Interview of J-P. Serre by Mariana Cook, Paris 2007

I prefer to close my eyes when I think about mathematics. The best work is done by night, in half sleep. Sometimes I go to bed thinking, "Ah, I have a nice lemma to prove - or disprove". (Should I explain what a "lemma" is ? A mountain climber needs holds to get from one level to the next one. Lemmas are the "holds" of a mathematician.) Of course one has to write things down later, if only for publication. Sometimes you then find that what you have thought was wrong but that's rare.

My thesis (1950-1951) is a typical case. There was a simple-looking but very powerful new idea (the "loop space fibering", found at night, in a train). This basic idea was not enough : there was a technical part which required a rather difficult lemma. During three days I could only see the proof of that lemma when I was flat on my bed, my eyes shut. After that, I understood it clearly enough so that I could write it down ; my thesis was essentially done.

At that time, I was working in the branch of mathematics called "topology". Two years later I started doing something else : several complex variables (the favorite topic of my thesis adviser Henri Cartan). It did not last long. After one year, I was attracted by algebraic geometry, and then number theory, group theory, etc. The end result is that, even now, I am not a specialist of any subject !

I feel I should tell you about the several conjectures I made in the last fifty years. (What is a "conjecture" ? It is something that you cannot prove, but that you expect to be true and interesting.). I made quite a few of these, including some completely wrong ones (when I was less than thirty - I got more careful with age). Several of these conjectures have been much studied by a lot of people. Among them are the ones on Galois cohomology (made in 1962) which are called "conjecture I" and "conjecture II". Conjecture I is now a theorem (proved three years after I made it). As for Conjecture II, it is still open after forty-five years, but most special cases have been proved. Perhaps it will turn out to be false in the other cases ? I don't think so, but what I really hope for is that it will be decided : yes or no !

If you look on the Web at "Serre's conjecture" you shall probably find a different one (on "Galois representations") which I made in the early 70s (and in a refined form in the mid 80s). It became very popular for two reasons : it is related to Fermat's last theorem (bad reason) and it is a first step towards a "Langlands program in characteristic p " (good reason). It looked out of reach until about five years ago, when suddenly somebody had a brilliant idea and solved a big part of it. Now, with the help of several different people, the conjecture is dead : it has become a theorem. Indeed, in a few weeks I am going to a two-week conference near Marseille in which the proof will be summarized and explained (even two weeks are not enough to give complete details).

At the time I made the "Serre's conjecture" (in its refined form), I had decided to write it in a setting with which I was familiar, and which could be explained easily. But, on a higher level, I knew that it should be done in a different way. The two ways were of course similar, but not a priori equivalent.

There was a conflict between my conscious decision of making things “easy” and my unconscious feeling that it was not “the right way”. This conflict haunted me ; it made me very unhappy ; there was an horrible night where I had the impression that there were two parts of my brain which were fighting each other and spinning endlessly. Then, a few months later, I found an example showing that the two points of view are not equivalent, and the correct one is not the one I had chosen. But I also saw that, in all the really interesting cases, they are equivalent. Curiously, finding this “counter-example” made me incredibly happy : the two sides of my brain had reconciled. Happy End.