

Puis faire demi-tour, Denise Vella-Chemla, novembre 2024

Pour aller à la rencontre des parties imaginaires des zéros non triviaux de la fonction ζ de Riemann, on écrit finalement ce tout petit programme, qui montre la valeur $1/2$ intervenir dans le calcul à côté de la transformation des degrés en radians.

Voici le programme python de cette idée saugrenue :

```
import math
from math import log,pi
import matplotlib.pyplot as plt
import mpmath
from mpmath import zetazero

for k in range(2,111):
    res1 = k*log(k)*180/(2*pi)
    res2 = 2*((-1)*res1)**0.5).imag
    print(f'f({k:d}) = {res1:10f} | 2 sqrt(oppose(f({k:d})) = {res2:10f}
          | zz_{k-1:d} = ',end='')
    res = zetazero(k-1).imag
    print(res)
```

Voici le résultat du programme. On n'a pas d'explication, le programme trouve "un peu" les parties imaginaires des zéros, et la formule est si simple.

f(2) = 39.714408	2 sqrt(oppose(f(2)) = 12.603874	zz_1 = 14.1347251417347
f(3) = 94.418771	2 sqrt(oppose(f(3)) = 19.433864	zz_2 = 21.0220396387716
f(4) = 158.857632	2 sqrt(oppose(f(4)) = 25.207747	zz_3 = 25.0108575801457
f(5) = 230.534999	2 sqrt(oppose(f(5)) = 30.366758	zz_4 = 30.4248761258595
f(6) = 307.980766	2 sqrt(oppose(f(6)) = 35.098762	zz_5 = 32.9350615877392
f(7) = 390.223536	2 sqrt(oppose(f(7)) = 39.508153	zz_6 = 37.5861781588257
f(8) = 476.572896	2 sqrt(oppose(f(8)) = 43.661099	zz_7 = 40.9187190121475
f(9) = 566.512627	2 sqrt(oppose(f(9)) = 47.603051	zz_8 = 43.327073280915
f(10) = 659.642039	2 sqrt(oppose(f(10)) = 51.366995	zz_9 = 48.0051508811672
f(11) = 755.641034	2 sqrt(oppose(f(11)) = 54.977851	zz_10 = 49.7738324776723
f(12) = 854.247981	2 sqrt(oppose(f(12)) = 58.455042	zz_11 = 52.9703214777145
f(13) = 955.245024	2 sqrt(oppose(f(13)) = 61.814077	zz_12 = 56.4462476970634
f(14) = 1058.447928	2 sqrt(oppose(f(14)) = 65.067593	zz_13 = 59.3470440026024
f(15) = 1163.698854	2 sqrt(oppose(f(15)) = 68.226061	zz_14 = 60.8317785246098
f(16) = 1270.861057	2 sqrt(oppose(f(16)) = 71.298276	zz_15 = 65.1125440480816
f(17) = 1379.814920	2 sqrt(oppose(f(17)) = 74.291720	zz_16 = 67.0798105294942
f(18) = 1490.454927	2 sqrt(oppose(f(18)) = 77.212821	zz_17 = 69.546401711174
f(19) = 1602.687302	2 sqrt(oppose(f(19)) = 80.067154	zz_18 = 72.0671576744819
f(20) = 1716.428158	2 sqrt(oppose(f(20)) = 82.859596	zz_19 = 75.7046906990839
f(21) = 1831.602006	2 sqrt(oppose(f(21)) = 85.594439	zz_20 = 77.1448400688748
f(22) = 1948.140556	2 sqrt(oppose(f(22)) = 88.275490	zz_21 = 79.3373750202494
f(23) = 2065.981730	2 sqrt(oppose(f(23)) = 90.906143	zz_22 = 82.910380854086
f(24) = 2185.068859	2 sqrt(oppose(f(24)) = 93.489440	zz_23 = 84.7354929805171
f(25) = 2305.349994	2 sqrt(oppose(f(25)) = 96.028121	zz_24 = 87.4252746131252

f(26) = 2426.777351		2 sqrt(oppose(f(26))) = 98.524664		zz_25 = 88.8091112076345
f(27) = 2549.306822		2 sqrt(oppose(f(27))) = 100.981321		zz_26 = 92.4918992705585
f(28) = 2672.897569		2 sqrt(oppose(f(28))) = 103.400146		zz_27 = 94.6513440405199
f(29) = 2797.511672		2 sqrt(oppose(f(29))) = 105.783017		zz_28 = 95.8706342282453
f(30) = 2923.113829		2 sqrt(oppose(f(30))) = 108.131657		zz_29 = 98.8311942181937
f(31) = 3049.671093		2 sqrt(oppose(f(31))) = 110.447654		zz_30 = 101.317851005731
f(32) = 3177.152642		2 sqrt(oppose(f(32))) = 112.732473		zz_31 = 103.725538040478
f(33) = 3305.529584		2 sqrt(oppose(f(33))) = 114.987470		zz_32 = 105.446623052326
f(34) = 3434.774777		2 sqrt(oppose(f(34))) = 117.213903		zz_33 = 107.168611184276
f(35) = 3564.862676		2 sqrt(oppose(f(35))) = 119.412942		zz_34 = 111.02953554317
f(36) = 3695.769198		2 sqrt(oppose(f(36))) = 121.585677		zz_35 = 111.874659176993
f(37) = 3827.471596		2 sqrt(oppose(f(37))) = 123.733126		zz_36 = 114.320220915453
f(38) = 3959.948357		2 sqrt(oppose(f(38))) = 125.856241		zz_37 = 116.226680320858
f(39) = 4093.179096		2 sqrt(oppose(f(39))) = 127.955916		zz_38 = 118.790782865976
f(40) = 4227.144477		2 sqrt(oppose(f(40))) = 130.032988		zz_39 = 121.370125002421
f(41) = 4361.826130		2 sqrt(oppose(f(41))) = 132.088245		zz_40 = 122.946829293553
f(42) = 4497.206581		2 sqrt(oppose(f(42))) = 134.122430		zz_41 = 124.256818554346
f(43) = 4633.269189		2 sqrt(oppose(f(43))) = 136.136243		zz_42 = 127.516683879596
f(44) = 4769.998088		2 sqrt(oppose(f(44))) = 138.130346		zz_43 = 129.578704199956
f(45) = 4907.378131		2 sqrt(oppose(f(45))) = 140.105362		zz_44 = 131.087688530933
f(46) = 5045.394846		2 sqrt(oppose(f(46))) = 142.061886		zz_45 = 133.497737202998
f(47) = 5184.034390		2 sqrt(oppose(f(47))) = 144.000478		zz_46 = 134.756509753374
f(48) = 5323.283510		2 sqrt(oppose(f(48))) = 145.921671		zz_47 = 138.116042054533
f(49) = 5463.129504		2 sqrt(oppose(f(49))) = 147.825972		zz_48 = 139.736208952121
f(50) = 5603.560189		2 sqrt(oppose(f(50))) = 149.713863		zz_49 = 141.123707404021
f(51) = 5744.563871		2 sqrt(oppose(f(51))) = 151.585802		zz_50 = 143.111845807621
f(52) = 5886.129311		2 sqrt(oppose(f(52))) = 153.442228		zz_51 = 146.000982486766
f(53) = 6028.245707		2 sqrt(oppose(f(53))) = 155.283556		zz_52 = 147.42276534256
f(54) = 6170.902661		2 sqrt(oppose(f(54))) = 157.110186		zz_53 = 150.053520420785
f(55) = 6314.090162		2 sqrt(oppose(f(55))) = 158.922499		zz_54 = 150.925257612241
f(56) = 6457.798562		2 sqrt(oppose(f(56))) = 160.720858		zz_55 = 153.024693811199
f(57) = 6602.018559		2 sqrt(oppose(f(57))) = 162.505613		zz_56 = 156.112909294238
f(58) = 6746.741176		2 sqrt(oppose(f(58))) = 164.277097		zz_57 = 157.597591817594
f(59) = 6891.957747		2 sqrt(oppose(f(59))) = 166.035632		zz_58 = 158.849988171421
f(60) = 7037.659899		2 sqrt(oppose(f(60))) = 167.781523		zz_59 = 161.188964137596
f(61) = 7183.839537		2 sqrt(oppose(f(61))) = 169.515068		zz_60 = 163.030709687182
f(62) = 7330.488834		2 sqrt(oppose(f(62))) = 171.236548		zz_61 = 165.5370691879
f(63) = 7477.600214		2 sqrt(oppose(f(63))) = 172.946237		zz_62 = 167.184439978175
f(64) = 7625.166341		2 sqrt(oppose(f(64))) = 174.644397		zz_63 = 169.094515415569
f(65) = 7773.180110		2 sqrt(oppose(f(65))) = 176.331280		zz_64 = 169.911976479412
f(66) = 7921.634633		2 sqrt(oppose(f(66))) = 178.007131		zz_65 = 173.411536519592
f(67) = 8070.523232		2 sqrt(oppose(f(67))) = 179.672182		zz_66 = 174.754191523366
f(68) = 8219.839426		2 sqrt(oppose(f(68))) = 181.326660		zz_67 = 176.44143429771
f(69) = 8369.576929		2 sqrt(oppose(f(69))) = 182.970784		zz_68 = 178.3774077761
f(70) = 8519.729633		2 sqrt(oppose(f(70))) = 184.604763		zz_69 = 179.916484020257
f(71) = 8670.291606		2 sqrt(oppose(f(71))) = 186.228801		zz_70 = 182.207078484366
f(72) = 8821.257084		2 sqrt(oppose(f(72))) = 187.843095		zz_71 = 184.874467848387
f(73) = 8972.620462		2 sqrt(oppose(f(73))) = 189.447834		zz_72 = 185.598783677707
f(74) = 9124.376290		2 sqrt(oppose(f(74))) = 191.043202		zz_73 = 187.228922583502
f(75) = 9276.519263		2 sqrt(oppose(f(75))) = 192.629377		zz_74 = 189.416158656017
f(76) = 9429.044219		2 sqrt(oppose(f(76))) = 194.206531		zz_75 = 192.026656360714
f(77) = 9581.946131		2 sqrt(oppose(f(77))) = 195.774831		zz_76 = 193.079726603846
f(78) = 9735.220105		2 sqrt(oppose(f(78))) = 197.334438		zz_77 = 195.265396679529
f(79) = 9888.861370		2 sqrt(oppose(f(79))) = 198.885508		zz_78 = 196.876481840958
f(80) = 10042.865275		2 sqrt(oppose(f(80))) = 200.428194		zz_79 = 198.015309676252

f(81) = 10197.227289	2 sqrt(oppose(f(81))) = 201.962643	zz_80 = 201.264751943704
f(82) = 10351.942989	2 sqrt(oppose(f(82))) = 203.488997	zz_81 = 202.493594514141
f(83) = 10507.008062	2 sqrt(oppose(f(83))) = 205.007396	zz_82 = 204.189671803105
f(84) = 10662.418300	2 sqrt(oppose(f(84))) = 206.517973	zz_83 = 205.394697202163
f(85) = 10818.169591	2 sqrt(oppose(f(85))) = 208.020860	zz_84 = 207.906258887806
f(86) = 10974.257924	2 sqrt(oppose(f(86))) = 209.516185	zz_85 = 209.576509716856
f(87) = 11130.679380	2 sqrt(oppose(f(87))) = 211.004070	zz_86 = 211.690862595365
f(88) = 11287.430129	2 sqrt(oppose(f(88))) = 212.484636	zz_87 = 213.347919359713
f(89) = 11444.506429	2 sqrt(oppose(f(89))) = 213.958000	zz_88 = 214.547044783491
f(90) = 11601.904622	2 sqrt(oppose(f(90))) = 215.424276	zz_89 = 216.169538508264
f(91) = 11759.621132	2 sqrt(oppose(f(91))) = 216.883574	zz_90 = 219.067596349021
f(92) = 11917.652461	2 sqrt(oppose(f(92))) = 218.336002	zz_91 = 220.714918839314
f(93) = 12075.995185	2 sqrt(oppose(f(93))) = 219.781666	zz_92 = 221.430705554693
f(94) = 12234.645957	2 sqrt(oppose(f(94))) = 221.220668	zz_93 = 224.007000254604
f(95) = 12393.601500	2 sqrt(oppose(f(95))) = 222.653107	zz_94 = 224.983324669582
f(96) = 12552.858605	2 sqrt(oppose(f(96))) = 224.079081	zz_95 = 227.421444279679
f(97) = 12712.414131	2 sqrt(oppose(f(97))) = 225.498684	zz_96 = 229.337413305525
f(98) = 12872.265001	2 sqrt(oppose(f(98))) = 226.912009	zz_97 = 231.250188700499
f(99) = 13032.408202	2 sqrt(oppose(f(99))) = 228.319147	zz_98 = 231.98723525318
f(100) = 13192.840780	2 sqrt(oppose(f(100))) = 229.720184	zz_99 = 233.693404178908
f(101) = 13353.559842	2 sqrt(oppose(f(101))) = 231.115208	zz_100 = 236.524229665816
f(102) = 13514.562551	2 sqrt(oppose(f(102))) = 232.504301	zz_101 = 237.769820480925
f(103) = 13675.846126	2 sqrt(oppose(f(103))) = 233.887547	zz_102 = 239.555477573328
f(104) = 13837.407840	2 sqrt(oppose(f(104))) = 235.265024	zz_103 = 241.049157796217
f(105) = 13999.245019	2 sqrt(oppose(f(105))) = 236.636810	zz_104 = 242.823271934223
f(106) = 14161.355040	2 sqrt(oppose(f(106))) = 238.002984	zz_105 = 244.070898497078
f(107) = 14323.735327	2 sqrt(oppose(f(107))) = 239.363617	zz_106 = 247.136990074898
f(108) = 14486.383356	2 sqrt(oppose(f(108))) = 240.718785	zz_107 = 248.101990060148
f(109) = 14649.296646	2 sqrt(oppose(f(109))) = 242.068558	zz_108 = 249.573689644707
f(110) = 14812.472765	2 x sqrt(oppose(f(110))) = 243.413005	zz_109=251.014947795016
f(111) = 14975.909323	2 x sqrt(oppose(f(111))) = 244.752196	zz_110=253.069986747999

