q_{2x} + \frac{1}{2}p_{2x} - \frac{1}{2}q_{2x}$

$q^4 = Q_{2x} - \frac{1}{2}p_{2x} + \frac{1}{2}q_{2x}$

$pp = QQ - PP$

$qq = PP + QQ$

$P = (4)\sin 1 + (3)\sin 3 + (5)\sin 5 + \dots$

$(1) - (3) + (5) - (7) \dots = \sqrt[4]{\frac{1}{2}} = 0.84089634$

$P_{45^\circ} = \sqrt[4]{\left(\frac{1}{2}\sqrt[4]{2} - \frac{1}{4}\sqrt[4]{2}\right)}$

v. 41887

$\sin 45^\circ \cdot P_{45^\circ} = \frac{1}{2}\sqrt[4]{\left(\sqrt[4]{8} - \sqrt[4]{2}\right)}$

$= \frac{1}{2}\sqrt[4]{\left(\sqrt[4]{2} \cdot (\sqrt[4]{2} - 1)\right)}$

~~$= \frac{1}{4}\sqrt[4]{2} \cdot \sqrt[4]{2}$~~

$\overset{P}{(1)} - \overset{P}{(3)} + \overset{P}{(5)} \dots = 0.8393264$

$\overset{Q}{(1)} - \overset{Q}{(3)} + \overset{Q}{(5)} \dots =$

$$P, Q \ 90^\circ = \sqrt{\frac{1}{2}} = 0,8408963$$

$$r \ 90^\circ = \sqrt[4]{2} = 0,1892071$$

In the
of the
in the

1773
June
p. 181
1773
July
August

In the
1773
1773

1 =
B =
C =
D =
E =

* for the
list

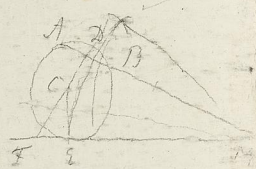
July
August

See
for

Die Höhen der Berge in England sind durch die Beobachtung der Höhen der Berge in England ...
 Die Höhen der Berge in England sind durch die Beobachtung der Höhen der Berge in England ...
 Die Höhen der Berge in England sind durch die Beobachtung der Höhen der Berge in England ...

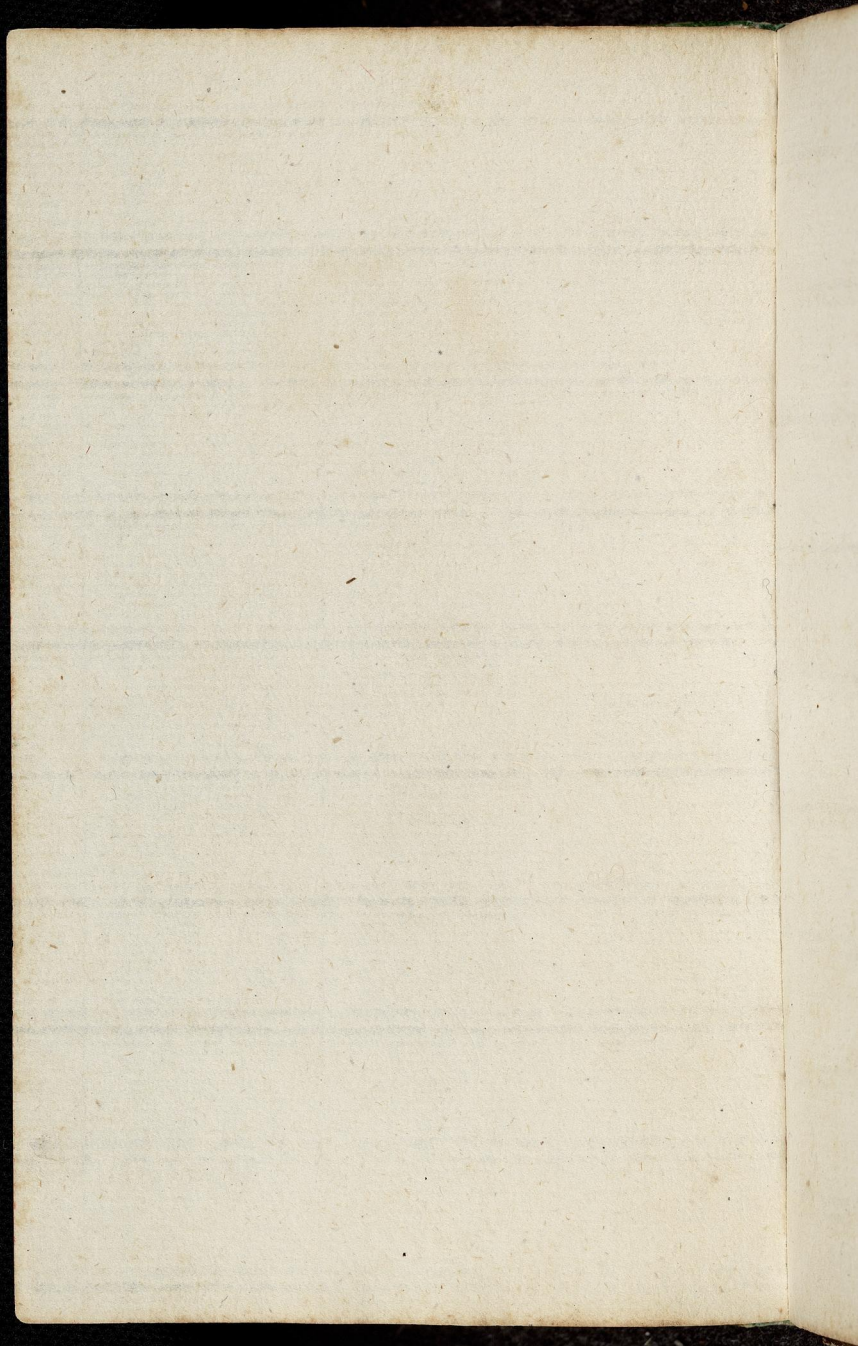
Die Höhen der Berge in England sind durch die Beobachtung der Höhen der Berge in England ...
 Die Höhen der Berge in England sind durch die Beobachtung der Höhen der Berge in England ...
 Die Höhen der Berge in England sind durch die Beobachtung der Höhen der Berge in England ...

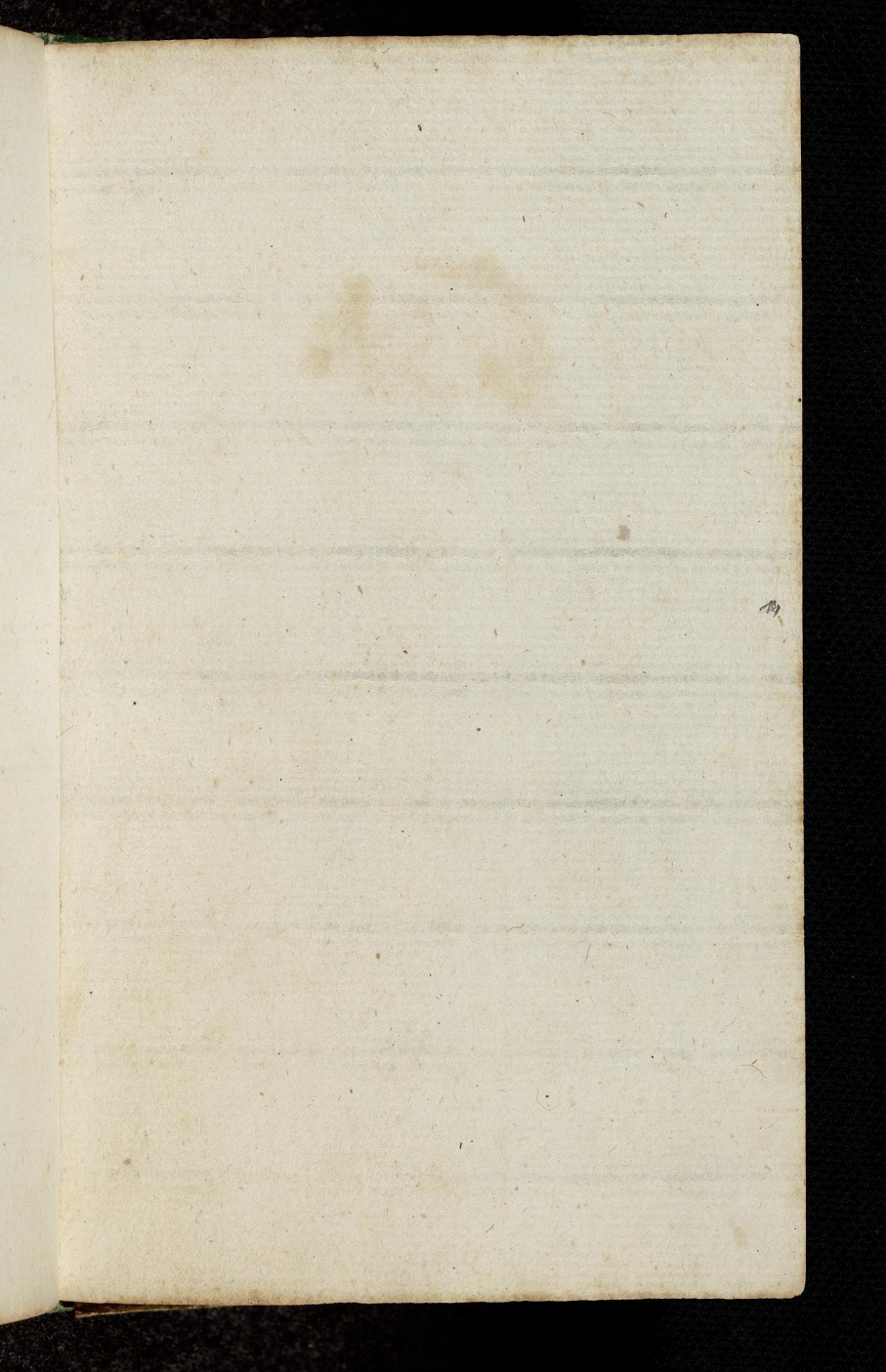
$$\begin{aligned}
 & a^6 - ab^5 + a^5b - ab^4 + a^4b^2 - ab^3 + a^3b^4 - ab^5 + b^6 \\
 & a^6 + 6ab^5 + 15a^2 \\
 & 7a^5b + 14a^4b^2 + 21a^3b^3 + 14a^2b^4 + 7ab^5 \\
 & 7a^6b + 21a^5b^2 + \dots + 21a^2b^5 + 7ab^6 \\
 & = (a+b)^7 - a^7 - b^7 \\
 & a^7 + 7a^6b + 21a^5b^2 + 35a^4b^3 + 21a^3b^4 + 7a^2b^5 + b^7 \\
 & a^7 + a^6b + a^5b^2 + a^4b^3 + a^3b^4 + a^2b^5 + b^7 \\
 & - a^7 - a^6b - a^5b^2 - a^4b^3 - a^3b^4 - a^2b^5 - b^7 \\
 & a^6 \quad a^5 \quad a^4 \quad a^3 \quad a^2 \quad a \quad 1
 \end{aligned}$$



Problems quod proponit Newton Arithm Univ. no 37 circuli
 Describere qui p[ro]p[ri]etate datam lineam tangat ac per dua data puncta
 ead multa elegantius solvitur utriusq[ue] producendo AB donec fecerit
 rectam FE in M ac pro determinatis hinc AD & MD
 pro determinanda ME = \sqrt{AD^2 + MD^2}. Centrum est ubi
 Perpendicularum super l Perpendiculi e D in AM occurrunt







Markel
Fels

Duppe
Bolge
Wurm

v. M
v. Z
Sch

Soff
Offe

dae
Ried
Sch

Wing
Thie

Agge
Papp

7
O

Corne

o

Maskelyne Herschel Butler Lagarde
 Fuhs Schubert
 Duprat Gernain Lagrange Laplace Delisle Bouvard
 Bolyai
 Wurm Lieb Christmann v. Scaun

- v. Murr
 v. Zsch Ludman
 Schütz Kudeus
 Seyffer Schröder Schönknecht Joe Heyne
 Olbers Harding Schroter Bessel

Bode Leoz
 Roding Benzenberg ^{Poussin Pettes} ^{Reposit de}
 Schulze Pfaff Bruns ^{Be my own names Theoria}
 Klügel Nollweyde
 Thiele Kinnel Brückner

Bugge

Piazzi

7+ 4+

Bruno

13



- 1) pins auf
- 2) Dr. Schumacher
- 3) Edelhoff
- 4) Mayer
- 5) v. Zuccon
- 6) Elert
- 7) Engel
- 8) Richter
- 9) Gumbel
- 10) König
- 11) Albrecht
- 12) Jünger
- 13) n. Lorenz

20

[Faint handwritten text, possibly a signature or title]

Exemplare meiner DEMONSTR. NOV. abgegeben
 bis zum letzten Nov. 1799 — an

nach

Academie zu Berlin
 Academie zu Petersburg

Bartels

Butler

Chauvelot Lady Drake

Eschenburg

Euler

Facultät zu Helmstedt

Fuß Fischer

Helwig Lalgrange

Herszog

Hindenburg Hobert, Ideler

Kästner

Klingel

Leiste

Mahner

Mollweyde Nationalinstitut

Olbers

Pfaff *

Rohde

Schubert

Schubse Seyffer

Societät zu Göttingen

Societät zu London

v. Stamford

Tamfen

v. Tempelhoff

Volkmar

Wood

v. Zach

v. Zimmermann

Berlin

Braunschweig

Bremen

Cambridge

Gotha

Göttingen

Halle

Hamburg

Helmstedt

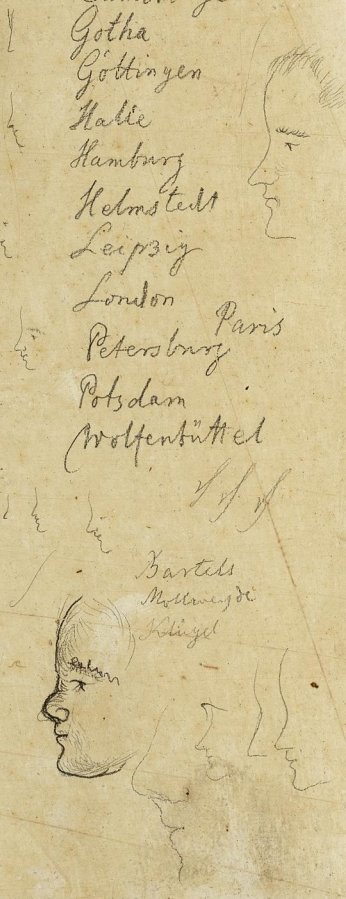
Leipzig

London

Petersburg Paris

Potsdam

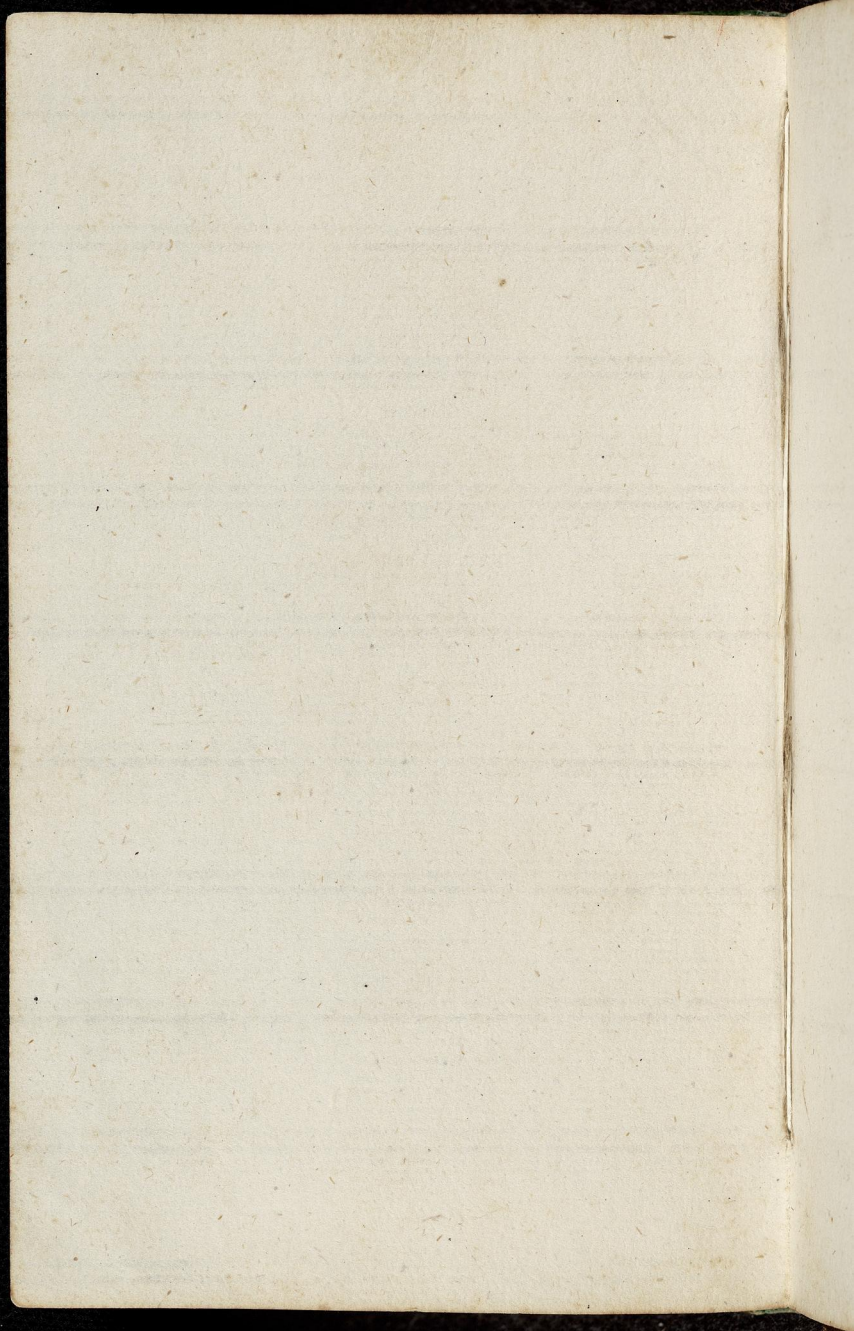
Wolfenbüttel



Bartels

Mollweyde

Klingel



Erweise die That, dass $b \frac{a-1}{a}$, wenn a eine Primzahl ist, alle
 mal eine ganze Zahl ist. Bestimme statt $b = t+1$ ⁶⁶ $\frac{66}{7}$ laßt
 sich zeigen, daß die Zahl für $b+1$ richtig ist, folglich man seine
 Richtigkeit hier & vorwärts setzt.

Einige Primzahlen können zerfallen in Zahlen dieser
 Form $(aa+1)$ sein z. B. 2, 5, 13, 17, 29, 37, 41 etc.
 andere nicht z. B. 3, 7, 11, 19, 23, 31, 43 etc. Daß jene
 zwei Grundlegenden Gesetz ist zu bestimmen

~~Eine n, n' genau auf einander folgende Primzahlen
 die zwischen zwei n mit n' a dieser Form
 fallen. $n' - k + n$ oder $n' - k + n' - n$ ²²
 k jede beliebige ganze Zahl, betrachten kann.~~

Es würde hier möglich sein, eine dieser Primzahlen b ,
 hauptsächlich eine hochgehoben, vorwärts einzuwerfen
 zu sehen, durch Induktion, nicht dieser Art.

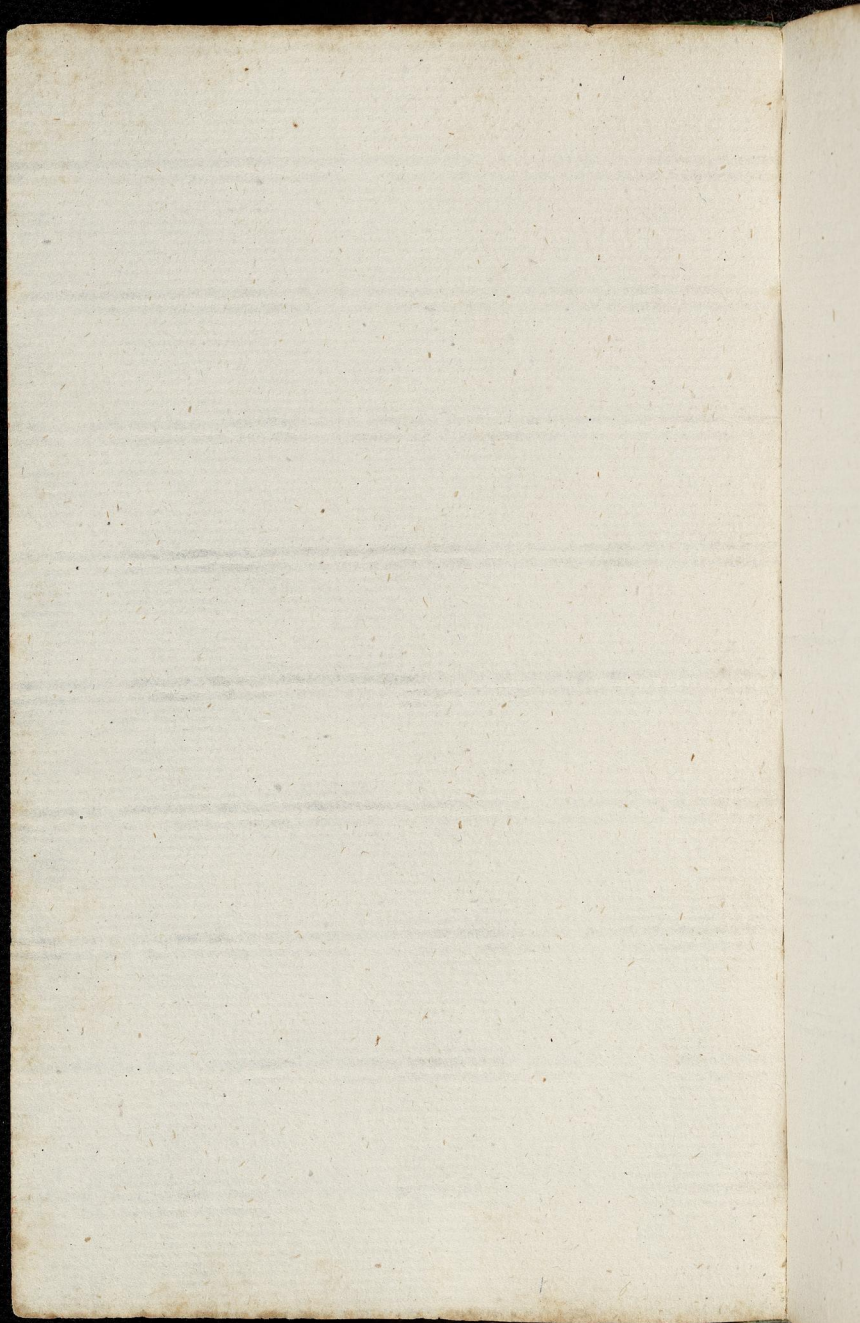
2; 5; 17; 17; 29; 37; 41; 53; 61; 73; 83; 101; 109; 109

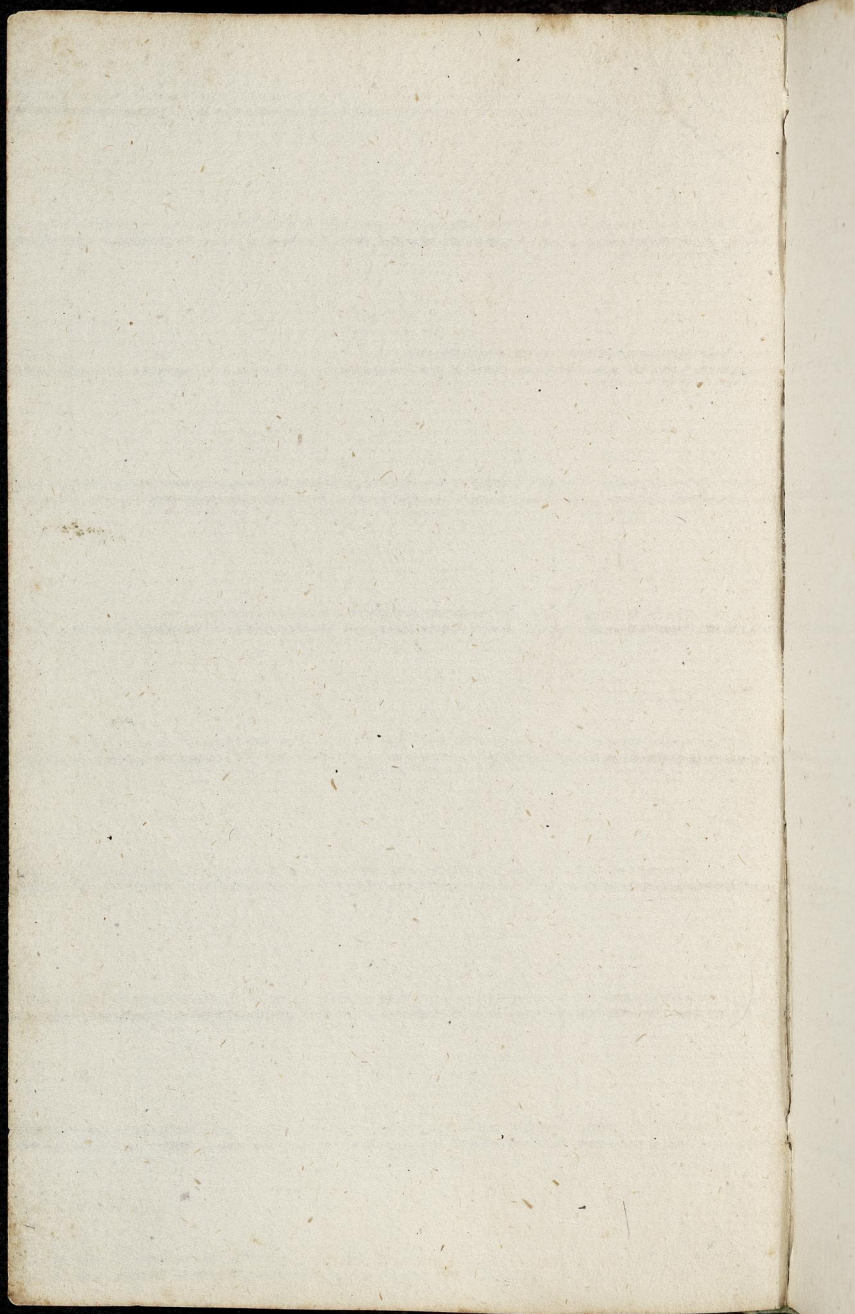
Die Zahlen $2, 5, 17, 17, 29, 37, 41$ sind für $a=1$
 aus $aa+4$ ²² 2 anderen nicht.

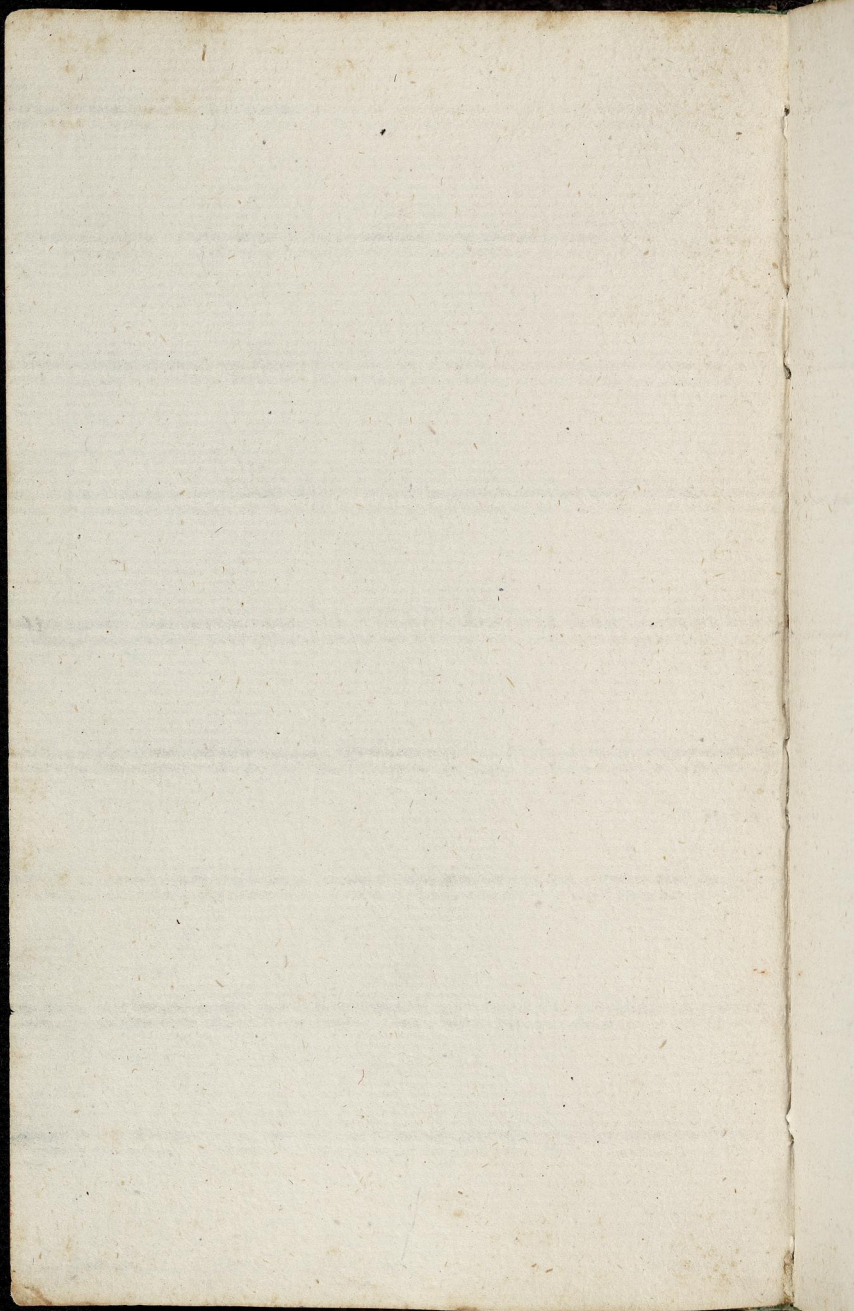
Jedoch unter diesen Zahlen alle Primzahlen a der
 Form $4n+1$. Zu den anderen die nur die Form
 $4n-1$.

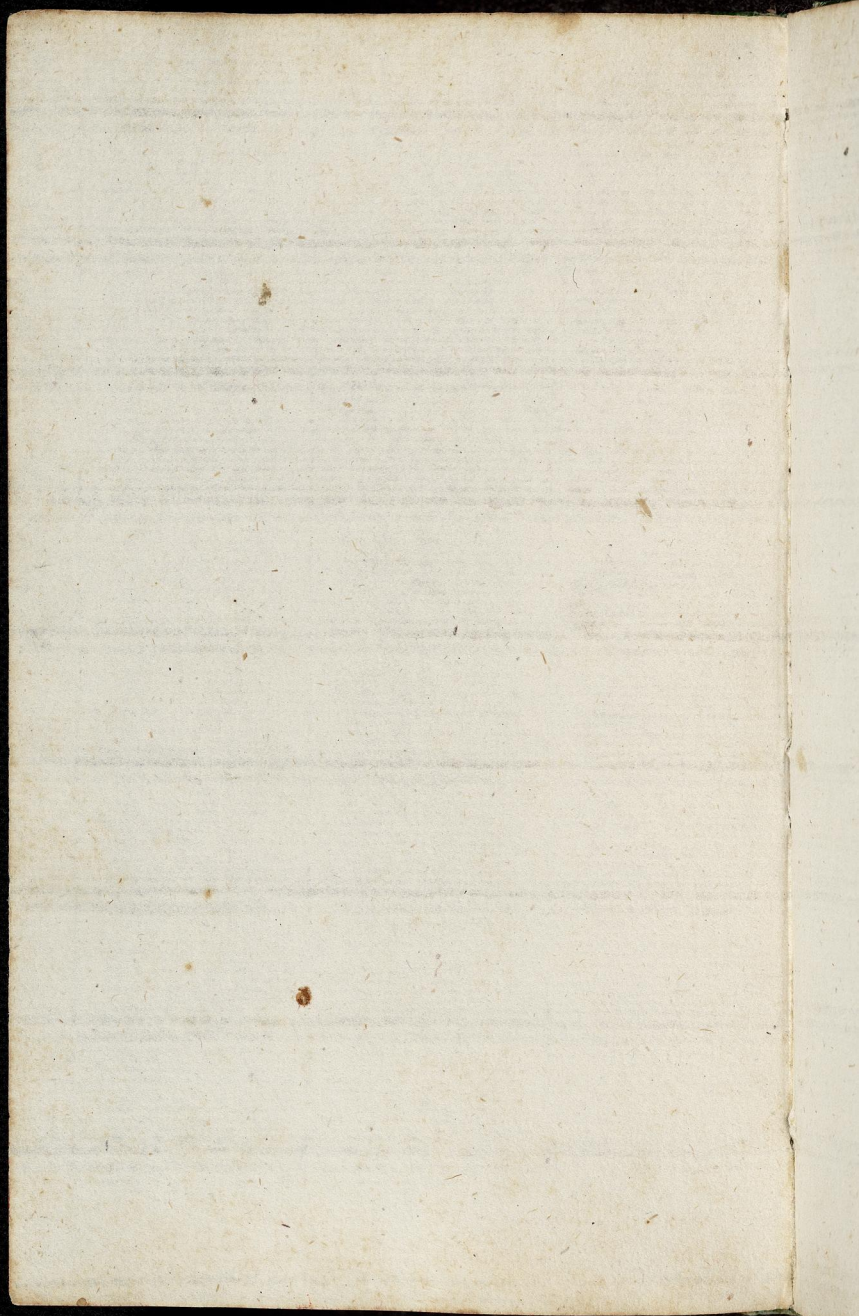
Auf alle Zahlen von der Form $aa+1$ fallen unter
 die Zahl k eine Zahlen von der Form $4n+1$ zu
 zerfallen.

Handwritten notes in the left margin, including the words "at fin", "Lullay", and "Dell".









Quantitates imaginariae :

Quantitas criterium generale, secundum quod functiones plurium variarum complexae ab in-complexis diagnosis possint.

$$\begin{array}{r|l} 110 & x \\ \hline 20 & \frac{1}{6} \\ 20 & \frac{2}{2} \\ 8 & \frac{2}{3} \end{array} \quad \begin{array}{l} 56 \frac{2}{3} \\ 52 \frac{1}{3} \\ 12 - \\ \hline \end{array}$$

$$\begin{array}{r|l} 18 & 2 \\ 54 & 1 \\ \hline & \frac{1}{2} \end{array} \quad \begin{array}{l} 72 \ 2 \\ \hline \end{array}$$

97. 4. 15.

640 4
651 4
6928 4
7291 5
7303 3
7372 7
7386 2
7344 2
7386 2
7382 2
7387 3
7393 2
7402 1
7445 2
7447 1
7448 2
7451 1
7452 1
7453 1
7471 1
7483 1
7487 1
7518 1
8183 99
8759 6
17224
17305
17315
17405

11.1.2.3 4. 5. 6

1824

97. 4. 19. C. d. A. 7291.

5343 1-8-0
5397 2*

4 2

8
9.9
12.12.

17.18
24 26.43

6740 4 7 6 10.12.

6911. 4 11 2 13.30

6928. 4 15 5 4.17

7291. 5 20 4 4.15

7303. 3 23 2 27 1343

7312. 7. 30 6. 4 5.6

7336. 2. 32. 0 30

7344 2. 34. depend on ... # 6.7 5.3

7356. 2. 36. 6 19 43.7

7382. 2. 38. 4 7. 15 51.9

7387. 3. 41. the first ... 2 + 20 75.11

7393. 2. 43. # 1 7. 26 84.15

7402. 2. 44. # 3 8. 4 96.21

7445. 2. 46. n.m. # 9.16 1.2

7447. 1. 47. L 6 18 3.3

7448 2 49 + 0 19 46.5

7451 1 50 G + 3 22 55.6

7452 1 51 B # 4 23 61.8

7453 1 52. # 5 24 66.11

7471 1 53. L 2 10.12 92.13

7483 1 54 # 0 24 104.15

7487 1 55 # 4 28 112.17

7518 1 56 L 0 11.28

0
1 ..
2
3 ..
4
5 ..
6 ...

29
E

8113; 99VII.16. D.

8759 D

1824

47

365

17224 W. z. g. L.

17305. 1

17315. 1.

17405 semb

17155

17197

