Annex to the proposal denitac.pdf (Denise Vella-Chemla, 23.7.2019)

We try to decompose an even number *n* into a sum of 2 prime numbers p1 + p2.

We cannot refer to ÿ (ÿ1) as we did in [1]. However, to obtain a reduction in the number of Goldbach decomposers of n, we can use the cardinal P *n* of the set of prime numbers less than or equal to *n* and multiply it
2 2

by the product Q pÿ ÿ *n* $1 - \frac{1}{p}$ which counts how many chances has the prime number p1 of not not sharing its remainder with *n* according to each modulus *p* less than ÿ *n* (the fact of not sharing its remainder with *n* allows p1 to have a complement to *n* (called p2) which is also prime).

The reduction [1] of *ÿ (x)* (the number of prime numbers less than x) by is given log x 69; for x ÿ 17 in [2], page (Corollary 1, (3.5), of Theorem 2, whose proof is given in paragraph 7 of [2]).

We therefore have | P *n* | > log $\frac{not/2}{2\,(n\,2)}$.

The reduction of Q pÿ ÿ *n* $1 - \frac{1}{p}$ is also provided in [2], page 70 (this is the corollary (3.27) of Theorem 7 whose proof is given in paragraph 8 of [2], with *ÿ* the Euler-Mascheroni constant).

(3.27) $\frac{ÿÿ e}{\log x}\ 1 - \frac{1}{\log2 x\ !} < Y_{pÿx}\ 1 - \frac{1}{p\ !}$ for $1 < x$.

By multiplying these expressions together, we obtain that the number of decompositions of Goldbach of *n* must be greater than:

$$\frac{n\,/}{2 \log (n/2)}\ \frac{ÿÿ e}{\log ÿ \overline{n}}\ 1 - \frac{1}{\log2 ÿ \overline{n}\ !}$$

which is strictly greater than 1 from 24.

Bibliography

[1] http://denisevellachemla.eu/denitac.pdf.

[2] JB Rosser and L. Schoenfeld, *Approximate formulas for some functions of prime num bers,* dedicated to Hans Rademacher for his seventieth birthday, Illinois J. Math., Volume 6, Issue 1 (1962), 64-94.

1. This lowering is to be distinguished from the prime number theorem, independently proven by Hadamard and La Vallée-Poussin, and which provides an asymptotic tendency for *ÿ (x).*