An interview with Jean-Pierre Serre
My first half-century at Collège de France

Jean-Pierre Serre was Professor at Collège de France, holder of the Chair of *Algebra and Geometry* from 1956 to 1994.

You taught at Collège de France from 1956 to 1994, in the Chair of *Algebra and Geometry*. What memory do you keep of it ?

I held this chair for 38 years. It's a long time, but there are precedents : if we believe the Collège de France Directory, in the 19th century, the chair of physics was occupied only by two professors : one stayed 60 years, the other 40. It is true that there was no retirement at this period and that the professors had substitutes (to whom they paid part of their salary).

As for my teaching, here is what I said in an interview from 1986 [1] : "Teaching at Collège de France is a wonderful and formidable privilege. Wonderful because of the freedom in the choice of subjects and the high level of the audience : CNRS researchers, foreign visitors, colleagues from Paris and Orsay - many are regulars who have been coming regularly since five, ten or even twenty years. Fearsome too : you need every year a new course subject, either on your own research (which I prefer), or on those of others ; as an annual course lasts about twenty hours, that makes a lot !"

How was your inaugural lesson ?

When I arrived at Collège de France, I was a young man of thirty. Inaugural lesson appeared almost to me as an oral exam, in front of professors, family, fellow mathematicians, journalists, etc. I tried to prepare it. After a month, I had managed to write half a page.

The day of the lesson arrives, a fairly solemn moment. I began by reading the half page in question, then I improvised. I don't know very well anymore

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1. M. Schmidt, Men of Science, 218-227, Hermann, Paris, 1990.

what I said (I only remember talking about Algebra, and the ancillary role which it plays in Geometry and in Number Theory). According to the report published in the *Combat* newspaper, I spent my time wiping mechanically the table that separated me from the public ; I did not feel comfortable when I took a stick of chalk and started writing on the blackboard, that old friend of mathematicians.

A few months later, the secretariat pointed out to me that all inaugural lessons were written and mine was not. As it had been improvised, I proposed to start it again in the same style, by mentally putting me in the same situation. One evening, they gave me opened a college office and I was loaned a tape recorder. I tried to recreate the original atmosphere, and I probably did a lesson again somewhat similar to the original. The next day, I brought the tape recorder at the Secretariat ; I was told that the recording was inaudible. I estimated that I did everything I could and I stayed there. My inaugural lesson remained the only one that was never written.

Generally, I don't write my presentations ; I don't consult my notes (and often I don't have one). I like to think in front of my listeners. I have the feeling when I explain math, of talking to a friend. In front of a friend, we don't want to read a text. If we forgot a formula, we give structure ; it's enough. During the presentation I have in mind a quantity of things that would allow me to speak much longer than expected. I choose according to the audience, and the inspiration of the moment.

The only exception : the Bourbaki seminar, where a sufficient text must be provided in advance so that it can be distributed during the meeting. It is also the only seminar which applies such a rule, which is very restrictive for speakers.

What is Bourbaki's place in French mathematics of today ?

The seminar is the most interesting. It meets three times a year, in March, May and November. It plays both a social role (opportunity to meetings) and mathematics (presentation of recent results - often under a clearer form than that of the authors) ; it covers all branches of mathematics.

The books (*Topology, Algebra, Lie Groups,...*) are still read, not only in

France, but also abroad. Some of these books became classics : I'm thinking in particular of the one on root systems. I recently saw (in the AMS *Citations Index*[2]) that Bourbaki came to 6$^{\text{th}}$ place (with citations) among French mathematicians (from plus, globally, numbers 1 and 3 are French, and are called both Lions : a good point for Collège de France). I kept a very good memory from my collaboration in Bourbaki, between 1949 and 1973. It taught me many things, both in substance (by forcing me to write things that I did not know) and on the form (how to write so as to be understood). It also taught me not to trust the "specialists" too much.

Bourbaki's working method is well known : distribution of texts to different members and criticism of them by reading aloud (line to line : it is slow but effective). Meetings ("congresses") take place 3 times a year. The discussions were very lively, sometimes even passionate. At the end of the congress, the papers were distributed to new writers. And we started again. The same chapter was often written four or five times. The slowness of the process explains why Bourbaki did not finally publish that very few works in forty years of existence, since the years 1930-1935 until the late 1970s, when production declined.

Regarding the books themselves, we can say that they have filled their mission. People often believed that these books dealt with subjects that Bourbaki found interesting. The reality is different : his books deal which what is useful for doing interesting things. Take the example of number theory. Bourbaki's publications speak very little about it. However, its members appreciated it very much, but they considered that it was not part of the *Elements* : you first had to understand a lot of algebra, geometry and analysis.

In addition, we often blamed on Bourbaki about everything we did not like in mathematics. He was criticized in particular for the excesses of "modern maths" in school curricula. It is true that some officials of these programs claimed to be from Bourbaki. But Bourbaki was not there for nothing : his writings were intended for mathematicians, not students, let alone adolescents. Note that Bourbaki has avoided pronouncing himself on this topic. His doctrine was simple : we do what we choose to do, we do it as best we

2. AMS : American Mathematical Society.

can, but we don't explain why we made it. I really like this point of view which favors work over speech - never mind if it sometimes leads to misunderstandings.

How do you analyze the evolution of your discipline since that from your beginnings ? Do we do math today as we did fifty years ago ?

Of course, we do math today like there are fifty years ! Obviously, we understand more things ; the arsenal of our methods has increased. There is continuous progress (or sometimes progress by jolts : some branches remain stagnant for a decade or two, then suddenly wake up when someone introduces a new idea).

If we wanted to date "modern" mathematics (a dangerous term), it would probably go back to around 1800 with Gauss.

And going back further, if you met Euclid, what would you say to yourself ?

Euclid seems to me to be rather someone who put the math of his time in order. He played a role analogous to that of Bourbaki fifty years ago. It is not a coincidence that Bourbaki chose to title his works *Elements of mathematics* : it is by reference to the Euclid's *Elements*. (Note also that "Mathematics" is written in the singular. Bourbaki teaches us that there are not several distinct mathematics, but only one mathematic. And he teaches it to us in his usual way : not by great speeches, but by omitting a letter at the end of a word). Coming back to Euclid, I don't think he produced contributions really original. Archimedes would be a more appropriate contact. He is the great mathematician of Antiquity. He did extraordinary classic things, both in mathematics and in physics.

In philosophy of science, there is a very strong current in favor of a thought of rupture. Are there no breaks in mathmatics ? We have for example described the emergence of the probability as a new way of representing the world. What is its meaning in mathematics ?

Philosophers like to speak of "rupture". I guess that adds a little spice to their speeches. I don't see anything like that in mathematics : no catastrophe, no revolution. Progress, yes, I already said it ; it's not the same thing. Sometimes we work on old questions, sometimes on new questions. There is no frontier between the two. There is a great continuity between the mathematics of two centuries ago and that of now. The time of mathematicians is the "long duration" of my late colleague Braudel.

As for probabilities, they are useful for their applications at the same time mathematical and practical ; from a purely mathematical point of view, they constitute a branch of the theory of measurement. Can we really talk about them as "a new way of representing the world" ? Surely not in mathematics.

Do computers change something in the way to do math ?

We used to say that research in mathematics was inexpensive : pencils and paper, and those are our needs. Attoday we have to add computers. It remains inexpensive, in the sense that mathematicians rarely need very important computing resources. Unlike, for example, particle physics, whose calculation needs are commensurate with the very large equipment required to the collection of data, mathematicians do not mobilize large data centers.

In practice, it changes material working conditions for mathematicians : you spend a lot of time in front of your computer. It has different uses. First, the number of mathematicians has grossly increased. When I started 55 or 60 years ago, the number of productive mathematicians was a few thousand (worldwide), the equivalent of the population of a village. At present, this number is at least 100,000 : a city. This increase has consequences for how to contact and get informed. The computer and Internet access manage the exchanges. This is all the more precious since mathematicians are not slowed down, like others, by experimental work : we can communicate and work very quickly. I take an example. A mathematician has found a demonstration but a lemma is missing of a technical nature. Using a search engine - like Google - it identifies colleagues who have worked on the issue and sends them an email. In this way, it is likely to find in a few days or even within hours the person who actually demonstrated the lemma he needs. (Of course, this only concerns problems auxiliaries : points of detail for which we wish to refer to existing references rather than doing the demonstrations yourself. On the really difficult questions my mathematician would have little chance of find someone who can help him).

The computer and the Internet are therefore tools to speed up our work. They also make manuscripts accessible worldwide whole, without waiting for their publication in a newspaper. It's very useful. Note that this acceleration has also disadvantages. E-mail produces informal correspondence that is less readily retained than paper. We rarely throw letters when we delete or lose easily emails (when you change computers, for example). We recently published (in bilingual version : French on one page, and English on the opposite page) my correspondence with A. Grothendieck between 1955 and 1987 ; this would not have been possible if it had been electronic.

In addition, some demonstrations use the computer to correct a series of cases that it would be impractical to deal with by hand. Two classic cases : the 4 colors problem (coloring cards with only four colors) and the Kepler problem (stacking of spheres in the 3-dimensional space). This leads to demonstrations which are not really verifiable ; in other words, they are not real "demonstrations" but only experimental facts, very likely, but that no one can guarantee.

You mentioned the increase in the number of mathematicians. What is the situation today ? Where does mathematics go ?

The increase in the number of mathematicians is an important fact. We could fear that this would be at the expense of quality. In fact, there nothing like that. There are many very good mathematicians (in particular among young French people - a very good omen).

What I can say about the future is that despite this great number of mathematicians, we are not short of material. We let's not run out of problems, when just over two centuries ago, at the end of the 18^{th} century, Lagrange was pessimistic : he thought that "the mine was dried up", that there was not much left to find. Lagrange wrote that just before Gauss revived mathematics in an extraordinary way, him alone. Today, there is a lot of land to explore for young mathematicians (and also for the less young, I hope).

According to a commonplace of the philosophy of science, the great mathematical discoveries are made by young mathematicians. Is

this your opinion ?

I don't think the term "great discovery" applies to me. I have mostly done "useful" things (for other mathematicians). In all case when I got the Abel award in 2003, most of the work that was cited by the jury were made before I was 30 years old. But if I had arrested at that time, I probably would not have been given this price : I also did other things afterwards (even "guess-work" about which many people have worked and still work).

In my generation, many of my colleagues have continued beyond 80 years old, for example my old friends Armand Borel and Raoul Bott, all dead two recently at 82. There is no reason to stop, as long as health permits. The subject must still lend itself to it. When we are very wide, there is always something to do, but if we are too specialized, we can get stuck for long periods, either because we demonstrated everything there was to demonstrate, or on the contrary because the problems are too difficult. It is very frustrating.

Mathematical discoveries give great joys. Poincaré, Hadamard, Littlewood[3] explained it very well. As far as I'm concerned, I keep especially the memory of an idea that helped unlock the homotopy theory. It happened one night back from vacation, in 1950, in a train berth. I was looking for a fiber space with such and such properties. The answer came : the space of laces! I couldn't help but wake up my wife who was sleeping in the bunk below to say : that's it! My thesis came out of there, and much more. Of course, these sudden discoveries are rare : it happened to me maybe twice in sixty years. But these are luminous moments, truly exceptional.

Is Collège de France a place where you interact with others disciplines ?

Not for me. Even among the mathematicians of Collège de France, there is no collective work. It should be noted that we work in branches often very separate. It's not bad : Collège de France is not supposed to be a club. A number of modern commonplaces - like *collective work, interdisciplinarity and teamwork* - do not apply to us.

3. J.E. Littlewood, A Mathematician's Miscellany, Methuen and Co, 1953. This book explains well the unconscious part of the creative work.

What have you thought of the dialogue between a specialist in neurosciences, Jean-Pierre Changeux, and the mathematician Alain Connes, which is reproduced in the book *Matter of thought*?

This book is a fine example of deaf dialogue. Changeux does not understand not what Connes says, and vice versa. It's quite amazing. Personally, I'm on Connes' side. Mathematical truths are independent of us [4]. Our only choice is how to express them. If desired, we That's what he has can do this without introducing any terminology. Consider for example a troop of soldiers. Their general likes to arrange them in two ways : rectangle, or in 2 squares. It's up to the sergeant to place them. He realizes that just put them in a row by 4 : if there are 1 left that he could not place, or he will manage to put them all in a rectangle, or he will manage to distribute them in two squares.

[Technical translation : the number n of soldiers is of the form $4k+1$. If n is not first, we can arrange the soldiers in a rectangle. If n is prime, a theorem due to Fermat says that n is the sum of two squares.]

What is the place of mathematics compared to others science? Is there a new demand for mathematics, from these sciences?

No doubt, but we have to separate things. On the one hand there is theoretical physics, which is so theoretical that it straddles mathematics and physics, physicists considering it to be mathematics, while mathematicians take the opposite view. It is symbolized by string theory. Its most positive aspect is to provide mathematicians a large number of statements, which they must demonstrate (or possibly demolish).

In addition, especially in biology, there is everything related to systems with a large number of elements that must be treated collectively. There are branches of mathematics which deal with these questions. That responds to

4. A few years ago, my friend R. Bott and I were going to receive an Israeli award (the Wolf Prize) presented in the Knesseth, in Jerusalem. Bott had to say a few words about mathematics. He asked me : "What can I say ?". I said to him, âIt's very simple ; you have not only to explain this : the other sciences seek to find the laws that God has chosen ; the mathematics seeks to find the laws which God had to obey.". The Knesseth appreciated.

a request. There are also requests that concern logic : it is the case of data processing, for the manufacture of computers. You must also mention cryptography, which is a source of interesting problems relating to number theory.

Regarding the place of mathematics compared to other sciences, you can think of math as a big warehouse full of shelving. Mathematicians place things on the shelves whose they guarantee that they are true ; they also give the manual and the way to reconstruct them. The other sciences come to serve themselves in function of their needs. The mathematician doesn't care of what they do with his products. This metaphor is a bit trivial, but it pretty much reflects the situation. (Of course, we don't choose to do math for putting things on the shelves : we do math for fun to make).

Here is a personal example. My wife, Josiane, was a specialist in quantum chemistry. She needed to use the linear representations of certain groups of symmetries. The available works were not satisfying : they were correct, but used very heavy notations. I wrote for her a presentation tailored to her needs, and then I published in a book entitled *Linear Representations of Finite Groups*. I have done my job as a mathematician (and husband) : put things on the shelves.

Does truth in mathematics have the same meaning as elsewhere ?

No. It is an absolute true. This is undoubtedly what makes the unpopularity of mathematics in the audience. The man in the street is willing to tolerate absolute when it comes from religion, but not when it comes from mathematics. Conclusion : to believe is easier than to demonstrate.