

Scottish Squares and Fixed Points in the Complex Plane
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1. Presentation of the Problem

Still seeking a justification for Goldbach's conjecture, we reflect on how to visualize the Goldbach components in the complex plane.

We use a square with side length n . The odd prime numbers p_k between 3 and $n - 3$ inclusive are positioned on the x -axis (note the set of these prime numbers E_n). We denote by F_n the “complement of E_n ”, the set of numbers $n - x$ with $x \in E_n$. E_n and F_n have the same cardinality.

We draw a network of lines:

- the vertical lines have equations $x = p_k$ for each of the prime numbers in E_n ;
- the horizontal lines have equations $y = n - p_k$ for these numbers.

The set R of intersection points of the network of lines $E_n \times F_n$ has cardinality $\#(E_n \times F_n) = (\# E_n)^2$. We have

$$R = \{z \in \mathbb{C} \mid \Re z \in E_n, \Im z \in F_n\}.$$

Let us give as examples the sets E_{24} , F_{24} , E_{36} and F_{36} . We have

$$\begin{array}{l|l|l} E_{24} = [3, 5, 7, 11, 13, 17, 19] & \# E_{24} = 7 \text{ (\# } E_{24} \text{ est impair).} & F_{24} = [21, 19, 17, 13, 11, 7, 5] \\ E_{36} = [3, 5, 7, 11, 13, 17, 19, 23, 29, 31] & \# E_{36} = 10 \text{ (\# } E_{36} \text{ est pair).} & F_{36} = [33, 31, 29, 25, 23, 19, 17, 13, 7, 5] \end{array}$$

2. Goldbach components of n and symmetry S

As a reminder, we recall that in the complex plane, the image of the point with affix z by a symmetry with respect to the line determined by the points a and b is $z' = \alpha \bar{z} + \beta$ with $\alpha = \frac{a - b}{\bar{a} - \bar{b}}$ and

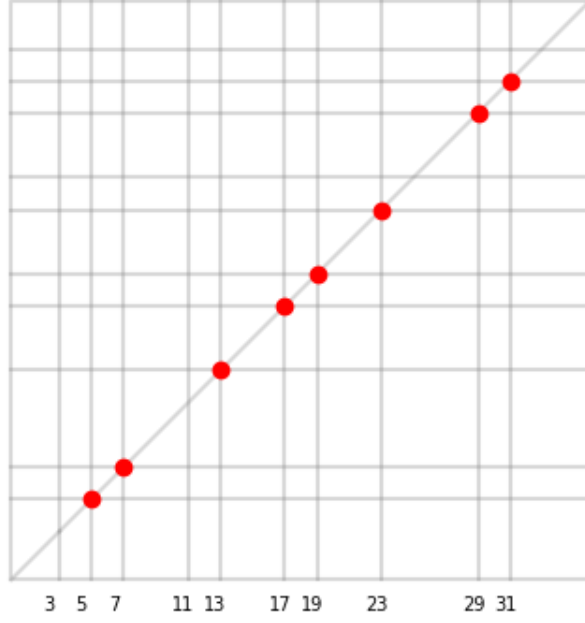
$$\beta = \frac{b\bar{a} - a\bar{b}}{\bar{a} - \bar{b}}.$$

The NW-SE diagonal, denoted D in the following, with equation $y = n - x$, is determined by the points $a = ni$ and $b = n$. The orthogonal symmetry with respect to D , denoted S , maps a point (x, y) , with complex affix $x + iy$, onto the point $(n - y, n - x)$, with complex affix $(n - y) + i(n - x)$. Every point of the lattice has its image by S which belongs to the lattice. For the symmetry S , we have

$$\alpha = \frac{-n + ni}{-n - ni}, \quad \beta = \frac{-2n^2i}{-ni - n}.$$

Below, let's draw the lines associated with $n = 36$; the intersections of these lines belong to what is called in the following the network of points. The intersection points have as affixes complexes of the form $x + iy$ with $x \in E_n$ and $y \in F_n$.

n=36



The file of graphs associated with the even numbers n between 6 and 102 can be viewed at:

<https://denisevellachemla.eu/touscarrespourdg.pdf>.

On the graphs, the Goldbach decomposing lines of n are colored red on the diagonal D' with equation $x = y$. To simplify the visualizations, the vertical and horizontal lines associated with the divisors of n have not been shown because they can never provide Goldbach decomposing lines of n .

Note: Even numbers that are doubles of a prime number (such as $38 = 2 \times 19$, or $94 = 2 \times 47$) trivially satisfy Goldbach's conjecture; they are the sum of two identical prime numbers. Their graph shows a red dot at the intersection of the diagonals of the square, at the center of the square.

- If $\#(E_n \times F_n) = (\# E_n)^2$ is odd, let us apply the symmetry S to R , with respect to the line D . The fixed points of this symmetry are on the line D . Since S is an involution (see the elements on the notion of involution in the appendix), it has a fixed point that is on the line D with respect to which the symmetry S is realized. Indeed, if $\# E_n$ is odd, its square, which is the number of points in the lattice R is also odd (the square of an odd number is an odd number because $(2k+1)^2 = 4k^2 + 4k + 1 = (4k^2 + 4k) + 1 = 2k' + 1$) and an involution on a set of odd cardinality admits a fixed point. The fixed point in question is of the form $p + px$ and n admits as Goldbach decomposition $p + (n - p)$ with p and $n - p$ both prime ;
- If the number of prime numbers between 3 and $n - 3$ (both inclusive) is even, let us remove an element p_k from E_n and consider the domain

$$R' = \{z \in \mathbb{C} \mid \Re z \in (E_n \setminus \{p_k\}) \text{ et } \Im z \in (F_n \setminus \{n - p_k\})\}.$$

which is of odd cardinality; we have reduced ourselves to case 1, the symmetry S , which is an involution on a set of odd cardinality, admits a fixed point. The fixed point in question is

of the form $p + px$ and n has the Goldbach decomposition $p + (n - p)$ with p and $n - p$ both prime.)

Appendix: Reminders on the concept of involution

In mathematics, an involution is a function f such that $f \circ f = \text{Id}$, that is, $f(f(x)) = x$ for all x in the domain. In other words, an involution is a function that is its own inverse.

Case of an involution on a finite set: For an involution acting on a finite set of n elements, a point x is a fixed point of f if $f(x) = x$. The non-fixed elements of an involution can be grouped into pairs (x, y) such that $f(x) = y$ and $f(y) = x$.

If the set has an odd number of elements, then it is impossible to group all the elements into pairs (because an odd number cannot be divided into an integer number of pairs). Therefore, there must be at least one element that is not in a pair, i.e., a fixed point.

An involution on a finite set with an odd number of points must have at least one fixed point. This follows directly from the structure of involutions and the parity of the number of elements. In an odd-sized set, it is impossible to pair all the elements, so at least one element must remain fixed.

Author Biographies

Denise Vella-Chemla is a retired teacher with a strong background in research and computer science. She began a PhD at LIRMM (France) on natural language processing for medical data. She later worked for eight years as a research engineer at Syseca (Thomson group), contributing to several R&D projects, including one on air-traffic flow management for the Orly airport co-authoring a research paper with Daniel Diaz. She then joined the French Ministry of Education as a teacher and later as a digital education advisor. Now retired, she continues her personal research in mathematics.

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