An effective model of spacetime
to geometrize the Standard Model
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Cern, 2004

"Okay, so, I mean, I am a mathematician and you know, what I want to explain, of course I was asked this question, you know : "what is fundamental in mathematics ? and so on" and uh, I mean, somehow you know, the way I I view things because I am in front of an audience of mainly physicists is to try to pinpoint somehow there are differences of approach between mathematics and physics.

And I mean the first thing in mathematics, you know is that if you think about it, I mean the main point of mathematics is that it's really a factory of new concepts. So somehow, if you want, the first fundamental thing is that through the kind of process of distillation in the alambic of the human mind, what we produce are concepts which a priori are extremely simple because they are like elementary particles of thoughts. So for instance, if you think about a number like the number 3, what is the number 3, I mean the number 3 is a quality which is common to all those sets which become empty after you remove one element remove another element and remove another element, okay ? That's the number 3. So uh so more I mean I agree entirely with what Groot was saying about communicate, communicating with other uh civilizations and so on so forth and in fact there was a mathematician who wrote an entire book which is called Lincos[1]. It's a language which just by, I mean, you can communicate just by saying that "there is a signal or there is no signal". And just starting from there, you can communicate very elementary concepts like numbers, addition, multiplication and so on so forth. And then you find your way.

So, if you want, the first thing that we sort of fabricate, I mean the elementary particles of thoughts, are these new concepts. And the second thing is that the way that we have to proceed, if you want, in our investigations is in fact quite different from physicists, in the sense that we try to understand things from above. I will give you examples very quickly. But I wanted to take two concepts which are not trivial, which are center stage in mathematics and which are constantly evolving, just to specify something, to specify the subjects you know and these two concepts are the following : *(writing SPACE and SYMMETTRY on the blackboard)* the first concept is the concept of *space* and the second concept is the concept of *symmetry*.

Okay now, so these really also occupy center stage in mathematics as well as in physics. But I mean the way a mathematician proceeds is quite different from the methodological approach, if you want, of a physicist. So typically what happened, I mean, for a space, I will just put up two transparencies. For a space, you know, I mean it was of course, architects and painters in the middle age and so on, were using perspective ; it's a very well understood thing. I don't know this, I don't know how to set it up, it doesn't matter, I mean you know the picture okay ? *(projecting (sic !)*

Dans le cadre du Symposium de Richard Dawkins, Gerard't Hooft et Alain Connes au Cern en 2004.
Référence youtube de la conférence : Alain Connes : "*An effective model of spacetime to geometrize the Standard Model*". https://www.youtube-nocookie.com/embed/vBWPxGsXHD0.
Transcription et traduction à l'aide d'outils downsub, google traduction, Latex : Denise Vella-Chemla, octobre 2024.

[1] abréviation pour lingua cosmica.

The Cene with perspective dashed lines on it) Now mathematicians, and in particular Desargues, try to put this sort of empirical rules of perspectives and so on on a good footing. So they tried to devise a good mathematical model for that and what did they devise ? They devise what is called projective geometry so projective geometry, what is it ? I mean it's a geometry in which the axioms, if you want the conditions that will be fulfilled by the geometry, are incredibly simple. What are they ? There will be points and lines and there are just 4 axioms :

- two points determine one line ;
- one line has at least three points okay ;
- two lines which are coming from two points which are on two coplanar lines (i.e. two lines which meet) meet in one point ;
- and the final axiom is that the geometry (the fourth axiom which is the final one) is that the geometry is generated by finitely many points.

In other words, there are finitely many points means that if you iterate the operation of taking lines generated by two points, you get all the points. That's all. Now mathematicians completely classified these geometries. They found all of them. And what they found is that there are a projective space over what is called a field, okay. What is a field ? You know the field of rational numbers, you know the field of real numbers but if you do projective geometry with a field of real numbers, you miss the whole thing. I mean you would say you know we live in a real space that is three dimensional so you miss the whole thing because we do planar geometry ; with complex numbers instead of real numbers, all circles pass by two points which were found by Poncelet, which are at infinity and which have complex coordinates which are called the cyclic points. And then everything become miraculously simpler, and better, okay ? So this is where mathematicians differ from physicists : they classify all cases, okay, that's a very very major difference.

Now the second point is that when they classified all cases, they classified all case instances which are called local, which are locally compact. What did they found ? They found found the real numbers, they found the complex numbers but they found others : they found others like p-adic numbers, they found fields of functions over a finite field and they have the full list and the full list is *endian*[2] And you can look at it, okay, and you know there are no others. So the main, if you want, difference in approach is that in mathematics you sort of display the world scenery. And then you can go. And in particular what they found in the case of projective geometry is that the field K doesn't need to be commutative.

So if you would sort of restrict yourself to the commutative case you will be impaired. Okay now in this picture you can already see this duality between the angel of geometry and the devil of algebra. What is The duality ? The duality is the following : where is the angel of geometry, well you know, you draw pictures, it's nice, you see, in one flash, you see the whole picture and so on and in one flash, for instance you can appreciate a theorem. So let me put up a theorem which is well known which is due to Morley which is the only theorem on triangles that was not known to the Greeks, okay. It's a beautiful theorem, I showed it to my father, he understood it immediately : you take any triangle, okay, any triangle the triangle ABC is completely arbitrary you trissect

[2] *mal audible.*

each angle, okay, you decompose each angle into three equal pieces you intersect the tri-sectors, you get a small triangle in the middle ; this triangle is always equilateral. That's a beautiful result, you know, which was found by Morley. Now, try to prove it, you will have a hard time, okay. So I mean I have shown you the angel of geometry, I will show you the devil of algebra, okay. So the angel of geometry tells you there is something very beautiful, now, prove it, okay. And it was I mean actually I mean it's a known fact that there is no nice geometric proof of that, but here is the algebraic statement which comes from the devil of algebra and which is equivalent, I mean, which is in fact much stronger than this.

What is the statement ? The statement is now you think about the group of affine transformations of the line : x goes to $ax + b$, I mean this is not much complicated thing[3] okay ? Now these transformations have a fixed point, they have an x which is fixed by the transformation okay that's what I call fixed. And now the theorem which you can give in high school in fact it was given in the french exam after I found the proof, it was given in Capes [4] okay, with some detail, so what is the statement ? It's that you take this group of affine transformations, and you write the equivalence between two conditions, condition a) and condition b). Condition a) is that $f^3g^3h^3$ is equal to one where f, g and h are elements of the group. Condition b) is that there is a j which is, if you want, the amplitude part of the product fgh whose cube is equal to one that's nothing and then that if you take $\alpha + j\beta + j^2\gamma$ is equal to zero, you are still in the line where α is a fixed point of fg, β is a fix point of gh and γ is a fix point of hf. Now this, you can check it just by matrix computation, it's very very simple. Now what do you do ? Now, you take the line, not the real line, you take the complex line, and you take f and g and h to be the rotation around the point A by angle $2a$, around the point B by angle $2b$ and C by angle $2c$ okay ? And you do it, fine, you do the computation, you find that $f^3g^3h^3$ is equal to 1 : this is just because the sum of the angles of a triangle add up to π, that's exactly the same thing, so you deduce that you have the second condition, which is $\alpha + j\beta + j^2\gamma = 0$ but the fix point of fg is exactly the intersection of the two tri-sectors, same for the fix point of gh, same for the fix point of hf. And that condition $\alpha+j\beta+j^2\gamma = 0$ is a wellknown condition to get an equilateral triangle, that we learn in high school in fact, okay ? So you see, here is the angel of geometry ; here is the devil of algebra. But the devil has a lot of power, why ? Because this theorem which is on that side, now it is true for any field. For instance, it is true for the field with four units, okay it's true for any field. So now the picture that you had here, what did it do ? It began a brain process which is going from the visual areas of the brain to the linguistic areas of the brain and the algebra is the linguistic areas. And from these linguistic areas, you get a statement which is purely linguistic which is algebra manipulation of symbols and so on. But now it has tremendous power because now it doesn't only apply to your visual original picture ; it applies to a much much much broader class of examples, okay ?

Now the second lesson that we learn from this axiomatic business, which is crucial in mathematics is that you know there are questions which to physicists and to many people could look completely esoteric questions. And the typical question was that when you look at the Euclide axioms, okay, when you look at these axioms you know, when you look at the axioms of Euclide, there are many more than the axioms I showed you for the projective geometry, there are plenty, plenty of axioms. But now if you look at these axioms, you find that there is one which is disturbing, you have to

[3] *mal audible*

[4] examen en France pour devenir Professeur de mathématiques en collège.

have some sensitivity to understand that it's not that when you will have removed it, the list should be much shorter because it's a very long list, okay, so there was this axiom, if you want, which was saying precisely that the sum of the angles of a triangle is equal to π that was one of the axioms of Euclide. It is called the axiom of the unique parallel but it is really that axiom that in fact the sum of the angles of a triangle is equal to π okay. Now in the Klein's model, so mathematicians thought a lot, they tried to prove this axiom. Legendre, for a long part of his life, tried to prove this axiom but when trying to prove this axiom, they found that there was a new world, which was unknown to them which they discovered because it was consistent. So what they found is that if you drop this axiom, that the sum of the angles of a triangle is equal to π, it is still consistent, okay, you find a consistent world, and eventually you realize it, by how, it's so simple, the points of the geometry are just now the points which are inside an ellipse ; you don't allow all the points you only allow those points which are inside an ellipse. The straight lines are still the usual straight lines but now you see that outside a given line which is a line D, you can pass several parallels because these two lines not meet, okay. The problem is that this is not enough to define the geometry, you have to define the distance between points that's the log of the cross ratio between the four points like A, B, a, b, okay. And then you find that all of the euclidean axioms are true but the axiom of the unique parallel is not true. So what do you gain out of that, you say "oh okay I cannot remove this axiom from this list. Not at all, that's not what you gain : what you gain is two big openings on the idea of geometry ; the first opening is the opening of Galois, okay, which was I mean then the idea of Lie groups and Sophus Lie, which is the idea of symmetry so that's the first big opening in in which what you would do is to say that this non euclidean geometry is beautiful because of its symmetries, because of the fact you want that you can transport rigid bodies around and so on and so forth.

But in fact there was a much wider opening, and the much wider opening came from Gauss and Riemann ; and what they did was to, uh you know, what they did was to model geometry in a situation which you know very well as the $g_{\mu\nu}$s, like the surface of the Earth and so on and so forth, but I mean the main new input, the main new ingredient there is that indeed instead of having to do geometry in a completely if you want sort of harmonious way in which you can move rigid bodies and so, now what you want, you want to have a manifold of points which has some dimension which is not fixed plus a line element okay. And the line element is prescribed arbitrarily essentially ; if you want, the length of the line element is prescribed just as a quadratic form locally. Now what is very important of course is that the usual notions continue to make sense in this new geometry, so the notion of straight line continue to make sense and of course I mean, you probably all know that the main reason why this geometry was so successful is that you get Newton's law in a given potential V when instead of letting time pass, you know, as in Minkowski space time, what you do is you let time pass in a different way, according to the height at which you are in the newtonian potential. And now if you go ahead, I mean a little bit of I think this probably has some use in Cern, but I mean typically for instance when people dug the tunnel under the channel, they used the correction which is given by this in order to position themselves with GPS, with respect to satellites. I mean this correction is non trivial correction ; time really passes differently, when you are at a different place in newtonian potential okay.

Now as I said okay this theory was incredibly uh successful and it was incredibly successful as a model of general relativity and I just flash you know these things which you know also pretty well, I suppose, so this is you know the curvature potential, the Einstein equations and I mean these which are checked with incredible precision in the the binary pulsar story and so on. Okay so this is just to flash it okay. So now if you want the mentality of mathematicians is quite different, it's very very different, in the sense that instead of sort of trying to solve a specific, a completely specific physics problem, what we do is that we try to look in an harmonious manner to the general concepts of geometry and so on. Now it turned out that uh the usual idea of geometry which is modeled by this riemannian geometry has recently evolved a lot and it has evolved in a spectral way.

So I mean here I show you know a spectrum and so on and what I mean by that is the following : what I mean is that you know there was a fundamental discovery of course you all know, which is due to Heisenberg and which is that when you try to model say in the riemannian world, in the idea of manifolds, when you try to model a microscopic system like an atom which is an interaction with radiation okay, the only data that you have is spectral okay. So what you have is the spectra of the emitted light or the absorbed light and so on okay. Now when people try to model this and when you try to apply the classical model of this, you find laws which are not compatible with the experimental laws which are the Ritz-Rydberg law. Now the Ritz-Rydberg law tells you that the spectral rays combine, they combine according to a certain combination law, which is called the Ritz-Rydberg combination principle and which tells that a line which is always labeled by two indices like i and j, initial and final state combined with j, k to give you the line transition i, k. Now Heisenberg got the genuine idea of transporting what you would get as the product of the coordinates on the classical space in that setup starting from the experimental law. And you got a new law and this law is the matrix composition law. So it is the law of matrices and what he immediately noticed of course is that this law is not commutative and mean what you have is that the coordinates that you would normally use as coordinates on a space no longer commute okay. So mathematicians when they are faced with that, okay, they are they have to take seriously this message and they have also to take seriously the message of Desargues because the Desarguian geometry is the geometry that came from projective space : the field that was there was not a commutative field. So in fact it was mandatory to extend all of our concepts of geometry, if you want, to these non-commutative situations in which the coordinates on a space the space we are looking at are no longer commuting okay. and there...

Okay I just want to mention it extremely briefly but let me tell you an example of a surprise that came very long ago, actually in the 70s, so the surprise that came is the following well we're talking about surprise, but the great surprise was the following ; it is that if you take an algebra of coordinates on non-commutative space, it evolves with time ; it has a god-given time evolution which only comes from its non-commutativity, which only comes from the difference between left and right, okay. So I mean there is, if you want, a canonical map from the real line which is the one parameter group of automorphisms, automorphism classes, which means that this algebra if you look at it like 10 minutes afterwards it has rotated okay it has a completely canonical time evolution okay. So that was the first surprise. Now what was very important was to extend to this framework the riemannian ideas, the ideas of measurement of distances and so on and I will just flash the transparency I will just put this transparencies to tell you what is the difference and to tell you in what way the measurement of distances differ in this new geometry from the way it is in the old geometry.

Now the way it differs is extremely simple to explain : you know that at the end of the 18th century people have tried to unify the unit of length. So they asked people around like Laplace and Lagrange “what can we do to get a good unit of length ?”. So the first answer was well take the length of a pendulum which beats for one second okay. But if you do that and you climb on Mont-Blanc, you know, you will have a difference so it’s no good okay. And also, you have to define the second okay. So they thought a lot, they thought a lot and then what they thought is “well, let’s take the largest available object okay which is the earth okay, on which we can measure, and take a very tiny portion of that 1/40 millions of that, okay they define it exactly, and then we make a meter, we make a meter, geometry is meter, and we deposit it somewhere okay it was in Pavillon de Breteuil in Paris and that’s the unit of length. But think about it, you know, and they thought about it for a long time. After a while they realize that the meter was shrunking, there was something wrong, and also if you want to measure something, you have to go to Paris, compare it, it’s not practical okay.

So in 1960, they came up with a new definition. And the new definition is that there is a certain Orange Line in Krypton okay and then the unit of length should be a multiple of this Orange Line in Krypton. Now... Then it was changed to cesium, krypton was replaced by cesium and of course, you can convert length to time, using the speed of light. So there is no problem okay. Now what is the advantage ? The advantage is obvious because if we were to communicate with some other civilization, we cannot tell them to unify unit of length by coming to Paris and compare it to the meter okay. But we can tell them : “come on, I mean, look at your periodic table of elements, take that number the n ones okay, take that spectral array and do it and it will be done. Now what happens is that in non-commutative geometry, which is this different geometry, what happens is that precisely, the unit of length is exactly of this spectral nature. And in fact, the unit of length is exactly what physicists call the fermion propagator, it is inverse of the Dirac operator. And this theory applies in a much much much wider framework than ordinary riemannian geometry ; it contains riemannian geometry, but for instance, the Einstein action which I put before, now I mean the Hilbert-Einstein action, you know, the one which when you differentiate with respect to $g_{\mu\nu}$ gives you the Einstein equation. That equation, what is it ? I mean what is this action ? It is just the area, the two-dimensional area of a four dimensional space. So you take a four dimensional space, okay, and instead of writing integral of ds to the four, which would give you the four dimensional volume, you sort of take something bizar, you write integral of ds to the square. Now if you know the Einstein action, you know that $1/G$ has dimension of one over length squared. So in fact you are just measuring exactly the area of that space you are looking at a two dimensional area. So the only thing I want to mention is that when you take this standpoint, then okay, and I mean this apparently is not yet widely known, but I mean when you take this standpoint then the lagrangian, which is the standard model part of this gravitation acquires an incredibly simple form : this lagrangian which is a combination of the two things, first of all you know, when you look at this lagrangian, you look at the symmetries ; the symmetries contain a different morphism group, which comes from the gravitation part, but it also contains a group of gauge transformations of second kind which come from the standard model. Now if you try to look at a space which has as this group of symmetries different morphisms, what you will find is that there is no manifold which works, and this is Kaluza-Klein idea, that it doesn’t work. Because if you take a Kaluza-Klein model, in fact, you will get a much larger group, which is always a simple group which doesn’t have a normal subgroup as the gauge transformations by diffeomorphisms. But, there is a very nice and simple non-commutative space, which does the job and the reason why it does the job is that precisely, when you take a non-commutative space, it has morphisms which are relatively trivial which are called inner in mathematics and because it's not commutative its x goes to u inverse, you see, because u doesn't commute with x. You cannot push the u around and get x. So it's a non trivial automorphism. But it's a mild one and this mild ones are called inner in mathematics and they correspond exactly to the internal symmetries in physics, okay.

Now it turns out that the action which combines the Einstein action plus the Standard model action is obtained by this very simple space which is the product of ordinary space-time by a finite space when we take as an action just the number of eigenvalues of the line element, okay, so you take your lineelement and you count the number of eigenvalues of this line element which are bigger than the given length ; you expand it and you find this okay ? So, I mean what I want to conclude on is the following : Dirac had proposed one strategy which was to change quantum mechanics ; I am proposing another strategy : and that as a strategy is to change geometry. And there are plenty of good reasons from the mathematical side to do that, it already had quite an impact in mathematics if you want by doing that, by applying geometry to spaces which are not commutative.

There are many examples of such spaces which are not commutative at all in which this new geometry applies, in which you can make computations that you couldn't do otherwise. But so what I'm proposing, I'm proposing this different strategy which is to change geometry to make geometry quantum and of course, okay, which is pretty much what is going on in this either, and then to say once it is like that, okay, then we can, you know, begin to do what one would do normally for being gravity but just because the space will be slightly more complicated, this pure gravitational theory will actually contain ordinary gravity and the matter fields, okay ? So it's a proposal okay, it's a proposal, which is such that for instance, what happens in this proposal is that the Higgs boson is not at all natural, the Higgs boson is computed from a gravitational computation and what you spit is a usual doublet of Higgs boson, a complex doublet of Higgs bosons, okay. And what plays the role of the metric, if you want, what plays the role of the metric in the finite space is exactly the Yukawa-? matrix in the standard model. So it contains both if you want Yukawa masses and the Kobayashi-Maskawa mixing matrix and this is incorporated in the geometry of the finite space okay okay. So I think I was brief enough but I think I will stop here.

Le maître de cérémonie : Thank you very much. Unfortunately I'm not a mathematician so I can't ask you any questions. Anybody else ? Yes, sir.

Un auditeur : You know by the way that in order to compare masses, you have to go to Paris.

AC : no, no, to England, the Kilo is in England. They are thinking about changing it, of course, I mean, the actual kilogram actually is losing weight, this is well known.

L'auditeur : I have a more serious question I mean since mass is generated presumably by the Higgs mechanism I'm wondering if you can make some particular predictions from your model...

AC : to dis from okay so I mean this model I mean Higgs formation is a big desert, which I don't believe in of course. Okay this model favors a Higgs of 160 G yeah exactly and it gives ... (?) 5, which is not so bad. And it gives a certain number of things you know but I mean what it does it gives you the quartic self-coupling of the Higgs at a certain sort of unification so it favors ... of that other of magnitude. So if the Higgs is 133 okay then it's not so good for this model, you see what I mean, and okay. But I mean what it does however, you see, the point of this model is that we have, I mean the psychological process we are in is a little bit crazy because in this very complicated lagrangian of the standard model, we have isolated the electromagnetic part and we say "This is Minkowski space". Of course you know the Minkowski space, if we look, it just comes from the electromagnetic part of that lagrangian but that lagrangian has many more pieces, it has a weak part and it has a strong part. And what I am proposing is a larger geometry, a larger framework which can absorb this model as being pure geometry and the answer you get is not Minkowski space : it's something which is slightly more elaborate which has a slightly non- commutative piece okay. So it it is as simple as that.

I just want... I'm not saying it is a final model, but I'm saying that 200 GV or one teV okay, that's what we see concretely it's a refinement of Minkowski space, it's more refined it has this slightly non-commutative aspect. Okay, that's what I'm saying, I'm not saying you know, this is a theory of everything or this is a final model, no no, no no, I'm extremely practical and you see what I mean, down to earth ; what I'm saying is that there is a more general framework for geometry, which applies to many many more mathematical cases ; it's motivated by mathematics, not by physics but then, of course, it should be compatible with physics. Now the way it's compatible with physics is precisely that the line element is defined as the inverse fermion propagator, okay. Then you look at it and you you just see what it is and you know very well that in the standard model the fermion propagator is not just pure Dirac it's Dirac with UKW (?) coupling, you see what I mean. And then okay, you get this more fancy geometry that's all, that's all I'm saying. But the action principle, I mean the principle that gives the lagrangian is unbelievably simple. Now it's crystal clear. It's not that and what you find, you know, you find that I have nightmares many time that you find that the sign in front of the Einstein action is correct, the sign in front of the Yang-Mills self-coupling is correct, the sign in front of the minimal coupling is correct, the sign in front of the Higgs quartic coupling which you find is correct okay and so on and so forth. So maybe it's an accident, you can say it's a mathematical curiosity and so on and so forth. But it might be it's not an accident, and you know in these days where we have a sort of a preponderant theory from the sociological point of view and so, I think it's important to know that it's not the only one, that there are other things, I mean of course it's not at the intersection of non-commutative geometry with string theory is zero, it's not zero, we know there is some intersection. But you see I am proposing it as an effective model : what I am proposing is that there is a more flexible notion of geometry which gives you an effective model of space-time at the energies that we know, of course it's not the final model, and when energies will get higher we will have to know whether it fits or not with this idea of geometry that's all.

Un autre auditeur : Okay thank you, which problems is your proposal solving ? I mean it is just predicting the Higgs mass ?

AC : No I mean the model is not really predicting the Higgs mass because it's only predicting the Higgs mass and the expense of taking a big desert in between, which I don't believe in okay. No I mean which problem is it solving ? It is solving, if you want, the problem of writing the standard model action on the on the back of an envelope and you I mean if you show me the standard model action on the back of an envelope. I know it, I mean, I saw it many times, it's not a thing you can write in the back of an envelope, you know, I mean, or on the back of a Timbre-Poste, of a stamp, no no, I mean I want to write it on the back of a stamp. So on the back of a stamp, okay, what I am telling you is that you take the line element like this... You want to write no no no no no no no it is redictioning you see I don't believe in a theory I mean I I would hate to live in a world...

Un autre auditeur : Do you say anything about the cosmological constant, you say something about ?? Okay like many other theories ?

AC : Yeah, sure, sure, theories... okay, sure. yeah but I mean you don't pay the price of grand unifying theories in the sense that you are not saying that you know there is this small gauge group, and so no, you just... what you get is the standard model as such. As you said the Higgs phenomenon arises from the geometry, let me explain...

Le même auditeur : Does it say anything for theological constant ?

AC : No no, okay, let me explain why the phenomenon arises from geometry, so simple to explain okay. I mean I am oversimplifying it, okay, but imagine that you have a space-time which has two sides : the upper side and the lower side okay. Now if you take a function there okay you can differentiate it on one side, you can differentiate it on the other side, but you can also differentiate it across. Now this is exactly how the Higgs occurs okay. So the Higgs occurs because functions when you differentiate them on this side or that side, they will give you gauge fields of course, but when you differentiate them across, they will give you something which has no spin of course, because the spin would rotate in a space, it will not rotate transversally, that's how you get the Higgs. And you get the Higgs with the right quantum numbers, you get it with the correct quantum numbers, maybe it's an accident okay you know.

Un autre auditeur : Is the Higgs a consequence of gauging variance ?

AC : No, not all, no, it's a consequence of geometry what I giving you is a purely geometric principle that gives you this, that's all okay I you know I understand physicists you know I understand that when Minkowski writes his model of space-time, people can tell him "what do you bring to relativity ?" and he cannot say anything ; of course, it doesn't bring anything to special relativity, it is the same, they are the same equations, it's exactly the same thing, okay. What I am telling you is that there is an extremely simple geometric model that gives you exactly the standard model from an extremely simple principle, out of a certain geometry. The geometry is not so simple, the geometry is more delicate than Minkowski geometry, that's so, okay. I'm not saying that I solve a physical problem.

Le maître de cérémonie : It cannot fit on the back of an envelope, probably not, probably not, but it may fit in the coffee break so we're we're going to have a general question answer session, after a short break.