good morning good afternoon everybody wherever you are today we have a special lunch of our seminar first of all because of the 60th anniversary of birthday of P who is U the founder of our seminar and uh spiritual moments of many activ around the seminar uh maybe I would ask Professor D brosi from cisa to give her special address well actually I'm not well
prepared but just want to tell you that it's not only that it's anniversary of it but also 25 years of this seminar notive geometry though it was not of course at the beginning on on Zoom but just in warso at impan do Blackboard seminar so there are two anniversaries and the 60th birthday of P of course I can just tell that we have a special vegan wine at the end of the
of the seminar of of Alan and of course AI won compl okay okay thank you thank you uh second I'm thrilled to announce um alen who is a founding father of all our subject noncommutative geometry uh today he will speak about classial Theory and Z spectral triples so it's a double Delight to introduce you so please Al the floor is yours take
it away okay thank you well the the first thing uh let me try to turn the pages the first thing is happy birthday thank and I mean it's a a great occasion for me to first of all congratulate you for your beautiful mathematical achievements so this is really what I want to say but also I want to thank you and I want to thank you for these 25
years of seminar on noncommutative Geometry which is really something extremely important for our subject and um I mean this you know this means a lot in the sense that probably the quality of human beings which I appreciate the most is persistence so I am so grateful to you for that and of course I am also grateful to the other organizers of this because of the way it is going and well you know in a minor way for inviting me but I mean okay this is a this is a minor fact okay so so I mean I will I will explain the following things I mean one one aspect of this seminar which I really like is the fact that somehow you know we are not limited by time and this gives a um a possibility to adjust the tempo so that even if one wants to try to say a lot of things okay I mean one
can take a Tempo which is reasonable uh I think one of the things which is difficult in giving a talk is is to adopt the right Tempo so I hope I won't go too fast anyway I will reach whatever I can reach and um the if you want the General topic is somehow to understand first abstract very abstract science which explain what is
really going on with this work on Zeta which I am doing in joint work with Kya konani and H movich and the second aspect is an aspect Which is far more concrete in a way and which is if you want this construction of Zeta spectral triples and this will be done both at the infrared level and at the ultraviolet level and when I say that it's more concrete I mean that thanks to
the modern computers we have as mathematicians now the same one aspect that we didn't have before and which we share with physicist which is to to experiment so when we have some idea we can then experiment it with the computer it can take a lot of time a lot of computer time and so on but it's possible and one thing which I want to
tell to P also the following you see when I got 50 I considered myself as totally old and you know outed and all that but I took the decision that I should learn how to use a computer and okay and and I mean this worked out in the sense that uh you know it's like to make a comparison it's exactly like buying a good pair of glasses it's not like you know it's not that it's going
to completely change things but it improves really it improves a lot of things going on especially now with the modern computers which are so efficient and do you think we can use artificial intelligence in the future somehow to help us yeah let me tell you this I small part the small part is the

following if you are put off by the C Language by mathematical language and all that don't worry one second just ask
CH she will give you the program and you will just have to implement it it's nothing now okay so you don't have to learn all right so now let me start the stuff okay so what about class field Theory there is a very beautiful saying of about class field Theory by CL chal in 1940 and I mean so here I have show I show you in French but let me show you the English translation so the English translation is the following it says that the object of class field theory is to show how the Aban extensions of an algebraic number field can be determined by elements derived from the knowledge of this field or if one wishes to present it in dialectical terms our field processes within itself the elements of its own surpassing the French is even better in that because in French you say Z and this this has some
kind of poetical feeling but the idea is the following the idea is that you have all this Tower of field extensions which are you know very hard to think about but they can be derived at least their galoa group can be derived from the knowledge of the field itself and the knowledge of the field itself means that since the work of Kum we invented ideal numbers the work of D kin we invented ideals then ideal classes and eventually
chevalet invented the edels now the edels are when you divide the group of adels which is dl1 of the adels by the multiplicative group of the field and when you divide all that by the connected component of identity you obtain the galwa group of the maximal Aban extension this is the main result of class field Theory but when you think about this result you you you have to realize one thing what you realize is that in fact it would be wrong to reduce galwa Theory to groups and gwa groups this is not correct when you look in fact at the papers that galwa wrote when he was 17 when galwa was 17 he wrote a paper in which he explained that for an equation which he called primitive Without Really defining what it
meant then for this equation to be solvable it's necessary and sufficient that you can label the roots by the finite field that galwad invented the finite field which has a number of elements which is a prime part power so the degree of the equation has to be a Prime power so you can label the roots by a finite field in such a way that the gall group is contained in the semidirect product of the aine group of
the finite field by the fenus it's unbelievable that galwa 17 stated this theorem and it took up to the 50s 1950s to understand what he meant by A Primitive equation uh what he meant which is very subtle is that the Galla group is acting by transitively on the roots of course it's always acting transitively but it's acting transitively on pairs of roots okay so what you understand by
looking at this example is that it's not enough to know the gwa group abstractly you also have to keep in mind how the gall group is acting on Roots so in general if you want gwa Theory cannot be reduced just to groups it has to be complemented by the way these groups are acting on Spaces now when you look at the elel class Group which play this fundamental role okay which is
this D1 of adels by K cross okay we don't see the slides you don't see see we see we see oh I don't maybe I can see the slides yeah okay I think it's a problem with your new computer yeah it's a problem with my Compu okay so so what I was saying is the following is that after all you have this eel class Group which is the quotient of gl1 of a ring because after if you want
the the idel class group it was found that this group is in fact the gl1 group of a ring and this ring is the ring of adels so the term adels I think was coin by Andre veale and uh so if you want this group is not an arbitrary group it's really the gl1 of a ring so now the natural task is to see how this group The D class group is acting on some spaces what are the natural spaces on which it's

acting now there is an
obvious space on which this group is acting because it's gl1 of a ring so it's acting on the ring itself it's acting on on AK the Ring of Adel but on the other hand the Del class group is divided by K Cross by the multiplicative group of K so you have to divide by K and this is what I was doing in 1996 when I introduced the Adel class space which is the quotient of the Ring
of adults I mean which is a space locally compact Space by the action of the group now this space is a very delicate space because of ergodicity of course I knew this ergodicity since the very beginning and it is this ergodicity which makes this space resemble the space of leaves of a foliation and be if you want the Prototype of a non-commutative
space now immediate advantages of this space immediate advantages are that you see a class field theory is based on the link between the local class field Theory which is much easier and the global class field Theory now when you look at the local class field Theory what you get is that what replaces the maximal Aban extension is in fact the corresponding group is just the multiplicative group of the completion
of the field that the place V uh but this then this group is related to theel class group in a very septile manner because it turns out to be a locally compact sub group of theel class group now from the Adel class space this becomes obvious because this subgroup is just the isotropy subgroup of the Adel classes which have a zero at the place V so if you want this uh way of relating the local to Global becomes geometric
and it becomes exactly the inclusion of an isotropy subgroup inside the full group moreover when you look at at the action of this isotropic group on the transverse space to the corresponding periodic orbity what you find out is that it gives you immediately the term very mysterious term which enters in the veil explicit formula in the reman veil explicit formula for a reason which is ultra
simple which is the computation of the trace namely the integral of the diagonal values of the Schwarz kernel of the eration of scaling which is if you want the operation of the Del class group on the Del class space so this is the second point the third point is that it gave at in 1996 the spectral realization of zeros of Zeta and L functions but as an absortion spectrum and we shall come
back to that now very recently with Katya consani we put a short note in January of this year in which we related the Nots the primes and the Adel space and Kya has given in this seminar a talk about this topic so I won't talk in details about it but I will explain uh further uh Improvement that we did and this this Improvement is the following you see there has been for many
years uh similarity I mean you know in mathematics it's very important to know these theories which look alike there was a looking alike between two theories one of them was galwa Theory the second was a theory of covering spaces in topology and dimier told me an interesting anecdote he told me that once in the carton seminar in the beginning of the 50s there was a
discussion about this similarity and Pier Samuel was was negative and he was telling to cartan no this cannot work because this and that and then kartan stopped him and told him no it works okay and then you know grend came with theal Theory we shall come back to that later but we won't need it because what we can construct with Katya is a direct construction from an abelan extension of Q to a finite cover of the
scaling site and what what we shall see is that it it it gives a perfect translation from the properties of the galwa extension to the property of the cover exactly as one should expect in the analogy between extensions and covering spaces so I mean it all goes like this uh we have the scaling site which is the quotient of the Adel class Space by the subgroup which if you want is the group which correspond when you take the fix point to the reman data function if you don't take the fixed

point if you take a bundle over it then you get L functions but if you want to have the reman zeta function you have to divide by this subgroup of the Del class Group which is the maximal compact subgroup now this space is a space which you can explain in terms which do not require anything about adels or anything like that this
space is a noncommutative space and as non-commutative space it is the space of rank one subgroups of R okay and I mean here you just have to think a little bit because but when I say r one subgroup well for instance the group of rational numbers is a rank one subgroup what does it mean rank one it means that any two elements are commensurable now when you consider r one subgroups of you could say oh wow I
mean this is a SP which I can understand and all that but I stop you immediately because that's a noncumulative space so what does it mean to be a noncumulative space what does it mean in in simple terms it means that you cannot construct an injection from this space to the real numbers even though it has the cardinality of the Continuum it's impossible to construct an injection to the real numbers because of
ergodicity now in this space what you have is you have the per iodic orbits which have length log P for each prime number and which were very obvious from the beginning of the theory and uh now what we are going to see is that well in a NE Manner and we we shall make this more precise you have to think of this non-commutative space as having a visible part so it has a visible part which is
the what remains of theel class Group which is just the positive the group of positive real numbers that's something which will play the role of the generic orbit and it has for each prime this periodic orbit what is it well it is the collection of all subgroups of rank one which as abstract groups are the same as a group of rational numbers whose denominators are a power of P
when you look at these subgroups okay they form a periodic orbit they form an orbit of length log p and uh so we shall gradually understand that this visible part of the scaling site is in fact the points of the hypothetical curve that people are trying to construct since ages that would play the role if you want of the curve of algebraic geometry
in the case of function Fields so now there is a simple manner to construct a cover when you have a finite Aban group and when you take a homomorphism from this group the star to this uh subjective continues of course uh homomorphism from the art star to this group G now if you take such a subjective continuous morphism you can take the following you can if you want instead of dividing fully by Z Star you
can take the same Adel class space and cross it over Z Star by G so in doing that you get a space on which G is acting it's acting transitively on the fibers and it's a space which projects on the quotient of course it projects on the scaling side which is the quotient so now here is the functor from Aban extensions of Q to covers of the scaling site so if you start with the finite extension of Q uh well then it
turns out if you want that you can of course you can put this Aban extension of Q inside the complex numbers the morphism from this finite Aban extension to complex numbers is not unique but the range of this inside the complex numbers is unique and so what you have you have a canonical map from the Star to the galwa group of this extension I will come back to that essentially I mean this is if you want
the chronic vber theorem and it is a class field theory in the case of the rational numbers so once you have this morphism from Z Star to G what you have is you have an associate bundle an Associated fiber bundle and which is constructed just from the previous construction using this morphism now

one what one can see first of all is that it defines a
contravariant functor from finite Aban extensions to finite covers when I was talking about this analogy between uh extensions and covers of course it's contravariant because when you will take a first cover it will map down whereas the extension will be included so it's contravariant functor now it turns out there is a very simple definition of ramification for covers and the ramification is the same
as the ramification for the field extension but now comes the wall point now the wall point is that if you take an unramified Prime then the monodromy of the periodic orbit in in the finite cover is the element of the gwa group which is given by the fenus and moreover the connected components of the inverse image of this periodic orbit are circles labeled by
the place of the finite extension over the prime P so the picture is the following the picture is that you have this periodic orbit Associated to the prime and now in the finite cover oh what is it yes in the finite cover what you have you have a certain number of connected components when you try to lift this periodic orbit each of these components corresponds to a place of the field
extension over the place given by P now for each of these field extensions what you have is you have a a cover of the periodic orbit and this cover has a monodromy and this monodromy is generated exactly by the fenus element now the way this this comes about is by Artin reciprocity so what happens is that you have the chronic V theorem which gives
you that every finite Aban extensions of Q is contained in the group of roots of unity okay and the gwa group is obtained by raising to power the roots of unity okay so if you want the galwa group of the extension of the Aban extension what you do is you put this extension in an extension by roots of unity there is an optimal way of doing it which is by the conductor and then what you have is you have a way to understand the gwa group
by this canonical map from the no if you want the element of Z mod m z which are prime to M as a multiplicative group they map to the gwa group now the fundamental result of class field theory in the case of the rational numbers is that the fruse is equal to the image of the number P under this map now the fenus is well defined this is the reason for the frin
to be well defined is the following is that when you take a prime which is unramified then uh everything with which happens in in in an extension when you take if you want one of the places over the prime everything that happens in in this extension is in fact occurring in the residual field so it's occurring by an extension of the residual field so it's like the residue field FP is extend to
fq and then the G group is generated by what gwa knew namely the fren automorphism of fq over FP which is Raising to power now if you want to understand something about the previous theorem the previous theorem is really the fact that when you take this fiber bundle if you want this bundle over the scaling site then what happens over a periodic orbit it's really a mapping Taurus it's really the mapping taus of
the fenus element so once you know that then you can prove the theorem but it's quite amazing that if you want the Adel class space and the role of the Prime P fits exactly with Artin reciprocity we don't have to add anything it's like the field new everything what was going to happen for covers provided you don't just work with groups with a gwa group you also work with spaces namely the space okay
and the scaling side in this occurence now uh so just to tell you the way you should think about the periodic orbit let me show you a picture so if you want when you look at the samplified situation where you take only two places where you have the the aredian place and the pic numbers what you find is the following picture you find that the the way the Adel class space looks it looks like

there is this
dense generic orbit which is if you want approaching more and more to the periodic orbit and it sort of sees it if you want as a kind of limit limit cycle in a way so if you want to have this periodic orbit and you have this T thing which is corresponding to Ells and the very strange thing is that now this phenomenon of being dense if you
want in this periodic orbit that the generic orbit is dense in the periodic orbit reproduces itself when you take several primes so I won't enter in the details but the picture is the following the picture is now if you take several primes you will have several of these periodic orbits but the generic orbit will be dense in each of them now you can say wow how could this possibly happen well it happens because of for
noncommutative geometers this should be obvious in the sense that for noncommutative geometers what you have to imagine is that for each of the periodic orbits you have the density okay so you can you can plot the the the picture for each of them and then you can say but oh but how can it be the same you know generic orbit which is dense in each of them well you just
take the equivalence relation which identifies the green for each of them with the green of the other and then it's done and to do this identification of course you have to use noncommutative geometry or you have to use you know equivalence relations which are okay with an open set and so on you can use here you could use 3x3 matrices actually and you would do it it would work perfectly okay so then if you want it
happen that grend add also extended G Theory to spaces and by inventing theal Theory and in our little not with with Kya consani what we we we formulated the link with grendal Theory by actually you know uh saying what I have said now about finite covers but saying it about the projective limit of these covers and then comparing the fundamental theal fundamental group with the picture that we have from the uh Adel class space and
then we understood something in a way you know when you try to work on a specific problem mightbe at the end of the day you have some revelation of something conceptual and here the Revelation coming from Rend and all that the Revelation was the following the Revelation was that from the start this work you know that I was beginning in 1996 with the Del class space and so on
what is it well it is the extension of class field Theory from groups to spaces and in particular it applies to the grendal theory in other words the granal theory has now a counterpart in class field Theory so not only the groups have a counterpart but also the spaces and in particular the spaces of the grendal theory
and this is extremely satisfactory let me tell you why because I always had the you know like being eat by a picado when you are bull or something like that by the idea that these people in atal theory are much more advanced that we are because they have all this theory about spec Z and all that and they say you know spec Z is threedimensional and so on and so forth but now it Falls in our realm because we can translate
what they were saying about spec Z we can translate it in our language thanks to class field theory in this way and uh and we learned something we learned something because when we look at the way for instance you know at the beginning ofal Theory there were great contributions by Artin Michael Artin I mean the son of Emil Artin and also by mford and um you know
about Barry me and and Berry Meer of course about the the way if you want one can do computations with this stuff and one computation was precisely that the spec of the integers is something which should be start off as three-dimensional and the the proof when you look at it because of

course one should look at the proof you know uh when you look at it what you find out is that it all relies
on two things it relies on class field Theory and it relies on the fact that you have a generic point in Spec Z and this generic point is dance which is an extremely strange thing so the generic point is such that it's everywhere dance in specy oh but then you come back then you say oh look I have this generic orbit and this generic orbit is dense in all my periodic
orbits so in fact when you think about it you find that in fact people have tried in fact there are traces of that on the internet people have Tred to picture spec Z and they all more or less came up with a picture not with the periodic orbits but a picture with points in which the generic point is dense like that now what is the role of the periodic orbits well the role of the periodic orbit is that of course spec Z
when you look at spec Z you you are not taking points of the hypothetical curve over an algebraically closed field you are taking the places but the places when you look at the case of function Fields they are just the orbits of the fenus so in fact these are the orbits of the fenus and they have length log P the fenus is given by the scaling group and here is the picture of what should be S of as spec Z okay so it's specz taking
the points of spec Z over an hypothetical algebraically closed field and moreover it reconciles us completely with something which would have thought which we could have thought totally paradoxical which is that when you look at the Adel CL space you say but this is an extremely strange space because the dels are Dan in the adults but in fact this is a blessing because the generic orbit or the generic point
is dense in Spec Z and so when you involve on that by doing more thinking and so on what you find out is that it guides us because it says that the spectral realization as an absorption Spectrum as I had found it from the start in fact should be S of as a relative H1 so in fact it should be S of as the action for this is the spectral realization of all L functions okay with
the crossing character and it should be S of as a realization in a relative H1 where the relative is relative to the generic Point namely to the generic orbit which is given by the eels so that's if you want the conceptual gain from kind of mastered what people were doing in in this approach by theal Theory and so what we can be totally reassured about is that with these finite covers
and so on we have the analog of theal site that people were using to Define ital chology and so on so we can be reassured about that okay on the other hand okay this realization this spectral realization was an as an absorption spectrum and so the absorption Spectrum I don't want to spend too much time about it but the only thing you have to know about it is that this was what led to the discovery of ilium because in the
spectral lines from the sun which were absor on line there were some lines which were never identified on Earth which were missing like here in the sunlight and so physicist or chemics I mean were extremely smart people they invented physical a chemical body pure one which because in honor of the sun they called it helium and which was characterized by
its kind of you know signature which was the Spectrum but then at the end of the 19th century there was an eruption of vovio they made a spectral analysis of the lava and I found that ilom was there which is an amazing fact now what we what we did with Katya in the paper in the called the scaling aonian we did we we did the step which was a crucial step in the invention of
Photography you see in the invention of Photography there was first The Invention by N4 nips what he did was to uh uh put a box in front of his Courtyard and let it take the light for eight hours and after that he looked because he had a plate which was sensitive to light and he got a

kind of image of the of his Courtyard after eight hours of impression okay okay and so then afterwards there were improvements which were done by successors of n for especially in France but there was a great contribution which was done in England by William Henry Fox TBO and what was the idea of William Henry Fox TBO it's a great idea his idea was the following his idea was to instead of taking um if you want a a plate which was sensitive to light and was which was if you want lighter when
there is light and dark when there is no light he took something which was much easier to fabricate which was something which was if you want which was dark when there was light and so so it was a negative it was if you want it so what it took he he was taking negatives negatives which is exactly the same thing as in an absorption Spectrum so it was exactly taking if you want the absorption Spectrum version of what he
wanted to see but then he get he got a beautiful great idea of course doing this negative was not a great idea it was just you know using another chemical to have an absorption Vision the great idea he had was to take a picture of the negative that's a fantastic idea and when he took a picture of the negative of course things were reversed and he got the positive as an outcome
and in this way he had invented photography and photography was then developed you know for years exactly following the principle of fox starboard now with Kya in in our paper we did the same thing so what we did we took the absorption Spectrum version that I had deviced and what we did we took a picture of it namely we substracted this negative from the white light this is explained in great detail
in our paper and and so what we got if you want we we substracted from the white light which had a cut off the pictures that I had obtained and then out of that we obtained if you want a um a picture which was the positive I will this picture gave if you want as a consequence a trace
formula which if you want as I I will come back to the previous but it gave a trace formula in which instead of having the cut off as I had in my paper of 1998 had no cut off anymore so the Lambda disappeared there was no limiting situation and uh so this formula was expressing the the veil terms if you want in the ran Veil formula in terms of operators
now the role of this rean Veil terms is that there is an equivalent way to formulate reman hypothesis which is the positivity or rather here it's written as negativity but you just put a minus in front of the veil terms now it turns out that it's it as we as we shall see very soon it's possible to localize this positivity so that the number of places which are involved is
finite and for instance one effect that this localization has is to totally um dispel if you want a belief that most mathematicians have I think that for most mathematicians if they were asked what is the difficulty in proving RH they would tell you that the difficulty is because there are infinitely many primes now it turns out that because one can localize this positivity which is
equivalent to RH that's a wrong statement and in fact there is a statement which is which is involving only finally many V at a time and which is true if and only if the hypothesis is true so what you have to do is to only consider fin many primes at a time so there is this Trace formula okay and this Trace formula involves pair of projections and the angle between these
pair of projections so I mean and it also involves a space which is called the sonin space and I mean sonin was a mathematician of the 19th century and I think he he he he he got his stesis in war show in fact I think he was and he was teaching in war show for sometime and he actually got his stesis in work show what was his first name uh oh this is a a name which I cannot pronounce let me look I mean I will tell
you a little bit later I have okay but I mean but he he was a great mathematician of the 19th

century okay and so in fact the the the trace formula that you get from this transformation from the absorption Spectrum into an emission ELR is a Formula which involves two terms it involves one term which is the sonin term and one term which is the prate term and what we did with Katya consani long time ago I mean some time ago this
was in 2021 okay looks long time ago was to prove that if you restrict functions test function with this condition on the support then you have the negative or the positivity property if you want and it comes from the Sun in space so if you want the the reserve of positivity is due to the Sun in space the other difficulty is not coming from sonin it's coming from this stupid little square here so in other words when you look at
the trace formula you find that positivity is guaranted by this infinite Square here but there is a guardian of the difficulty if you want which is here and which will prevent you from concluding in and it's coming from this little square and the reason why this Guardian has to be there is that if it were not there then the positivity would hold unconditionally but it doesn't hold
unconditionally what I mean is that you it's crucial in our proof with GAA that we have the support condition on this function because we only look at the archimedian place and when you look further so as I said I mean you localize the veil positivity when you look further and we did with the computer what you find is something which is absolutely mindblowing what you find is that when
you look at this Veil quadratic form which you could prove to be positive when you restrict to functions which have support in the small support so when you look at this Veil quadratic form and you restrict functions to have support between Lambda inverse and Lambda that's why I put a Lambda here then when Lambda here is Lambda Square here in fact so it's mu when this mu increases okay I
have put here the log of the smallest value so for instance when Lambda is equal to 5 you find that the smallest value of this Veil quadratic form is of the order of 10 to the power minus 123 so this means that it's number 0 0000 0 you have 123 zeros and then you have a small thing okay now you can say well okay first of all why don't you get exactly zero well you cannot get exactly zero this follows
from the Nal the or if you want it follows because if you assume RH then the smallest value of this Veil quatic form you know that it strictly positive it can be zero because uh if you want if it were zero or the point is that if it were zero then the for transform of this ion function would vanish at every zero of Zeta but there are too many zeros of Zeta for this allomorphic function which is of exponential type to vanish at all
these zeros so it's impossible so you know it's impossible you know it's impossible and you are in front of dmna because in the work with Kya we were using and in fact this had also been some time before by Yoshida we were using the computer so I mean here of course you cannot choose the computer it's impossible because how can you know that the computer is delivering a positive result if you have the first
Zer which are 123 of them this is impossible okay so if you want the difficulty is coming from the prolate part and the prolate part is coming from the analysis of the angle between two projections and this analysis I don't want to spend too much time on it because I gave many talks on that this analysis is between two projections which has been done by slan and his collaborators in the Bell lab in the 50s
and they were working in communication of signals and what they were trying to do was to find a way to optimize the transmission of signals knowing that when you transmit a signal you have if

you want a limited time and you have limited frequencies so this means that you have a limitation on the time which is this projection P Lambda and you have a limitation on the frequencies which is this projection P Lambda at and what
stepan and his collaborators found they found that they could diagonalize this operator and so they could compute the cosine square of this angle us using an operator which I will come later to which is the prolate operator and uh the bare fact which I ask you to contemplate is that you have the same minuscule behavior in other words when you look at the difference between one and the largest ion value of this operator okay
so this operator is an operator which is the product of three projection but it's it's the compression of a projection between another one so it's less it's between zero and one and so when you look at the Spectrum the spectrum is between zero and one now you look at the largest value and you look at the difference between this largest Dion value n one and you find exactly the same
behavior as you add for the minimum of the veil quadratic form namely when you look at the log of this difference you find the same numbers so you find for instance that you know when the MU which is Lambda square is 25 so when Lambda is five you again find 10 to Theus minus in fact you find even a little bit more smaller 10 minus 124 okay so what we did then was to look
so you can look at the decay of the corresponding value of the veil quadratic form and then there is a an understanding of why you get these extremely small I values of the ve quadratic form and the conceptual understanding is the following is that if you would look at the full Veil quadratic form not with restriction of support condition then it has a radical so and it has a radical which is given
by functions which are obtain in this way and which we shall soon relate to the REM sums of an integral so it turns out that when you take functions of this form they are uh because of the pon formula they are in the radical of the full Veil quadratic form and then if you try to force them to have compact support with for transform of compact support it's not possible but it's
almost possible due to the work of slean and his collaborators so experimentally what one finds is the following fundamental fact that when you look at the space of I vectors of K lowest value for the veil quadratic form q w Lambda this corresponds to something which is computable and which is given by the prolate functions of San and his collaborators okay and using this fact this was what we were doing with GAA a
few years ago so what we what we tried to define a spectral triple that would be a good spectral triple for the infrared we tried that and I mean what I will I will explain briefly what we did and then I will explain less briefly what is the the good spectral triple for the infrared but we are not yet there this is what we were doing in a paper with Katya for dedicated to V Jones so what we tried to do with Katya was the
following for triple we were taking the quotients of the multiplicative Group by powers of mu so if you want we were taking periodic boundary condition for the scaling operator which is given by IU byd and then we were compressing this operator on the orthogonal of this prolate projection so you know this was an attempt so what we had in mind was the following after all this prolate
projection gives you extremely small ion value of the veil quadratic form so we force them to be there and then we take the scaling because we know that it is a scaling operator which has to be the right operator okay and then we made experiments with this we made ex experiments and when we began making experiments we were quite excited because at the beginning of the
experiments we were getting something which was looking like the spectrum of of theta which was looking like if you want the the zeros of theta but after a while we found that when we were varying

the problem was the following the problem was that we just didn't have a single operator for each Lambda because we had to make K conditions and the number K was not fixed from Lambda and the number K is an
integer so when you jump from K to another you know something happen so so in fact what we found after a while was that it's not true that we were getting the spectrum of data but what was true is that when we were looking at the aen values these aen values were meeting when were changing the number of conditions and the places where they were meeting was here the first zero of Zeta and then we looked at
the place where they were meeting further on and so on and we got the second zero of Zeta then we understood that there was a quantization condition I go very fast here because this is not the point of my talk I'm just recalling something which is our paper and then so after a while what we found is that in fact we were getting zeros of Zeta but in order to get zeros of Zeta we had to adjust the wavelengths of our experiment
because uh since we were taking periodic boundary conditions we were forcing the Spectrum to be in a certain ltis and so we had to adjust the wavelengths if you want of our AP stus in order to catch the zeros of Zeta so this was good of course and we got a mathematical theorem okay so we got a conceptual explanation of that we got a corresponding mathematical theorem the corresponding mathematical theorem was that in
fact there was a notion there was an abstract notion which was the notion of data cycle so in other words there was a a way to introduce uh abstractly what are Thea Cycles so what are they they are circle of lengths fixed length circle of of given length which is such that when you look inside the circle to the space of ransoms rescal by mu remms of arbitrary functions you get a Subspace which is
not dense in theber space of L2 functions in the space so that's an extremely simple condition and then the theorem that we got was that if you take a Zeta cycle and if you take the action on The Zeta cycle of the multiplicative group then the spectrum is formed by imaginary parts of Zer of Zeta on the critical line and conversely if you take a critical zero of data then there is a corresponding data cycle but as I said
this was not fully satisfactory because it was not giving you an operator which has the write first zeros of data no it was giving you an apparatus which you could use in order to find the Zer of Zeta which is quite different okay so now here is the new idea which we have found I mean with Kya and Ari in our and the new idea is coming from a totally different place
and it's coming from the top L paradigma it turns out that when you look at topits topet matrices there is a fantastic result I think it's due to zego but I'm not sure I'm not sure about the paternity of this result but this result is like if you want uh an r i mean a tool for RH this is if you want the the second thing which I would like to say to when if you
have people and for instance they tell you oh yesan hypothesis coming from the Infinity of primes false if they tell you well reman hypothesis we don't have any tool false here is a great tool the great tool is the following theorem tells you that if you take a toet matrix any self ad joint which is positive okay and and if you if you assume it as a one dimensional kernel then the polom that you get from
the lowest Vector in this from the vector in this kernel has all its roots on the unit circle and this means for instance that you can take you know what people do with thean hypothesis in function Fields you can take their topist Matrix and out of their top L Matrix you can take the lowest Vector it always exist doesn't to be positive because you you just push the Matrix to be positive then this

Vector satisfies RH in other words the corresponding polinomial has all its roots on the unit circle and I tell you this for good reasons I tell you because I was Computing when I was trying to illustrate if you want the hypothesis for function Fields with music I was Computing with hyperelliptic curves and pooms and so on and I was always very struck to find all these pols which have all roots on the circle but what is true
is that in fact it's not something so great because it's something you get automatically from a templet Matrix so I find this extremely striking and I mean we we used it in our paper with Kya on the veil archimedian case we also used it in work with wter vom uh in the in the for operator system STS now okay so now what we have in the work with h and Kya we have a sister algebra proof of this
result which because it's a sister algebra proof has all the reasons to be possible to extend to The Continuous case and what is this proof well the proof is the following the proof is that you take this real symetric positive semi-definite Matrix okay and you take a a nonzero element of its kernel then what do you do well then you consider the ideal in the luron pols which is generated by this polinomial
the polinomial whose coefficients are the coefficients of the element in the Kel and then what you prove it's not so difficult you prove that there exist a unique linear form on the quotient by this idea which fulfills the condition that when you evaluate it on monomial so it's a kind of moment problem so you you you prove if you want that there is a unique linear form
satisfying the moment problem for this uh uh XJ which are the the terms which enter in the top Matrix moreover you prove that this linear form is positive on the quotient algebra okay and and and the proof is not really difficult I mean it comes from the positivity of the to matrix it's really easy statement now from C algebras what do you deduce and from the gns construction so from the gns
construction what you have you have a linear form which is positive which vanishes on this ideal it defines in fact a positive linear form on the full cister algebra which is the C algebra of convolution of the group z and so it find a positive measure on the Dual U one of Z now this measure is supported by the N values of the unitary and these I values are exactly the Zer of the polinomial I mean this is a very easy
check so what you have is that just by unitarity you get the RH namely you get that the roots are of modulus one so you get it from a completely conceptual abstract proof this is an essential step now what one has and what we use okay with ar and Kaa is a dictionary between the topest case and the veil quadratic form so in the toplet case you have this toplet Matrix which is this now in the case of the veil quadratic form you have
the distribution which is given by Veil there is a Nuance which is very important which is that in the case of the topist Matrix you can get a state on the sister algebra here you get a weight because of the singularity of the veil distribution at the at the origin so it's a little bit more difficult but then you have what corresponds to The topist Matrix is a Schwarz Kel given by the veil
distribution the topist quadratic form is the veil quadratic form of course the companion Matrix which plays a key role in the case of the topet theory is replaced by a certain operator the zeros of the polinomial are now the zeros of the for transform okay so I mean now we are getting really to something and the unitarity of the operator which is coming from the unitarity of the cister algebra notation
is coming from the selfness of an operator for the quadratic form and so now we have our candidate for the data spectral triple so here it is so now the space of the algebra is the same the algebra

is the algebra of smooth functions in the interval inverse Lambda okay where mu is the square of Lambda the ilber space is more delicate yes the space is the completion of the smooth functions for the inner product given by
the veil quadratic form but you have to shift the veil quadratic form to put it minimum to zero that's easy okay so that doesn't assume that it's positive we have shown that it's lower bounded so this is always possible and the operator is just scaling so if you want this is quite amazing because it's the same operator as we were using with Kaa but the ilber space structure is
changed the algebra is the same but we do not put the periodic boundary condition the only change is in the product okay and now what you have you have a a picture in the F transform why because the for transform of the C algebra of convolution of functions on R plus star is just multiplication of functions on the Dual Group which is r and what you find is that the spectrum of this operator is
given by the Zer of the for transform of what of the vector for the lowest value of the veil quadratic form okay and so now you can be back to the working uh table you go back to the computer and you say oh my God now I I was able to compute the vector for the lowest value of the veil quadratic form what will happen for it for a transform and it Zer okay and here I have to recount a story
which happened when we did the computation it's a crazy story so when we started doing the computation didn't look good at all in other words we were getting for functions these functions were coming from the lowest Vector of the vtic form and when we are looking at their zeros they were looking like nothing except for one value very
strange value around 22 oh there were a few first zeros which were looking like the zeros of Zeta and because they were looking so close to the zeros Zeta it was impossible to think that this was just a coincidence okay so then the idea was to redo the calculation so the computation had been done with 40 decimal places with a computer and the idea was to redo the
calculations with 80 decimal places with the computer and see what happens and when we R the calculation in this example temp with 80 decimal places the good zero that disappeared so in other words after doing that the good zero that disappeared so this was really a catastrophe on the other hand you know it was like there was a devil which had shown us that there was something in
this story okay so for several days we were totally mystified except that when there were these good zeros the function that was obtained had the first component which was Vanishing almost it was extremely small so then we got the idea of imposing that this first component would be zero as an additional
condition and then we redid the calculation with this additional condition and it worked marvelously it worked marvelously okay and this was the situation again for sometime perhaps two weeks or three weeks okay and after two or three weeks finally we understood that there was a computer mistake in the
calculation which was giving the wrong value of the for transform for this when this component was not zero so after correcting this computer mistake then we could go back to the working Table after you could say after wasting three weeks okay fine but you know this is what physicist do when they do experiments of course they do mistakes and the whole point is to
correct this mistake so after correcting this mistake we were back to the working table and we were back sorry we are back Computing Computing again the zeros of the F transform form of the vector for the lowest value and I must say I mean this is an answer to P I must say that in order to compute these zos I had to ask CH GPT why because I mean you know I didn't know of course I have Mathematica and

I've been using Mathematica for years but in order to compute several zeros of a function I didn't know how to make a good program and what what uh chbt told me is that oh well you know it's simple you have to to make a grid with a lot of numbers ask when there is a change of sign of the function between two and once you are in this interval use another program to focus on the zero that's what she told me okay okay
so then I used what she told me and I began to to compute so this is the graph of the F transform that you get for a small value of Lambda Lambda equals three okay but you look now at this graph more precisely what do you find oh this function is cting close to 14 close to 14 14 something is the first zero of Zeta so this function is is cutting close to 14 what about the next one you look at
the next one oh it's now closing close to 21 and then close to 20 25 okay of course these numbers are very small okay and now you ask using the program of CH GPT you ask okay and now you can continue with 30 and so on you ask CH GPT please compute with this mathematical program compute my zeros and here is the Zer that you compute and here are the true zeros the first one is 14 1347 in fact there are 15 decimal places which are the same as for OFA this is just for Lambda equal 3 okay and you keep going 21 022 2519 okay and it keeps going like this it goes astray after a while well which is normal after a while you can you know so it goes astray when you are reaching the end I mean around here I mean it goes astray but very small you know for
instance here is 72.0 648 and you're 72.0 672 I mean you know it goes astray but it's amazing it's amazing because when you compute the number of good decimal places that you get you know you get something which is unbelievable which is truly unbelievable and uh in order to check that what you have done for Lambda equals 3 continues in fact to hold for all values of Lambda
what you do is okay now you take the computer you use 16 Kels to do the computations and here are the computations of the zeros uh of the functions that you get for transform for when you take the square roots of these numbers so Lambda Square belongs to these uh sequences and what you get is that perhaps at the beginning okay it's not perfect but after a while it's really perfect so what you have what you
have in doing this if you want what we have achieved is now now we really have if you want a perfect candidate for the infrared of the zeros of the reman zeta function all the more because the next issue if you want is to completely get rid of the veil quadratic form because after all the construction was
really the main thing that it was using was the lowest value igen vector and because of the work on prolate functions we have a candidate for this lowest value Vector so in other words now if you want we have an understanding of the infrared part of the Zer of theta us using the prolate the prolate wave functions not yet the prolate operator I'm now coming to the prate operator namely to the ultraviolet Behavior and
the ultraviolet Behavior this is Joint work with h and uh uh I mean in this work if you want um the it was again something which which is a kind of Miracle what happened in the sense that what we did with Ari was to start with a problem which a prior had no relation whatsoever with the remand data function and I always take this example as a prototype example of a
work which you know if you wanted to write a proposal for the NSF or now for the European stuff and so on okay I mean it would be impossible it would be impossible because I mean to to think that we mathematicians can write proposals in which we are going to say what we are going to find when we are going to find and so on this is absolutely ridiculous I mean what matters is that if you want we we we
always try to work on conceptual problems but of course having no idea of what the results of our

investigations will be so if I have time I will come back to Z expansion and Zeta but I prefer to to really go to the uh prolate operator so going to the prolate operator if you want what happens is the following so what happens is that uh first of all there was this discovery of San and his collaborators that if you want the uh diagonalization of the angle operator between the projections P Lambda and P Lambda at was possible because they were commuting with the differential operator now if you want to understand why they commute with this differential operator it's very very important to have a mental picture and that's what I want to explain what is the mental picture between the fact that there is an operator differential operator that actually commutes with the two projections well the mental picture is the following let's take a semil classical picture so in the semi classical picture when you say that you are localizing between minus Lambda and Lambda in the Q space what you have is you have two lines in the face space which are bounding the interval from

minus Lambda to Lambda when you say that you try to localize the function so that the frequencies are localized between minus Lambda and Lambda what you have is are two lines which are the lines P equals minus Lambda and P equals Lambda okay now how can it be that by staring at this picture you will guess a differential operator that commutes with the projections P Lambda and P Lambda

at okay that's my that's my question let's just stare at this picture now when you stare at this picture you say what do I have I have four lines what is the equation of the Union of these four lines well the equation is $Q^s minus Lambda Square time p minus Lambda Square$ it's an equation of degree four and when you solve this equation you get

the four lines so the zeros of this product are exactly the four lines okay now what do we learn when we learn semi classical approximation and when we learn aonian mechanics we learn that if we take an aonian the flow lines of the aonian will preserve the levels of the aonian so in particular if I take the aonian which is is given by P sare - Lambda Square * Q s- Lambda Square the

flow lines will preserve these four lines this is the picture this is the picture of the flow lines of the P Square minus Lambda Square * q^s minus Lambda Square you see that these four lines are preserved but what does it mean that the flow preserves these lines it means that the flow commutes with the two projections okay so it exactly means that the flow

coming from the hamiltonian h Lambda of PQ = pus Lambda sare * Q^s- Lambda sare preserves preserves the the four lines and then commutes with the two projections okay now the prate operator okay is up to a constant which doesn't change of course the fact that it commutes a multiple of this aonian and then you are the constant of course this is a semi classical picture and you could argue yes p and Q do not

commute how do you quantize and so on but okay I mean but if you want by doing this you get really an understanding of why there is this commutation now of course there is a lot of work and uh what the discovery of of San and his collaborator was that this operator commutes but it commut with the compression now what we did with h was more delicate because we wanted to have an operator which was defined on the

full full line and which was commuting with these two projections and this was non-trivial it was non-trivial because when you work with this you find out that it's a very delicate problem of selfi joint extension because what you have is you have an operator which is symmetric when you take for fundamental domains the sh space of SCH functions on the line but it has for nman deficiency indices which are four and four so it's

a very non-trivial problem to find the good self agent extension so this was the start of our work

with ar and what we found is that the main discovery first discovery that we found with which was extremely exciting was that this self joint operator has this spectrum I mean I was not expecting this at all that's why I had abandoned this work in in the 90s and so what we found is that this operator has discrete Spectrum that's an amazing fact because if it has discrete Spectrum W this spectrum should be interesting and it should be interesting in each new part because there is an easy part of the Spectrum which corresponds to what people were doing with the prolate operator and all that which is the positive part of the spectrum but the new part which is extremely interesting is the negative
part of the spectrum and when we began to look with Ari at the negative part of the spectrum we found uh something which okay you know there are few moments in life which are worth living and uh what we found is that when we take the semi classical approximation so we take of course this thing and we look at you know we compute the area of where it is in certain
sector and so on we found that the formula for the number of values of the operator begins to resemble the formula that is in remans paper for the number of zeros of Zeta which are less than a and as I said you know this is a typical case where oh you begin to to say there is something there okay now we went much further of course and okay you can use elliptic integrals and so but this is technique the we can use
we use the Lil transform to transform it into a Schrodinger type problem and uh so this is just a Schrodinger type problem and then we used Fine Results of of the theory in order to compute to get a prec size formula for the number of Zer and using this formula now we add the formula very analogous to reman formula where the term was exactly as in reman with the remainder term which was
of the order of log e and which resemble the best estimate which is known by Tran for the reman zeta function where there is also a log e term okay and so what we did with ar and I have given talks on that so I I will not spend too much time on that but I just want to remind you about that so that you have the framework so if you want this was for an operator which was this prate operator and we gradually realized that the role
that this operator is playing is really like the square of the scaling so if you want uh in in the negative to positive picture we're getting with Kaa there was a problem with the picture the problem with the picture that the scaling does not preserve the decomposition into sonin and prolate part and the beauty of the prolate Operator Operator is that it preserves
the Sun in space and it preserves the prolate part so it commutes with this decomposition which of course is much better because it allows you to restrict it to sonine and when you restrict it to sonin essentially you obtain the negative Spectrum which is an amazing fact now with ar we knew this result only up to finitely manyen values but now as I will explain at the end of my talk with recent work by Ramis and his
collaborators they seem to know that it's exactly correct so now what we did with ar was to extract a square root of this prate operator in order to get a dra operator because I mean of course you know the spectrum of Zeta looks like a dra operator and I mean of course a d operator is much more suggestive of of spectral triple so it it took us a lot of time to get this
operator uh because and we used stuff which in fact is called the daru method and which was developed in the 19th century where where this allows if you to factorize the operator which is the prate operator into a product and then to use a 2x2 matrix technique I mean to factorize you need to solve RTI equation but all this can be done this is technique and then you get a direct type operator okay and when you look at

the Spectrum of this operator you find exactly what you would love to have namely that you have the number of zeros which is exactly as inan now when you do that this is abstract stuff now you have to go back to the working uh pad okay and you have to do computations that you do with a computer again okay and so I I mean when you do these

computations uh they are done in a very kind of unsophisticated manner which was the following the computation was done by taking the solution of the differential equation corresponding to a putative value near Lambda taking the solution of the differential equation corresponding to the putative value starting from Infinity namely verifying the boundary condition at infinity and see if these

two guys actually meet and you look at an intermediate region which is around a thousand so here for instance for minus 65 they don't meet but when you look for instance at minus 38 they meet up to sign of course but okay I mean up to sign workers you can of course change the sign of one of them so here they meet and then you you you keep doing that and for instance for minus 93 they meet and so what one is doing you know

by doing this if you want one is doing this experiment and one has the pencil and the paper and of course these numbers were in fact integers so what one does is to take note of when they meet and when they don't meet which is of course extremely ra okay now what happens is that very recently jaier raamis and these collaborators were fris rishun and jeant have applied their the beautiful

theory of Ramis on you know the STS phenomenon and differential galwas Theory and I mean they have uh among the results which they have which I mean I cannot talk about because I don't know their results but among their results they have been able to truly compute these numbers which I was Computing if you want in this extremely approximative Manner and then I show you the table of comparison here I had my poor integers

minus 39 okay okay and here here is that number so you see their number isus 39. 383 2 1 657 4467 okay the next one I had was minus 94 they have Min - 94. 28833 so you know this is of course extremely reassuring because it shows that well sometimes I'm really wrong I mean for instance I have I had minus 211 and I have minus 24 and then 06 and so on so of course

you know this means that my my hand sleeped on something okay then Min and so on but what happens then is the following is that then they compare it of course by doing the thing with the zeros of Zeta and they find the same type of agreement that we had they find the same type of agreement that we had so if you want they find marbly good agreement with the zeros of data in the ultraviolet

and uh I mean this if you want raise is the fundamental question on which we are working with K which is if you want one of the few questions that you have to keep in mind all the time and uh when you see something in the street and so mightbe it give you give you some idea but the key is to keep this question all the time in your mind okay and the question is the

following how on Earth are we going to marry the infrared realization which is so good which was using prolate with the ultraviolet realization which is so good in the ultraviolet and which is also using the prolate functions so this requires an idea okay in the best sense of an idea okay but what it says is that you know in all

this work we are digging hard hard on the data function and we are finding Marvels along the way so if you want and what we are finding each time is that the answer is more subtle than what we thought at the start and each time we have to understand a new degree of subtilty which is hidden behind a computation and for instance at the moment we know that there is another degree of subtility in the guas for the smallest T Vector for the veil quadratic form involving the prate functions okay so I think I will stop here because I'm supposed to talk for one hour and a half and uh so I have skipped some part but I think I have more or less reached what I wanted to say okay thank you thank you very much was fascinating okay start so so let me begin I don't really have questions but I have some answers uh so

so for starters a very very big thank you to Al for delivering this talk yes thank you you know my colleagues couldn't give me a better birthday present than your talk so I'm very that's that's too nice of you that's great great great great so big thank you you know to to lud Tom Andre and Mark many many others who who who helped to arrange all of this I'm I'm really very happy about it and now I cannot resist

this opportunity to mention two reasons why I love non communative geometry one is very obvious I mean because it's fantastic mathematics is so diverse so rich so multi-dimensional you are never Bor doing connective geometry you always can have a different angle you can always have a different type of diet I mean it's very healthy intellectual diet so so this is why I like it but that's obvious but another thing is the human

factor I mean while working on non to Geometry I had the honor and pleasure to interact with so many Brilliant Minds with so many wonderful human beings and this is an incredible adventure and and and and I'm I'm very proud and happy to be a member of this particular scientific Community it makes me very happy and gives us big push to go on I mean you never want to stop exactly yes yes thank you so much P I want to add

something you see I think what we are we are in the continuation of uh the dream of Eisenberg when he was in Eland and when he saw he saw all this I'm sure and he was scared by what he saw I mean because after all you know he had discovered the the key point which is metrix mechanics you know and uh and uh I mean of course I

mean so so I I think the what what I would like to to to say is the following it's the power of ideas you know I remember I remember once I was in a dinner and um we were uh celebrating Katherine Brak who was you know Secretary of the of the of the academy and it was a dinner where there were several tables and um and and the the people were asked to to it were several tables

so there were lots of people and people were asked to ask questions you know and okay and uh and so I was thinking about some question I should ask okay but before I was given the microphone there was a theologist who spoke and this Theologian um explained that um well we don't know what happens after death and so what he said is that even

though he had studied theology for all his life you know he didn't know he couldn't say anything about that okay and so I changed my intervention after he said that you know so I was eager to talk and so then came my turn to talk and um what I I tried to explain but in a very Malad manner but okay so what I tried to explain is the following I tried to explain that you know we we we are confused when we think of

ourselves as collection of cells and so on and the material body for the following reason that all the cells of the body actually change renew themselves after a relatively small amount of time like two years or something so what I explained was that we are not a collection of cells we are a scheme which is a scheme underlying this collection of cells and moreover I I said in a very blunt manner galwa is not dead yes because what I said is that you know galwa is more alive than anybody I know because of the Fantastic power of his idea yeah and and so I mean you know I I think this is a lesson and and so I mean you know what I wanted to say is Eisenberg is not dead because of the Fantastic power of his idea of his Discovery and um I I know that P you love galwa theori because

of course very
good reasons but but this is a fantastic thing I mean think about this guy I mean he he made this this great work when he was 17 he didn't do much work after because he was in jail and so on he didn't really have time to do much work he died when he was 20 and his ideas are like a wild horse you know they are alive they are powerful they are there everywhere yeah that's all I want to say yeah
yeah this is like conservation of information in Black ho yes yeah that's what it is it's the amount of information that we can bring to the human uh Community it's exactly this and in fact you know there was this birthday of di I should say um it was very interesting because um there was so Dix turn 100 years old in May and there was a a dinner which was organized in his place and it was
very interesting because there was um if you want a discussion between s and Di on this dinner and S was saying that okay we do mathematics not because of Glory or whatever no but because of the urge to understand okay and this I agree completely of course I mean you know there are situations in which I mean the world thinging is the urge to understand and then thep said okay that he said he was speaking of himself I do
mathematics to exactly to this to contribute you know to the human knowledge and to the progress and all that and I think you know this is so true I mean in a way I mean we are mathematicians and I think the best goal we can have is to contribute you know really to this progress which is a and nowadays with chat GPT and all that I mean we have access to so much that we have no excuse you know I mean there is
so much we can read we can access to and there are lots of things which I'm sure are already there I mean you know I'm sure completely sure okay are there some other comments yes please yes make a little comment Al said about the first coefficient being zero that condition yes it reminds me of cidality yes yeah it could have been
spum so it could be related to something like that yeah except that this condition was virtual because it was a mistake the comput yeah but that's what it said yeah yeah yeah yeah yeah yeah yeah caspit forms not just yeah yeah yeah sure no no I agree with you and in fact it's a good conditions that's what we found out in the topest case it's a good condition it was not a you know it's a condition that actually makes the
function more easily tractable but uh yeah but if you want the I I hope I try to convey the fact that um this remman data function is is a gold mine you know it's really I mean one has to explore one has to go and it's always always always more seple than what it looks like from the outside I me and there are misgivings there is another misgiving which I I I didn't explain but which is a very common misgiving because some
people tell you that the the reason for looking for the real hypothesis is to have a formula for the number of primes less than n now the story is there is that when I was in in high school at some point my teacher uh told in the class I think it was or something like that it's two years before Balor so my teacher said there is no formula for the number of primes less
than n of course I mean I knew that it was wrong because you can always concoct a formula stupid formula okay so I mean the next day I came to him with a a formula which was two pages long which was of course a stupid formula okay but okay but it was a formula and then okay then of course I mean you can you can say U and this is a usual misgiving of people there is no simple formula for the number of zeros
less than n now I CL this is wrong this is wrong and in fact I I published a paper in which I gave the following formula and you will accept that it's a simple formula if I can tell it okay and when

if when you write it down it takes less than a line it takes Alpha line okay so the formula is the following the number of prime less or equal to n so n is given to you okay what is it it's the sum from one to n

it's the integral part sorry of the sum from 1 to n of sin square of pi/ 2 mtip gamma of k k I claim this is the true formula that gives you for any n the number of prime less or equal to n warning if you want to use this formula for the computer computer should know a lot of decimals of the number Pi because this formula does

depend on a lot of decimal points of the number pi and if you replace Pi / 2 by a number X x what happens is that for almost all X in the measure Theory sense formula behaves like essentially like n /2 and for almost X all X in the bare sense it behaves like the number of primes up to a factor because there is a factor depending on what is so so you see that this is a misgiving that many people have that there is no simple

formula now challenge for physicist challenge for physicist of course I mean this is putting the bar very very high challenge for physicist write this sum of s Square as a fin integral after all it's a sum of exponential of imaginary numbers okay and get something out of that all right I mean and if you if you do that you will find out that it's it

shows you how subtle the find integral is see because here I have a sum of exponentials of I Pi / 2 * gamma of k k okay essentially I mean you know up to a small variant and it all depends on that and if you look at the F man integral you will find that it's extremely subtle because of the I Epsilon prescription of fan which many people totally

Overlook and which is in fact a way to VI rotate so if you want when you when you hear people talking about the fan integral in general they don't know what they are talking about because of this iepsilon prescription you know because of the of the V rotation so um there are plenty of you know things which are very interesting to learn

about and okay which uh which are connected to that and uh I think you know it's um I mean I remember also there was a conference in movi recently um and about grend I mean about topos Theory and all that and I mean people were saying that um you know gend was a like I mean they were not realizing that goik had

been inspired of course Very importantly in his work by the veil conjecture I mean you know people get inspired by problems it's very important Kum was when he discovered ideal numbers of course he was inspired by the faat problem so I mean you know there are all these reasons to uh I mean you know to to get some strong inspiration so there are these two sources of inspiration there is a source of galwa like you know great ideas which are fantastic power and stumbling blocks you know problems that you cannot solve because okay if we could solve every problem because this would be so boring I mean you know but it's not the case and there are stumbling blocks and there are stumbling blocks which are so great you know that by their by their strength they force

you to go deeper and deeper and deeper in the stuff I mean I think that's that's a richness you know I think I mean these type of problems which are you know sort of which have fascinated people and they are fascinating exactly because of this because they are you know they are and there is a of course the wrong side of the medal the wrong side is the following is that it was proved by a CH in that if you

list the mathematical truths namely the things which are true okay and among these you ask which ones are provable the proportion tends to [Laughter] zero so of course this is a warning you know that's a that's a big warning that we need to be we need to be careful about such things but yeah it contradicts Hilbert's EP yeah of course and it but it

contradicts it in a very concrete manner you see godal theorem is can be taken as abstract and so and you can say wow whatever I will work on it will be a problem which is solvable no I mean what chaitin gives you is you know a statement which uh and and probably what it says is that you have to use fancy stuff because for instance you know there are these statements which are improvable in piano arithmetic but which
are true when you use zfc but of course then you have to use fancy stuff so I mean okay we have to use something like the DI yes yes okay guys so thanks a lot for listening thanks a lot for your attention yes yeah yeah you have some question some more yes please more questions any comments if I could ask I would like to
ask some questions ah yes I was I was so sorry not to hear you you know yes because nor you have lots of nice questions yes please please I'm not sure about my internet connection so forgive me if if would it's okay I can hear you yes yeah okay so I would ask you about this uh behavior of this uh function accounting function so this grows like time log e so what what is the right way to think it about
it conceptually okay let me explain let me answer let me answer this is the part which I didn't show this is the part of the itat expansion for Thea you see what you are saying is that if it were a spectral triple of the usual kind if it grows like e what does it mean on the eat expansion what does it mean on the dimension Spectrum what it means on the dimension spectrum is that one is in the
dimension Spectrum but it has a pole of order two that's what it means and in fact I didn't show in my slides but I published an article on that that you can actually compute the first terms of the expansion in fact for Zeta you can compute all the terms of the expansion and you can compare it with the prolate that what we did with ar and you get the same terms and these terms are extremely subtle because the coefficient that you
get for the double pole it's something like you have the log of two pi and you have the ER constant and all that so it's it's really quite subtle yeah so this is the answer to your question so it means if you want of course it's not a spectral Triple A Circle it's something quite more subtle because of this double pole so this Dimension spectrum is like one plus Epsilon or one exctly yeah no
no plus plus it's one plus Epsilon yeah okay and another thing related to this spectrum is the fact that you have some spectr triple which is defined on the whole line but nevertheless the dur operator has Compact resolvent and this is that's okay no no that's okay because when you look at the corresponding metric you have to look at the corresponding metric and the corresponding metric is not the metric
of the usual line so I mean you know so this is the difference when you look at the Associated metric you will find that it doesn't look like a line you see there is a singular point and so on so so I mean this all depends on the on the associated metric yes yeah but that's a good question yes okay okay thank you and maybe maybe one last question uh uh related to your paper about the
second quantization and also of Fons this this notion of second quanti and as far as I remember also a function appeared there but I don't know whether connection yeah you are right of course no there what the it's like you know there is a devil which puts some some data function boom like it pops up so there it popped up for totally different
reason it popped up because of the fact that it gave if you want a very specific test functions to apply the spectral action and then when one was Computing the coefficient that one gets from this spectral action it was delivering the value of the data function at all integers okay which is a different story yeah so this was a this was quite surprising of course yeah extremely surprising

sure okay thank you other questions or comments may I ask a question okay please ask I have one question about your arithmetic site oh yes okay so there there were some approaches to analog of data function uh in characterist in positive characteristic in albic Geometry your arithmetic geometry and the solution to the problem was introducing intersection
theory on on arithmetic surface yes so now you have something of geometry your your arithmetic site with a place at Infinity yeah here something no no of course of course let me answer immediately so the answer is the following the answer is that what replaces the curve is what I showed uh namely it is a scaling site scaling site as a visible part which is
exactly the same as what happens in the case of finite characteristic with the points of the curve over the algebraic closure of the finite field it looks exactly right but it has a flesh around that because it's a noncommutative space that flesh is absolutely necessary otherwise you wouldn't get the correct fail terms which because they are more subtile cannot be obtained as in the case of
function Fields now what replaces the square you are right to mention the square of the curve what replaces the square of the curve and the analysis on the square of the curves are the Schwartz kernels of operators operating on the scaling side and among these Schwarz kernel there is the Schwarz Kel for the scaling operator which plays exactly the same role as the frin
correspondence in the function field so we have a full dictionary between at the geometric level which what corresponds to what and so on so if you want at the geometric level especially now after this business with the covers and all that we have really a full dictionary because with the covers what we get is not only the points over the algebraic closure of the ground field but we also get the aban extensions so if you want
at the geometric level we have the picture okay and we also have the what replaces if you want the intersection number of of divisors and all that but okay what is so if you want specific about the archimedian place all right is what requires all this analytic work so all the analytic work is coming from the uniqueness of the archimedian place and what the part of my talk which I
didn't show is entitled a first glance at the archimedian place in the sense that we hear the archimedian place from the it expansion and from this hearing we can guess what it looks like and and and and you know so this is another talk but in Cortona I gave this St of so the sound of infinity the sound of the archimedian place okay can we hear the shape of the
archan and we can that's okay maybe one more question how how could you compare this your approach with this uh philosophic IAL approach of defining everything over one element field no but that's the same so if you want no but it's the same because in what we do with Kya we were Guided by two things if you want the arithmetic site is like the
ultimate geometric point but the ultimate coefficients are what we developed with Kya namely coming from the gamma rings of seal and from the spectrum and so it's a combinatorial counterpart which now is gaining ground among the people working topology I mean with kadia we have a lot of trouble you know having this accepted but we wrote a paper which is very comprehensive which is in the honor of
manin and in which the title of the paper is inspired from andreil and it's called on the metaphysics of [Laughter] F1 this beautiful statements from Andre veale about the metaphysics of of the equations at the time of lrange before galwa on the metaphysics of this and that and F1 for a long time has been in the state of metaphysics but with
Kya okay we believe that we have found the ultimate coefficients and that they will gain ground they will gain ground it take time but they will gain if had a comment related to this point is that this uh idea that Vale had that function field as you know all I think at this point function field and

number field should be comparable in the sense of uh in the number field case to have an absolute the as we say now theory of
coefficients uh involved what alen spoke about today with the with a scaling site and it's a a billion cover abolute a billian cover uh so this is the geometric part if if we were in ajb Geometry we would have we would say the topological space like in scheme Theory we have a topological space and the Shi of structure Shi so uh what Alain spoke about today is a vast generalization of the notion of scheme but for the
topological space for the theory of efficience uh so what topologist have developed with the sphere spectrum is very interesting because uh it's characteristic free so when uh when we speak about the archimedian place in a way we are o over the archimedian play I mean with with be with the theory of uh uh of F1 as originally thought with a important structure uh that they sort of enlarge the category of uh um uh commutative rings in particular Z so we uh we we went below Z or in in a way categorical meant extend the category of of commutative rings with the with the EM Po and semi semi rings and in particular there is at the bottom this uh this Mudan but the boan has still in it has a very nice point of view which is something below the archimedian place in a way which recast information of the
archimedian place but the Boolean has still a characteristic attached to it so it's what we say characteristic one or what algebraist would call Iden poent structure but if we seek for absolute geometry in terms of coefficients it should be characteristic free and so this is what through we went through in the long uh the tour uh with the by implementing the some idea that green
Seagal had since the late 60 with this uh gamma Rings or gamma sets in particular so so these are structure that understood that the categorical level without moving to the homotopic more sophisticated viewpoint Point are very very good to represent an absolute theory of coefficients and in particular because any Cate any classical type of category that we use in algebra geometry like commutative Rings or even monoid
semi Rings they all fully Faithfully embed in this new vision where the set where the classical algebra in terms of sets with operation is replaced by a theor of categorical framework categorical type of construction with the fs and and operators among F so this is very very interesting and I think uh here here we have a group a strong group in otop theory at John Hopkins and they start to realize the importance of this
of this understanding basic understanding of the sphere Spectrum uh so they take immed medely the ototopical Viewpoint because they are interested in having a pi zero which is a group excuse me but um we instead we are concerned more at really fundamental understanding of this uh important uh structure in aotop theory which is the SE Spectrum in fact the localization of the spear Spectrum so to
have an H some something which is characteristic free that has to be Associated to the geometry that is represented by by the sking side of the space as us today excuse me to be so long thank you very much okay so so now we see how wide is the vision of alen of Quantic metaphysics so thank you very much for beautiful talk thank you guys thank you
guys I'm really grateful you thank you LLY thank you ludik yes L still L thank you okay so ludik will raise now a toast oh very good guys yes thank you so much thank you for coming thank you everybody thank you guys yes thank you