Des carrés écossais dans le plan complexe, Denise Vella-Chemla, août 2025

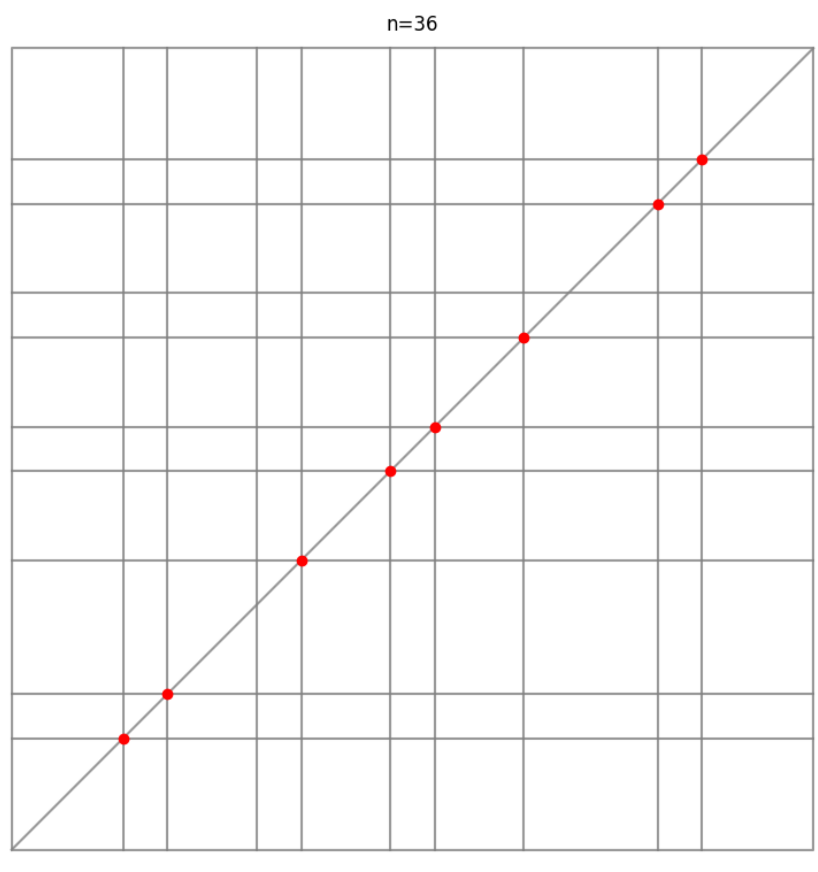
1. Présentation du problème

Toujours à la recherche d'une justification de la conjecture de Goldbach, on réfléchit à la façon de visualiser les décomposants de Goldbach dans le plan complexe.

On utilise un carré de côté de longueur n. Sur l'axe des abscisses x sont positionnés les nombres premiers impairs p_k de l'intervalle fermé $[3, n-3]$ et qui ne divisent pas n (on note l'ensemble de ces nombres premiers $\mathcal{P}_n$).

On trace un réseau de droites dont les droites verticales ont pour équations $x = p_k$ pour chacun des nombres premiers de $\mathcal{P}_n$. On trace également des droites horizontales $y = n - p_k$; ce choix a pour conséquence que si l'on souhaite énumérer les nombres premiers de $\mathcal{P}_n$ le long des axes des coordonnées, on doit le faire comme habituellement de gauche à droite sur l'axe des abscisses, mais de haut en bas sur l'axe des ordonnées y (en s'approchant de plus en plus de 0).

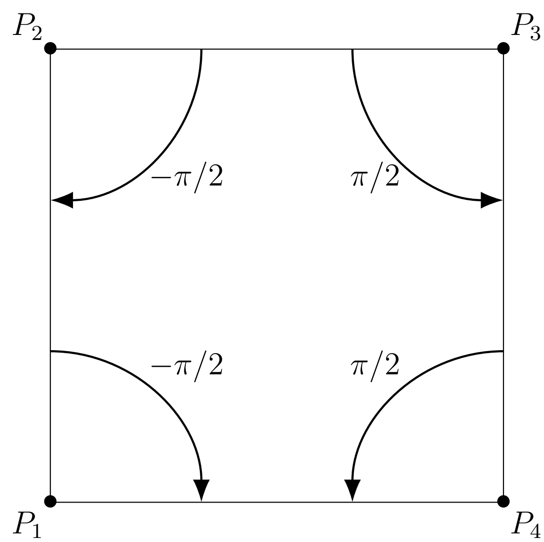
Donnons un exemple : pour $n = 36$, le seul nombre premier "manquant" dans $\mathcal{P}_n$, car il ne divise pas n est 3. Les autres nombres premiers de l'intervalle $[3, 33]$ sont ceux de la liste : $[5, 7, 11, 13, 17, 19, 23, 29, 31]$.



2. Première idée : les rotations autour des sommets du carré

On va ici particulariser 4 rotations du plan complexe, que l'on représente schématiquement sur le graphique ci-dessous :

- la rotation R_1 autour de $P_1 = (0,0)$ d'angle $\alpha = -\pi/2$;
- la rotation R_2 autour de $P_2 = (0,n)$ d'angle $\alpha = -\pi/2$;
- la rotation R_3 autour de $P_3 = (n,n)$ d'angle $\alpha = \pi/2$;
- la rotation R_4 autour de $P_4 = (n,0)$ d'angle $\alpha = \pi/2$;



Dans le plan complexe, la rotation de $z \in \mathbb{C}$ autour du point $a \in \mathbb{C}$ d'un angle α s'écrit

$$R_{a,\alpha}(z) = e^{i\alpha}(z-a) + a.$$

1) Pour $z = p + iq$ avec p et q premiers, $a = P_1 = 0$ et $\alpha = -\pi/2$, on a

$$\begin{aligned} R_{0,-\pi/2}(z) &= (-i)((p+iq)-0)+0 \\ &= -i(p+iq) \\ &= q - ip. \end{aligned}$$

2) Pour $z = p + iq$ avec p et q premiers, $a = P_2 = ni$ et $\alpha = -\pi/2$, on a

$$\begin{aligned} R_{ni,-\pi/2}(z) &= (-i)(z-ni)+ni \\ &= -i(p+iq) - n + ni \\ &= -ip + q - n + ni \\ &= -(n-q) + (n-p)i. \end{aligned}$$

3) Pour $z = p + iq$ avec p et q premiers, $a = P_3 = n + ni$ et $\alpha = \pi/2$, on a

$$\begin{aligned} R_{n+ni,\pi/2}(z) &= i(z-(n+ni)) + (n+ni) \\ &= i(p+iq) - i(n+ni) + n + ni \\ &= ip - q - in + n + n + ni \\ &= (2n-q) + pi. \end{aligned}$$

4) Pour $z = p + iq$ avec p et q premiers, $a = P_4 = n$ et $\alpha = \pi/2$, on a

$$\begin{aligned} R_{n,\pi/2}(z) &= i(z-n)+n \\ &= i(p+iq) - in + n \\ &= ip - q - in + n \\ &= (n-q) - (n-p)i. \end{aligned}$$

Si on préfère considérer les coordonnées dans le plan euclidien, la rotation de $\begin{pmatrix} x \\ y \end{pmatrix} \in \mathbb{Z}^2$ autour du point $\begin{pmatrix} a \\ b \end{pmatrix} \in \mathbb{Z}^2$ d'un angle α s'écrit

$$R_{a,\alpha}(z) = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} x-a \\ y-b \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix}.$$

Pour $P_1 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ et $\alpha = -\pi/2$, on a

$$\begin{aligned} R_{0,-\pi/2}(z) &= \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} x-0 \\ y-0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \end{pmatrix} \\ &= \begin{pmatrix} y \\ -x \end{pmatrix}. \end{aligned}$$

Pour $P_2 = \begin{pmatrix} 0 \\ n \end{pmatrix}$ et $\alpha = -\pi/2$, on a

$$\begin{aligned} R_{(0,n),-\pi/2}(z) &= \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} x-0 \\ y-n \end{pmatrix} + \begin{pmatrix} 0 \\ n \end{pmatrix} \\ &= \begin{pmatrix} y-n \\ -x \end{pmatrix} + \begin{pmatrix} 0 \\ n \end{pmatrix} \\ &= \begin{pmatrix} y-n \\ -x+n \end{pmatrix}. \end{aligned}$$

Pour $P_3 = \begin{pmatrix} n \\ n \end{pmatrix}$ et $\alpha = \pi/2$, on a

$$\begin{aligned} R_{(n,n),\pi/2}(z) &= \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x-n \\ y-n \end{pmatrix} + \begin{pmatrix} n \\ n \end{pmatrix} \\ &= \begin{pmatrix} -y+n \\ x-n \end{pmatrix} + \begin{pmatrix} n \\ n \end{pmatrix} \\ &= \begin{pmatrix} -y+2n \\ x \end{pmatrix}. \end{aligned}$$

Pour $P_4 = \begin{pmatrix} n \\ 0 \end{pmatrix}$ et $\alpha = \pi/2$, on a

$$\begin{aligned} R_{(n,0),\pi/2}(z) &= \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x-n \\ y-0 \end{pmatrix} + \begin{pmatrix} n \\ 0 \end{pmatrix} \\ &= \begin{pmatrix} -y \\ x-n \end{pmatrix} + \begin{pmatrix} n \\ 0 \end{pmatrix} \\ &= \begin{pmatrix} -y+n \\ x-n \end{pmatrix}. \end{aligned}$$

En résumé, les raisonnements similaires ci-dessus ont permis d'obtenir les résultats suivants pour les 4 rotations que l'on souhaite considérer :

- la rotation R_1 est telle que $R_1(p+iq) = q - ip$;
- la rotation R_2 est telle que $R_2(p+iq) = -(n-q) + i(n-p)$;
- la rotation R_3 est telle que $R_3(p+iq) = (n-q) - i(n-p)$;
- la rotation R_4 est telle que $R_4(p+iq) = (2n-q) + ip$.

Par programme, on constate que selon le choix de rotations qu'on a effectué, les décomposants de Goldbach de n sont les points qui ont leur image par la composée des deux rotations R_1 et R_2 qui appartient à l'ensemble des points du réseau.

$$p + iq \overset{R_2}{\mapsto} -(n-q) + i(n-p) \overset{R_1}{\mapsto} (n-p) + (n-q)i$$

Au contraire, les points qui ne sont pas des décomposants de Goldbach de n ont leur image par cette composition de rotations $R_1 \circ R_2$ qui n'appartient pas au réseau initial de points.

Pour l'exemple $n = 36$ dont on a fourni l'illustration, les images des points du réseau sont fournies ci-dessous. On a noté en regard des images celles qui ne sont pas des points du réseau (qu'on appelle M) car la partie réelle ou imaginaire de l'image en question est un nombre composé :

$5+5i \mapsto 31+31i$ $5+13i \mapsto 31+23i$ $5+23i \mapsto 31+13i$	$5+7i \mapsto 31+29i$ $5+17i \mapsto 31+19i$ $5+29i \mapsto 31+7i$	$5+11i \mapsto 31+25i$ ($\notin M$) $5+19i \mapsto 31+17i$ $5+31i \mapsto 31+5i$
$7+5i \mapsto 29+31i$ $7+13i \mapsto 29+23i$ $7+23i \mapsto 29+13i$	$7+7i \mapsto 29+29i$ $7+17i \mapsto 29+19i$ $7+29i \mapsto 29+7i$	$7+11i \mapsto 29+25i$ ($\notin M$) $7+19i \mapsto 29+17i$ $7+31i \mapsto 29+5i$
$11+5i \mapsto 25+31i$ ($\notin M$) $11+13i \mapsto 25+23i$ ($\notin M$) $11+23i \mapsto 25+13i$ ($\notin M$)	$11+7i \mapsto 25+29i$ ($\notin M$) $11+17i \mapsto 25+19i$ ($\notin M$) $11+29i \mapsto 25+7i$ ($\notin M$)	$11+11i \mapsto 25+25i$ ($\notin M$) $11+19i \mapsto 25+17i$ ($\notin M$) $11+31i \mapsto 25+5i$ ($\notin M$)
$13+5i \mapsto 23+31i$ $13+13i \mapsto 23+23i$ $13+23i \mapsto 23+13i$	$13+7i \mapsto 23+29i$ $13+17i \mapsto 23+19i$ $13+29i \mapsto 23+7i$	$13+11i \mapsto 23+25i$ ($\notin M$) $13+19i \mapsto 23+17i$ $13+31i \mapsto 23+5i$
$17+5i \mapsto 19+31i$ $17+13i \mapsto 19+23i$ $17+23i \mapsto 19+13i$	$17+7i \mapsto 19+29i$ $17+17i \mapsto 19+19i$ $17+29i \mapsto 19+7i$	$17+11i \mapsto 19+25i$ ($\notin M$) $17+19i \mapsto 19+17i$ $17+31i \mapsto 19+5i$
$19+5i \mapsto 17+31i$ $19+13i \mapsto 17+23i$ $19+23i \mapsto 17+13i$	$19+7i \mapsto 17+29i$ $19+17i \mapsto 17+19i$ $19+29i \mapsto 17+7i$	$19+11i \mapsto 17+25i$ ($\notin M$) $19+19i \mapsto 17+17i$ $19+31i \mapsto 17+5i$
$23+5i \mapsto 13+31i$ $23+13i \mapsto 13+23i$ $23+23i \mapsto 13+13i$	$23+7i \mapsto 13+29i$ $23+17i \mapsto 13+19i$ $23+29i \mapsto 13+7i$	$23+11i \mapsto 13+25i$ ($\notin M$) $23+19i \mapsto 13+17i$ $23+31i \mapsto 13+5i$
$29+5i \mapsto 7+31i$ $29+13i \mapsto 7+23i$ $29+23i \mapsto 7+13i$	$29+7i \mapsto 7+29i$ $29+17i \mapsto 7+19i$ $29+29i \mapsto 7+7i$	$29+11i \mapsto 7+25i$ ($\notin M$) $29+19i \mapsto 7+17i$ $29+31i \mapsto 7+5i$
$31+5i \mapsto 5+31i$ $31+13i \mapsto 5+23i$ $31+23i \mapsto 5+13i$	$31+7i \mapsto 5+29i$ $31+17i \mapsto 5+19i$ $31+29i \mapsto 5+7i$	$31+11i \mapsto 5+25i$ ($\notin M$) $31+19i \mapsto 5+17i$ $31+31i \mapsto 5+5i$

On ne sait pas démontrer que la diagonale principale d'équation $x = y$ contient au moins un point du réseau M des points de coordonnées deux nombres premiers impairs ne divisant pas n.

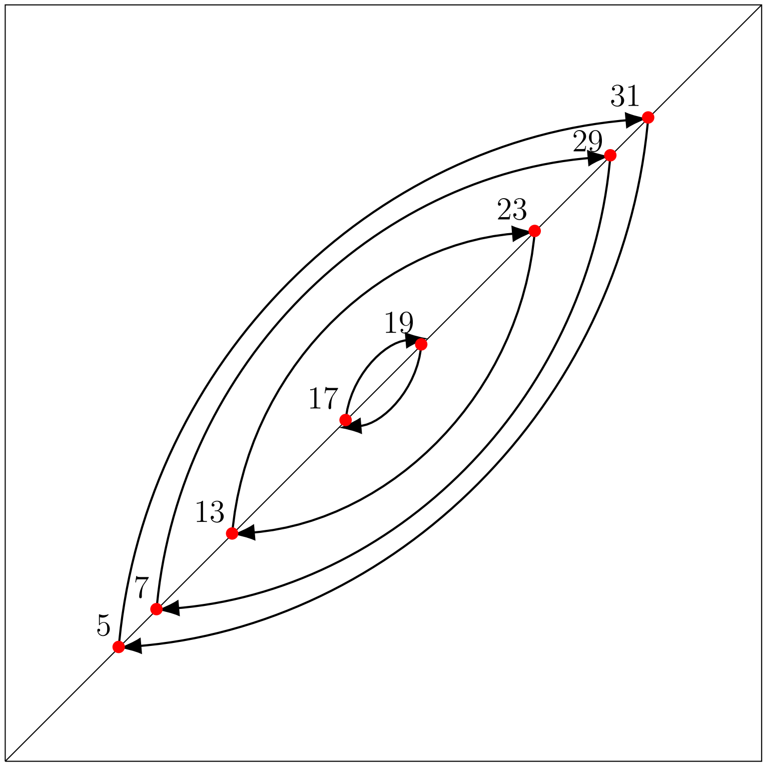
3. Seconde idée : rotations autour de centres appartenant à la diagonale "descendante"

En étudiant la figure 1, on voit qu'il s'agit, à la recherche de décomposants de Goldbach de n, de trouver qu'il existe forcément un point de la diagonale SO-NE, qui est fixe lorsqu'on lui applique la composition de deux rotations de centres deux sommets opposés par rapport au centre de la diagonale NO-SE et d'angle $\pi/2$.

On expérimente cette idée en utilisant le programme fourni en annexe 1.

Le résumé de son résultat sous forme d'un tableau des points et de leur image est fourni en annexe 2.

On montre sur un schéma expurgé ce qu'il advient des décomposants de Goldbach que l'on trouve sur la diagonale "ascendante", par les rotations qui les fixent (on a placé les points n'importe où).



4. Composition des rotations : cas général

La composée de 2 rotations planes est soit une rotation, soit une translation [1].

En effet, appliquer d'abord la rotation de centre a d'angle α, puis celle de centre b d'angle β, s'écrit

$$\begin{aligned}(R_{b,\beta} \circ R_{a,\alpha})(z) &= R_{b,\beta}(R_{a,\alpha}(z)) \\ &= R_{b,\beta}\left(e^{i\alpha}(z-a)+a\right) \\ &= e^{i\beta}\left(e^{i\alpha}(z-a)+a-b\right)+b \\ &= e^{i(\alpha+\beta)}(z-a)+e^{i\beta}(a-b)+b \\ &= e^{i(\alpha+\beta)}(z-a)+R_{b,\beta}(a).\end{aligned}$$

Pour écrire cette transformation sous la forme $e^{i(\alpha+\beta)}(z-c)+c$, il faut donc que

$$c\left(1-e^{i(\alpha+\beta)}\right) = R_{b,\beta}(a) - ae^{i(\alpha+\beta)}.$$

On doit distinguer 2 cas :

a) $\alpha+\beta = 0 \pmod{2\pi} \implies e^{i(\alpha+\beta)} = 1$, et la composée des 2 rotations est une translation de vecteur $R_{b,\beta}(a) - a$;

[1] Cf. https://fr.wikipedia.org/wiki/Rotation$_plane$.

b) $\alpha+\beta \neq 0 \pmod{2\pi} \implies e^{i(\alpha+\beta)} \neq 1$, et la composée des 2 rotations est une rotation d'angle $\alpha+\beta$ et de centre

$$c = \frac{R_{b,\beta}(a) - ae^{i(\alpha+\beta)}}{1-e^{i(\alpha+\beta)}}.$$

Remarque : l'ensemble formé de toutes les rotations planes et de toutes les translations, muni de la loi de composition interne forme un groupe non commutatif appelé le groupe des isométries directes.

5. Cherchons les centres

La mise en œuvre de la seconde idée nécessite de combiner systématiquement une rotation R_k de centre $C_k = p_k + (n-p_k)i$ d'angle $\pi/2$ à la rotation "inverse" R'_k de centre $C_{k'} = (n-p_k)+p_k i$ d'angle $\pi/2$.

Pour $z = p+iq$ avec p et q premiers, l'image de z par la composée des 2 rotations $R_k \circ R'_k$ ci-dessus est :

$$\begin{aligned}(R_k \circ R'_k)(z) &= i((i((p+iq)-C_k)+C_k)-C'_k)+C'_k \\ &= i(ip-q-iC_k+C_k-C'_k)+C'_k \\ &= -p-iq+C_k+iC_k-iC'_k+C'_k \\ &= (C_k+C'_k-p)+(C_k-C'_k-q)i.\end{aligned}$$

La combinée des deux rotations sera une rotation d'angle π et de centre $\dfrac{C_k+C'_k}{2} = \dfrac{n+ni}{2}$.

Le problème de cette seconde idée est que les décomposants de Goldbach *et* les non décomposants de Goldbach de n sont fixes par les composées des deux rotations par rapport à des centres symétriques l'un de l'autre par rapport à la diagonale "montante" (SO-NE).

6. Symétries par rapport aux diagonales

Pour mémoire, on rappelle que dans le plan complexe, l'image du point d'affixe z par une symétrie par rapport à la droite déterminée par les points a et b est $z' = \alpha\overline{z}+\beta$ avec $\alpha = \dfrac{a-b}{\overline{a}-\overline{b}}$ et $\beta = \dfrac{b\overline{a}-a\overline{b}}{\overline{a}-\overline{b}}$.

La diagonale NO-SE est déterminée par les points $a = ni$ et $b = n$.

On a pour cette diagonale $\alpha = \dfrac{ni-n}{-ni-n}$, $\beta = \dfrac{n(-ni)-(ni)n}{-ni-n} = \dfrac{-2n^2i}{-ni-n} = \dfrac{2n^2i}{n(i+1)}$.

La diagonale SO-NE est déterminée par les points $a = 0$ et $b = n+ni$.

On a pour cette diagonale $\alpha = \dfrac{-n-ni}{-n+ni}$, $\beta = 0$.

Malheureusement, on n'arrive pas à trouver une idée, dans la mise en œuvre de laquelle interviendraient ces symétries par rapport aux diagonales, et qui permettrait de distinguer les nombres premiers des nombres composés dans le réseau des points.

Annexe 1 : programme utilisé pour les expérimentations

```
import math
from math import sqrt
import numpy as np
import matplotlib.pyplot as plt
from numpy import exp, pi

def prime_sieve(N):
     is_prime = np.full(N, True)
     is_prime[:2] = False
     for p in range(2, math.isqrt(N) + 1):
          if is_prime[p]:
               is_prime[p*p::p] = False
     return np.nonzero(is_prime)[0]

class Primes():
     def __init__(self, N):
          self.__primes = prime_sieve(N)
     def __str__(self):
          return str(self.__primes)
     def __iter__(self):
          return iter(self.__primes)
     def __len__(self):
          return self.__primes.size
     def __getitem__(self, k):
          return self.__primes[k]
     def __contains__(self, x):
          k = self.index(x)
          return k < len(self) and self.__primes[k] == x
     def index(self, x):
          return np.searchsorted(self.__primes, x, side='left')
     def count(self, x):
          return np.searchsorted(self.__primes, x, side='right')
     def range(self, start, stop, step=1):
          return self.__primes[self.index(start):self.index(stop):step]
     def factors(self, n):
          if n in self:
               return np.array([n])
          else:
               P = self.range(2, n//2 + 1)
               return P[n % P == 0]

def rota(point, centre, angle):
     return((point-centre)*exp(1j*angle)+centre)

def syme(z,a,b):
     abar = a.real-a.imag*1j
     bbar = b.real-b.imag*1j
     zbar = z.real-z.imag*1j
     aaalpha = (a-b)/(abar-bbar)
     bbbeta = (b*abar-a*bbar)/(abar-bbar)
     return(aaalpha*zbar+bbbeta)
```

```
N = 104
P = Primes(N+1)
for n in range(36, 38, 2):
     trouve = False
     _,ax = plt.subplots(figsize=(10,10))
     milieu = n//2
     lf = P.factors(n)
     l1 = P.range(3, n - 3 + 1)
     l1 = np.array([x for x in l1 if x not in lf or x == milieu])
     l2 = n - l1
     listecomplexes = []
     for x in range(len(l1)):
          for y in range(len(l1)):
               listecomplexes.append(l1[x]+1j*l1[y])
     print('liste complexes = ',listecomplexes)
     for k in range(len(l1)):
          centre1 = l1[k]+(n-l1[k])*1j
          centre2 = (n-l1[k])+l1[k]*1j
          print('1 = ',centre1,' centre2 = ',centre2)
          for z in listecomplexes:
               print(z,' image :  ',rota(rota(z,centre1,pi/2),centre2,pi/2))
     lg = np.array([x in P for x in l2])
     print(f' n = n (facteurs = lf) '.center(80, '_') + f'1 = l1'+ f'2 = l2')
     print('lg = [',end='')
     for k in range(len(lg)):
          if lg[k]:
               print(l1[k],' ',end='')
     print(']')
     for k in range(len(l1)):
          plt.plot([l1[k],l1[k]],[0,n],color='gray',alpha=0.8,zorder=0)
          plt.plot([0,n],[n-l1[k],n-l1[k]],color='gray',alpha=0.8,zorder=0)
     plt.plot([n,l1[-1]],[n,l1[-1]],color='gray',alpha=0.8,zorder=0)
     plt.plot([0,l1[0]],[0,l1[0]],color='gray',alpha=0.8,zorder=0)
     plt.plot([0,n,n,0,0],[0,0,n,n,0],color='gray',alpha=0.8,zorder=0)
     plt.plot(l1,l1,color='gray',alpha=0.8,zorder=0)
     for x in range(len(l1)):
          plt.annotate(l1[x],xy=(l1[x]-1,-2),fontsize=7)
     plt.scatter(l1[lg],l1[lg],marker='o',s=36,color='red',zorder=0)
     plt.title('n='+str(n))
     plt.axis('equal')
     plt.xlim(-2,n+2)
     plt.ylim(-5,n+2)
     plt.show()
     nomfic ='carreRQ'+str(n)
     plt.savefig(nomfic)
     plt.close()
```

Annexe 2 : résultat du programme d'expérimentation

Dans les tableaux de calculs des images des points du réseau M ci-après, on a noté en rouge les points de la forme $p+pi$, notamment pour les décomposants de Goldbach de $n = 36 (5, 7, 13, 17, 19, 23, 29, 31), pour vérifier qu'ils sont fixés par l'application.$

centre1 $= 5+31i$; centre2 $= 31+5i$		
$5+5i \mapsto 5+5i$ $5+13i \mapsto 5-3i$ $5+23i \mapsto 5-13i$	$5+7i \mapsto 5+3i$ $5+17i \mapsto 5-7i$ $5+29i \mapsto 5-19i$	$5+11i \mapsto 5-i$ $5+19i \mapsto 5-9i$ $5+31i \mapsto 5-21i$
$7+5i \mapsto 3+5i$ $7+13i \mapsto 3-3i$ $7+23i \mapsto 3-13i$	$7+7i \mapsto 3+3i$ $7+17i \mapsto 3-7i$ $7+29i \mapsto 3-19i$	$7+11i \mapsto 3-i$ $7+19i \mapsto 3-9i$ $7+31i \mapsto 3-21i$
$11+5i \mapsto -1+5i$ $11+13i \mapsto -1-3i$ $11+23i \mapsto -1-13i$	$11+7i \mapsto -1+3i$ $11+17i \mapsto -1-7i$ $11+29i \mapsto -1-19i$	$11+11i \mapsto -1-i$ $11+19i \mapsto -1-9i$ $11+31i \mapsto -1-21i$
$13+5i \mapsto -3+5i$ $13+13i \mapsto -3-3i$ $13+23i \mapsto -3-13i$	$13+7i \mapsto -3+3i$ $13+17i \mapsto -3-7i$ $13+29i \mapsto -3-19i$	$13+11i \mapsto -3-i$ $13+19i \mapsto -3-9i$ $13+31i \mapsto -3-21i$
$17+5i \mapsto -7+53i$ $17+13i \mapsto -7-3i$ $17+23i \mapsto -7-13i$	$17+7i \mapsto -7+3i$ $17+17i \mapsto -7-7i$ $17+29i \mapsto -7-19i$	$17+11i \mapsto -7-73i$ $17+19i \mapsto -7-9i$ $17+31i \mapsto -7-21i$
$19+5i \mapsto -9+53i$ $19+13i \mapsto -9-3i$ $19+23i \mapsto -9-13i$	$19+7i \mapsto -9+37i$ $19+17i \mapsto -9-7i$ $19+29i \mapsto -9-19i$	$19+11i \mapsto -9-73i$ $19+19i \mapsto -9-9i$ $19+31i \mapsto -9-21i$
$23+5i \mapsto -13+53i$ $23+13i \mapsto -13-3i$ $23+23i \mapsto -13-13i$	$23+7i \mapsto -13+3i$ $23+17i \mapsto -13-7i$ $23+29i \mapsto -13-19i$	$23+11i \mapsto -13-73i$ $23+19i \mapsto -13-9i$ $23+31i \mapsto -13-21i$
$29+5i \mapsto -19+53i$ $29+13i \mapsto -19-3i$ $29+23i \mapsto -19-13i$	$29+7i \mapsto -19+33i$ $29+17i \mapsto -19-7i$ $29+29i \mapsto -19-19i$	$29+11i \mapsto -19-73i$ $29+19i \mapsto -19-9i$ $29+31i \mapsto -19-21i$
$31+5i \mapsto -21+53i$ $31+13i \mapsto -21-36i$ $31+23i \mapsto -21-13i$	$31+7i \mapsto -21+33i$ $31+17i \mapsto -21-7i$ $31+29i \mapsto -21-19i$	$31+11i \mapsto -21-6i$ $31+19i \mapsto -21-9i$ $31+31i \mapsto -21-21i$

centre1 $= 7+29i$; centre2 $= 29+7i$		
$5+5i \mapsto 9+9i$ $5+13i \mapsto 9+i$ $5+23i \mapsto 9-9i$	$5+7i \mapsto 9+71i$ $5+17i \mapsto 9-3i$ $5+29i \mapsto 9-15i$	$5+11i \mapsto 9+3i$ $5+19i \mapsto 9-5i$ $5+31i \mapsto 9-17i$
$7+5i \mapsto 7+9i$ $7+13i \mapsto 7+11i$ $7+23i \mapsto 7-9i$	$7+7i \mapsto 7+7i$ $7+17i \mapsto 7-3i$ $7+29i \mapsto 7-15i$	$7+11i \mapsto 7+3i$ $7+19i \mapsto 7-5i$ $7+31i \mapsto 7-17i$
$11+5i \mapsto 3+9i$ $11+13i \mapsto 3+11i$ $11+23i \mapsto 3-9i$	$11+7i \mapsto 3+7i$ $11+17i \mapsto 3-3i$ $11+29i \mapsto 3-15i$	$11+11i \mapsto 3+31i$ $11+19i \mapsto 3-5i$ $11+31i \mapsto 3-17i$
$13+5i \mapsto 1+9i$ $13+13i \mapsto 1+11i$ $13+23i \mapsto 1-9i$	$13+7i \mapsto 1+7i$ $13+17i \mapsto 1-3i$ $13+29i \mapsto 1-15i$	$13+11i \mapsto 1+31i$ $13+19i \mapsto 1-5i$ $13+31i \mapsto 1-17i$
$17+5i \mapsto -3+9i$ $17+13i \mapsto -3+11i$ $17+23i \mapsto -3-9i$	$17+7i \mapsto -3+7i$ $17+17i \mapsto -3-3i$ $17+29i \mapsto -3-15i$	$17+11i \mapsto -3+31i$ $17+19i \mapsto -3-5i$ $17+31i \mapsto -3-17i$
$19+5i \mapsto -5+9i$ $19+13i \mapsto -5+11i$ $19+23i \mapsto -5-9i$	$19+7i \mapsto -5+7i$ $19+17i \mapsto -5-3i$ $19+29i \mapsto -5-15i$	$19+11i \mapsto -5+3i$ $19+19i \mapsto -5-5i$ $19+31i \mapsto -5-17i$
$23+5i \mapsto -9+9i$ $23+13i \mapsto -9+i$ $23+23i \mapsto -9-9i$	$23+7i \mapsto -9+73i$ $23+17i \mapsto -9-3i$ $23+29i \mapsto -9-15i$	$23+11i \mapsto -9+3i$ $23+19i \mapsto -9-5i$ $23+31i \mapsto -9-17i$
$29+5i \mapsto -15+9i$ $29+13i \mapsto -15+i$ $29+23i \mapsto -15-95i$	$29+7i \mapsto -15+73i$ $29+17i \mapsto -15-36i$ $29+29i \mapsto -15-15i$	$29+11i \mapsto -15+3i$ $29+19i \mapsto -15-5i$ $29+31i \mapsto -15-17i$
$31+5i \mapsto -17+9i$ $31+13i \mapsto -17+i$ $31+23i \mapsto -17-95i$	$31+7i \mapsto -17+73i$ $31+17i \mapsto -17-36i$ $31+29i \mapsto -17-15i$	$31+11i \mapsto -17+3i$ $31+19i \mapsto -17-5i$ $31+31i \mapsto -17-17i$

centre1 $=$ $11+25i$; centre2 $=$ $25+11i$		
$5+5i \mapsto 17+17i$ $5+13i \mapsto 17+9i$ $5+23i \mapsto 17-1i$	$5+7i \mapsto 17+15i$ $5+17i \mapsto 17+51i$ $5+29i \mapsto 17-7i$	$5+11i \mapsto 17+11i$ $5+19i \mapsto 17+31i$ $5+31i \mapsto 17-9i$
$7+5i \mapsto 15+17i$ $7+13i \mapsto 15+9i$ $7+23i \mapsto 15-1i$	$7+7i \mapsto 15+15i$ $7+17i \mapsto 15+51i$ $7+29i \mapsto 158-7i$	$7+11i \mapsto 15+11i$ $7+19i \mapsto 15+31i$ $7+31i \mapsto 158-9i$
$11+5i \mapsto 11+17i$ $11+13i \mapsto 11+9i$ $11+23i \mapsto 11-1i$	$11+7i \mapsto 11+15i$ $11+17i \mapsto 11+51i$ $11+29i \mapsto 11-7i$	$11+11i \mapsto 11+11i$ $(*)$ $11+19i \mapsto 11+31i$ $11+31i \mapsto 11-9i$
$13+5i \mapsto 9+17i$ $13+13i \mapsto 9+9i$ $13+23i \mapsto 9-i$	$13+7i \mapsto 9+15i$ $13+17i \mapsto 9+51i$ $13+29i \mapsto 9-7i$	$13+11i \mapsto 9+11i$ $13+19i \mapsto 9+31i$ $13+31i \mapsto 9-9i$
$17+5i \mapsto 5+17i$ $17+13i \mapsto 5+9i$ $17+23i \mapsto 5-i$	$17+7i \mapsto 5+15i$ $17+17i \mapsto 5+51i$ $17+29i \mapsto 5-7i$	$17+11i \mapsto 5+11i$ $17+19i \mapsto 5+31i$ $17+31i \mapsto 5-9i$
$19+5i \mapsto 3+17i$ $19+13i \mapsto 3+9i$ $19+23i \mapsto 3-i$	$19+7i \mapsto 3+15i$ $19+17i \mapsto 3+5i$ $19+29i \mapsto 3-7i$	$19+11i \mapsto 3+11i$ $19+19i \mapsto 3+31i$ $19+31i \mapsto 3-9i$
$23+5i \mapsto -1+17i$ $23+13i \mapsto -1+9i$ $23+23i \mapsto -1-i$	$23+7i \mapsto -1+15i$ $23+17i \mapsto -1+5i$ $23+29i \mapsto -1-7i$	$23+11i \mapsto -1+11i$ $23+19i \mapsto -1+31i$ $23+31i \mapsto -1-9i$
$29+5i \mapsto -7+17i$ $29+13i \mapsto -7+9i$ $29+23i \mapsto -7-i$	$29+7i \mapsto -7+15i$ $29+17i \mapsto -7+5i$ $29+29i \mapsto -7-7i$	$29+11i \mapsto -7+11i$ $29+19i \mapsto -7+31i$ $29+31i \mapsto -7-9i$
$31+5i \mapsto -9+17i$ $31+13i \mapsto -9+9i$ $31+23i \mapsto -9-6i$	$31+7i \mapsto -9+15i$ $31+17i \mapsto -9+5i$ $31+29i \mapsto -9-7i$	$31+11i \mapsto -9+11i$ $31+19i \mapsto -9+31i$ $31+31i \mapsto -9-9i$

centre1 $=$ $13+23i$; centre2 $=$ $23+13i$		
$5+5i \mapsto 214+21i$ $5+13i \mapsto 21+13i$ $5+23i \mapsto 21+3i$	$5+7i \mapsto 214+19i$ $5+17i \mapsto 21+9i$ $5+29i \mapsto 21-3i$	$5+11i \mapsto 21+15i$ $5+19i \mapsto 21+7i$ $5+31i \mapsto 21-5i$
$7+5i \mapsto 19+21i$ $7+13i \mapsto 19+13i$ $7+23i \mapsto 19+3i$	$7+7i \mapsto 19+19i$ $7+17i \mapsto 19+9i$ $7+29i \mapsto 19-3i$	$7+11i \mapsto 19+15i$ $7+19i \mapsto 19+7i$ $7+31i \mapsto 19-5i$
$11+5i \mapsto 15+21i$ $11+13i \mapsto 15+13i$ $11+23i \mapsto 15+3i$	$11+7i \mapsto 15+19i$ $11+17i \mapsto 15+9i$ $11+29i \mapsto 158-3i$	$11+11i \mapsto 15+15i$ $11+19i \mapsto 15+71i$ $11+31i \mapsto 158-5i$
$13+5i \mapsto 13+21i$ $13+13i \mapsto 13+13i$ $13+23i \mapsto 13+3i$	$13+7i \mapsto 13+19i$ $13+17i \mapsto 13+9i$ $13+29i \mapsto 138-3i$	$13+11i \mapsto 13+15i$ $13+19i \mapsto 13+71i$ $13+31i \mapsto 138-5i$
$17+5i \mapsto 9+21i$ $17+13i \mapsto 9+13i$ $17+23i \mapsto 9+3i$	$17+7i \mapsto 9+19i$ $17+17i \mapsto 9+9i$ $17+29i \mapsto 98-3i$	$17+11i \mapsto 9+15i$ $17+19i \mapsto 9+71i$ $17+31i \mapsto 98-5i$
$19+5i \mapsto 7+21i$ $19+13i \mapsto 7+13i$ $19+23i \mapsto 7+31i$	$19+7i \mapsto 7+19i$ $19+17i \mapsto 7+9i$ $19+29i \mapsto 7-3i$	$19+11i \mapsto 7+15i$ $19+19i \mapsto 7+71i$ $19+31i \mapsto 7-5i$
$23+5i \mapsto 3+21i$ $23+13i \mapsto 3+13i$ $23+23i \mapsto 3+31i$	$23+7i \mapsto 3+19i$ $23+17i \mapsto 3+9i$ $23+29i \mapsto 3-3i$	$23+11i \mapsto 3+15i$ $23+19i \mapsto 3+71i$ $23+31i \mapsto 3-5i$
$29+5i \mapsto -3+21i$ $29+13i \mapsto -3+13i$ $29+23i \mapsto -3+33i$	$29+7i \mapsto -3+19i$ $29+17i \mapsto -3+9i$ $29+29i \mapsto -3-3i$	$29+11i \mapsto -3+15i$ $29+19i \mapsto -3+7i$ $29+31i \mapsto -3-5i$
$31+5i \mapsto -5+21i$ $31+13i \mapsto -5+13i$ $31+23i \mapsto -5+33i$	$31+7i \mapsto -5+19i$ $31+17i \mapsto -5+9i$ $31+29i \mapsto -5-3i$	$31+11i \mapsto -5+15i$ $31+19i \mapsto -5+7i$ $31+31i \mapsto -5-5i$

centre1 $=$ $17+19i$; centre2 $=$ $19+17i$		
$5+5i \mapsto 29+29i$ $5+13i \mapsto 29+21i$ $5+23i \mapsto 29+11i$	$5+7i \mapsto 29+27i$ $5+17i \mapsto 29+17i$ $5+29i \mapsto 29+5i$	$5+11i \mapsto 29+23i$ $5+19i \mapsto 29+15i$ $5+31i \mapsto 29+3i$
$7+5i \mapsto 27+29i$ $7+13i \mapsto 27+21i$ $7+23i \mapsto 27+11i$	$7+7i \mapsto 27+27i$ $7+17i \mapsto 27+17i$ $7+29i \mapsto 27+5i$	$7+11i \mapsto 27+23i$ $7+19i \mapsto 27+15i$ $7+31i \mapsto 27+3i$
$11+5i \mapsto 23+29i$ $11+13i \mapsto 23+21i$ $11+23i \mapsto 23+11i$	$11+7i \mapsto 23+27i$ $11+17i \mapsto 23+17i$ $11+29i \mapsto 23+5i$	$11+11i \mapsto 23+23i$ $11+19i \mapsto 23+15i$ $11+31i \mapsto 23+3i$
$13+5i \mapsto 21+29i$ $13+13i \mapsto 21+21i$ $13+23i \mapsto 21+11i$	$13+7i \mapsto 21+27i$ $13+17i \mapsto 21+17i$ $13+29i \mapsto 21+5i$	$13+11i \mapsto 21+23i$ $13+19i \mapsto 21+15i$ $13+31i \mapsto 21+3i$
$17+5i \mapsto 17+29i$ $17+13i \mapsto 17+21i$ $17+23i \mapsto 17+11i$	$17+7i \mapsto 17+27i$ $17+17i \mapsto 17+17i$ $17+29i \mapsto 17+5i$	$17+11i \mapsto 17+23i$ $17+19i \mapsto 17+15i$ $17+31i \mapsto 17+3i$
$19+5i \mapsto 15+29i$ $19+13i \mapsto 15+21i$ $19+23i \mapsto 15+11i$	$19+7i \mapsto 15+27i$ $19+17i \mapsto 15+17i$ $19+29i \mapsto 15+5i$	$19+11i \mapsto 15+23i$ $19+19i \mapsto 15+15i$ $19+31i \mapsto 15+3i$
$23+5i \mapsto 11+29i$ $23+13i \mapsto 11+21i$ $23+23i \mapsto 11+11i$	$23+7i \mapsto 11+27i$ $23+17i \mapsto 11+17i$ $23+29i \mapsto 11+5i$	$23+11i \mapsto 11+23i$ $23+19i \mapsto 11+15i$ $23+31i \mapsto 11+3i$
$29+5i \mapsto 5+29i$ $29+13i \mapsto 5+21i$ $29+23i \mapsto 5+11i$	$29+7i \mapsto 5+27i$ $29+17i \mapsto 5+17i$ $29+29i \mapsto 5+5i$	$29+11i \mapsto 5+23i$ $29+19i \mapsto 5+15i$ $29+31i \mapsto 5+3i$
$31+5i \mapsto 3+29i$ $31+13i \mapsto 3+21i$ $31+23i \mapsto 3+11i$	$31+7i \mapsto 3+27i$ $31+17i \mapsto 3+17i$ $31+29i \mapsto 3+5i$	$31+11i \mapsto 3+23i$ $31+19i \mapsto 3+15i$ $31+31i \mapsto 3+31i$

centre1 $=$ $19+17i$; centre2 $=$ $17+19i$		
$5+5i \mapsto 33+33i$ $5+13i \mapsto 33+25i$ $5+23i \mapsto 33+15i$	$5+7i \mapsto 33+31i$ $5+17i \mapsto 33+21i$ $5+29i \mapsto 33+9i$	$5+11i \mapsto 33+27i$ $5+19i \mapsto 33+19i$ $5+31i \mapsto 33+7i$
$7+5i \mapsto 31+33i$ $7+13i \mapsto 31+25i$ $7+23i \mapsto 31+15i$	$7+7i \mapsto 31+31i$ $7+17i \mapsto 31+21i$ $7+29i \mapsto 31+9i$	$7+11i \mapsto 31+27i$ $7+19i \mapsto 31+19i$ $7+31i \mapsto 31+7i$
$11+5i \mapsto 27+33i$ $11+13i \mapsto 27+25i$ $11+23i \mapsto 27+15i$	$11+7i \mapsto 27+31i$ $11+17i \mapsto 27+21i$ $11+29i \mapsto 27+9i$	$11+11i \mapsto 27+27i$ $11+19i \mapsto 27+19i$ $11+31i \mapsto 27+7i$
$13+5i \mapsto 25+33i$ $13+13i \mapsto 25+25i$ $13+23i \mapsto 25+15i$	$13+7i \mapsto 25+31i$ $13+17i \mapsto 25+21i$ $13+29i \mapsto 25+9i$	$13+11i \mapsto 25+27i$ $13+19i \mapsto 25+19i$ $13+31i \mapsto 25+7i$
$17+5i \mapsto 21+33i$ $17+13i \mapsto 21+25i$ $17+23i \mapsto 21+15i$	$17+7i \mapsto 21+31i$ $17+17i \mapsto 21+21i$ $17+29i \mapsto 21+9i$	$17+11i \mapsto 21+27i$ $17+19i \mapsto 21+19i$ $17+31i \mapsto 21+7i$
$19+5i \mapsto 19+33i$ $19+13i \mapsto 19+25i$ $19+23i \mapsto 19+15i$	$19+7i \mapsto 19+31i$ $19+17i \mapsto 19+21i$ $19+29i \mapsto 19+9i$	$19+11i \mapsto 19+27i$ $19+19i \mapsto 19+19i$ $19+31i \mapsto 19+7i$
$23+5i \mapsto 15+33i$ $23+13i \mapsto 15+25i$ $23+23i \mapsto 15+15i$	$23+7i \mapsto 15+31i$ $23+17i \mapsto 15+21i$ $23+29i \mapsto 15+9i$	$23+11i \mapsto 15+27i$ $23+19i \mapsto 15+19i$ $23+31i \mapsto 15+7i$
$29+5i \mapsto 9+33i$ $29+13i \mapsto 9+25i$ $29+23i \mapsto 9+15i$	$29+7i \mapsto 9+31i$ $29+17i \mapsto 9+21i$ $29+29i \mapsto 9+9i$	$29+11i \mapsto 9+27i$ $29+19i \mapsto 9+19i$ $29+31i \mapsto 9+7i$
$31+5i \mapsto 7+33i$ $31+13i \mapsto 7+25i$ $31+23i \mapsto 7+15i$	$31+7i \mapsto 7+31i$ $31+17i \mapsto 7+21i$ $31+29i \mapsto 7+9i$	$31+11i \mapsto 7+27i$ $31+19i \mapsto 7+19i$ $31+31i \mapsto 7+7i$

centre1 $=$ $23+13i$; centre2 $=$ $13+23i$		
$5+5i \mapsto 41+41i$	$5+7i \mapsto 41+39i$	$5+11i \mapsto 41+35i$
$5+13i \mapsto 41+33i$	$5+17i \mapsto 41+29i$	$5+19i \mapsto 41+27i$
$5+23i \mapsto 41+23i$	$5+29i \mapsto 41+17i$	$5+31i \mapsto 41+15i$
$7+5i \mapsto 39+41i$	$7+7i \mapsto 39+39i$	$7+11i \mapsto 39+35i$
$7+13i \mapsto 39+33i$	$7+17i \mapsto 39+29i$	$7+19i \mapsto 39+27i$
$7+23i \mapsto 39+23i$	$7+29i \mapsto 39+17i$	$7+31i \mapsto 39+15i$
$11+5i \mapsto 35+41i$	$11+7i \mapsto 35+39i$	$11+11i \mapsto 35+35i$
$11+13i \mapsto 35+33i$	$11+17i \mapsto 35+29i$	$11+19i \mapsto 35+27i$
$11+23i \mapsto 35+23i$	$11+29i \mapsto 35+17i$	$11+31i \mapsto 35+15i$
$13+5i \mapsto 33+41i$	$13+7i \mapsto 33+39i$	$13+11i \mapsto 33+35i$
$13+13i \mapsto 33+33i$	$13+17i \mapsto 33+29i$	$13+19i \mapsto 33+27i$
$13+23i \mapsto 33+23i$	$13+29i \mapsto 33+17i$	$13+31i \mapsto 33+15i$
$17+5i \mapsto 29+41i$	$17+7i \mapsto 29+39i$	$17+11i \mapsto 29+35i$
$17+13i \mapsto 29+33i$	$17+17i \mapsto 29+29i$	$17+19i \mapsto 29+27i$
$17+23i \mapsto 29+23i$	$17+29i \mapsto 29+17i$	$17+31i \mapsto 29+15i$
$19+5i \mapsto 27+41i$	$19+7i \mapsto 27+39i$	$19+11i \mapsto 27+35i$
$19+13i \mapsto 27+33i$	$19+17i \mapsto 27+29i$	$19+19i \mapsto 27+27i$
$19+23i \mapsto 27+23i$	$19+29i \mapsto 27+17i$	$19+31i \mapsto 27+15i$
$23+5i \mapsto 23+41i$	$23+7i \mapsto 23+39i$	$23+11i \mapsto 23+35i$
$23+13i \mapsto 23+33i$	$23+17i \mapsto 23+29i$	$23+19i \mapsto 23+27i$
$23+23i \mapsto 23+23i$	$23+29i \mapsto 23+17i$	$23+31i \mapsto 23+15i$
$29+5i \mapsto 17+41i$	$29+7i \mapsto 17+39i$	$29+11i \mapsto 17+35i$
$29+13i \mapsto 17+33i$	$29+17i \mapsto 17+29i$	$29+19i \mapsto 17+27i$
$29+23i \mapsto 17+23i$	$29+29i \mapsto 17+17i$	$29+31i \mapsto 17+15i$
$31+5i \mapsto 15+41i$	$31+7i \mapsto 15+39i$	$31+11i \mapsto 15+35i$
$31+13i \mapsto 15+33i$	$31+17i \mapsto 15+29i$	$31+19i \mapsto 15+27i$
$31+23i \mapsto 15+23i$	$31+29i \mapsto 15+17i$	$31+31i \mapsto 15+15i$

centre1 $=$ $29+7i$; centre2 $=$ $7+29i$		
$5+5i \mapsto 53+53i$	$5+7i \mapsto 53+51i$	$5+11i \mapsto 53+47i$
$5+13i \mapsto 53+45i$	$5+17i \mapsto 53+41i$	$5+19i \mapsto 53+39i$
$5+23i \mapsto 53+35i$	$5+29i \mapsto 53+29i$	$5+31i \mapsto 53+27i$
$7+5i \mapsto 51+53i$	$7+7i \mapsto 51+51i$	$7+11i \mapsto 51+47i$
$7+13i \mapsto 51+45i$	$7+17i \mapsto 51+41i$	$7+19i \mapsto 51+39i$
$7+23i \mapsto 51+35i$	$7+29i \mapsto 51+29i$	$7+31i \mapsto 51+27i$
$11+5i \mapsto 47+53i$	$11+7i \mapsto 47+51i$	$11+11i \mapsto 47+47i$
$11+13i \mapsto 47+45i$	$11+17i \mapsto 47+41i$	$11+19i \mapsto 47+39i$
$11+23i \mapsto 47+35i$	$11+29i \mapsto 47+29i$	$11+31i \mapsto 47+27i$
$13+5i \mapsto 45+53i$	$13+7i \mapsto 45+51i$	$13+11i \mapsto 45+47i$
$13+13i \mapsto 45+45i$	$13+17i \mapsto 45+41i$	$13+19i \mapsto 45+39i$
$13+23i \mapsto 45+35i$	$13+29i \mapsto 45+29i$	$13+31i \mapsto 45+27i$
$17+5i \mapsto 41+53i$	$17+7i \mapsto 41+51i$	$17+11i \mapsto 41+47i$
$17+13i \mapsto 41+45i$	$17+17i \mapsto 41+41i$	$17+19i \mapsto 41+39i$
$17+23i \mapsto 41+35i$	$17+29i \mapsto 41+29i$	$17+31i \mapsto 41+27i$
$19+5i \mapsto 39+53i$	$19+7i \mapsto 39+51i$	$19+11i \mapsto 39+47i$
$19+13i \mapsto 39+45i$	$19+17i \mapsto 39+41i$	$19+19i \mapsto 39+39i$
$19+23i \mapsto 39+35i$	$19+29i \mapsto 39+29i$	$19+31i \mapsto 39+27i$
$23+5i \mapsto 35+53i$	$23+7i \mapsto 35+51i$	$23+11i \mapsto 35+47i$
$23+13i \mapsto 35+45i$	$23+17i \mapsto 35+41i$	$23+19i \mapsto 35+39i$
$23+23i \mapsto 35+35i$	$23+29i \mapsto 35+29i$	$23+31i \mapsto 35+27i$
$29+5i \mapsto 29+53i$	$29+7i \mapsto 29+51i$	$29+11i \mapsto 29+47i$
$29+13i \mapsto 29+45i$	$29+17i \mapsto 29+41i$	$29+19i \mapsto 29+39i$
$29+23i \mapsto 29+35i$	$29+29i \mapsto 29+29i$	$29+31i \mapsto 29+27i$
$31+5i \mapsto 27+53i$	$31+7i \mapsto 27+51i$	$31+11i \mapsto 27+47i$
$31+13i \mapsto 27+45i$	$31+17i \mapsto 27+41i$	$31+19i \mapsto 27+39i$
$31+23i \mapsto 27+35i$	$31+29i \mapsto 27+29i$	$31+31i \mapsto 27+27i$

centre1 $=$ $31+5i$; centre2 $=$ $5+31i$		
$5+5i \mapsto 57+57i$ $5+13i \mapsto 57+49i$ $5+23i \mapsto 57+39i$	$5+7i \mapsto 57+55i$ $5+17i \mapsto 57+45i$ $5+29i \mapsto 57+33i$	$5+11i \mapsto 57+51i$ $5+19i \mapsto 57+43i$ $5+31i \mapsto 57+31i$
$7+5i \mapsto 55+57i$ $7+13i \mapsto 55+49i$ $7+23i \mapsto 55+39i$	$7+7i \mapsto 55+55i$ $7+17i \mapsto 55+45i$ $7+29i \mapsto 55+33i$	$7+11i \mapsto 55+51i$ $7+19i \mapsto 55+43i$ $7+31i \mapsto 55+31i$
$11+5i \mapsto 51+57i$ $11+13i \mapsto 51+49i$ $11+23i \mapsto 51+39i$	$11+7i \mapsto 51+55i$ $11+17i \mapsto 51+45i$ $11+29i \mapsto 51+33i$	$11+11i \mapsto 51+51i$ $11+19i \mapsto 51+43i$ $11+31i \mapsto 51+31i$
$13+5i \mapsto 49+57i$ $13+13i \mapsto 49+49i$ $13+23i \mapsto 49+39i$	$13+7i \mapsto 49+55i$ $13+17i \mapsto 49+45i$ $13+29i \mapsto 49+33i$	$13+11i \mapsto 49+51i$ $13+19i \mapsto 49+43i$ $13+31i \mapsto 49+31i$
$17+5i \mapsto 45+57i$ $17+13i \mapsto 45+49i$ $17+23i \mapsto 45+39i$	$17+7i \mapsto 45+55i$ $17+17i \mapsto 45+45i$ $17+29i \mapsto 45+33i$	$17+11i \mapsto 45+51i$ $17+19i \mapsto 45+43i$ $17+31i \mapsto 45+31i$
$19+5i \mapsto 43+57i$ $19+13i \mapsto 43+49i$ $19+23i \mapsto 43+39i$	$19+7i \mapsto 43+55i$ $19+17i \mapsto 43+45i$ $19+29i \mapsto 43+33i$	$19+11i \mapsto 43+51i$ $19+19i \mapsto 43+43i$ $19+31i \mapsto 43+31i$
$23+5i \mapsto 39+57i$ $23+13i \mapsto 39+49i$ $23+23i \mapsto 39+39i$	$23+7i \mapsto 39+55i$ $23+17i \mapsto 39+45i$ $23+29i \mapsto 39+33i$	$23+11i \mapsto 39+51i$ $23+19i \mapsto 39+43i$ $23+31i \mapsto 39+31i$
$29+5i \mapsto 33+57i$ $29+13i \mapsto 33+49i$ $29+23i \mapsto 33+39i$	$29+7i \mapsto 33+55i$ $29+17i \mapsto 33+45i$ $29+29i \mapsto 33+33i$	$29+11i \mapsto 33+51i$ $29+19i \mapsto 33+43i$ $29+31i \mapsto 33+31i$
$31+5i \mapsto 31+57i$ $31+13i \mapsto 31+49i$ $31+23i \mapsto 31+39i$	$31+7i \mapsto 31+55i$ $31+17i \mapsto 31+45i$ $31+29i \mapsto 31+33i$	$31+11i \mapsto 31+51i$ $31+19i \mapsto 31+43i$ $31+31i \mapsto 31+31i$