Interview d'Alain Connes

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Alain Connes was born on 1 April 1947 in Draguignan, in the south of France. After attending secondary school in Marseille, he entered the Ecole Normale Supérieure in Paris in 1966. He earned his doctorate in 1973, under the direction of Jacques Dixmier.

In the early part of his career, Connes held positions in the CNRS (Centre National de la Recherche Scientifique) and the University of Paris VI and was also as a visitor at Queen's University in Ontario and the Institute for Advanced Study in Princeton. He was appointed to the Léon Motchane Chair at the Institut des Hautes Etudes Scientifiques in Bures-sur-Yvette in 1979, and, in parallel, to the Chair of Analysis and Geometry at the Collège de France in 1984. In 2017 he retired from both positions. He has also held distinguished professor positions at Vanderbilt University and at the Ohio State University.

In addition to the Fields Medal in 1982, his honors include the Ampère Prize of the French Academy of Sciences (1980), the Crafoord Prize (2001), and the CNRS Gold Medal (2004).

Connes made major advances in operator algebras that revolutionized the subject and stimulated a great deal of further research. This led him to develop, starting in the late 1970s, an entirely new branch of mathematics, noncommutative geometry, which turned out to have deep connections to many fundamental questions in mathematics and physics. Through his dozens of collaborators and more than 250 publications, Connes has had a major impact on mathematics over the past several decades.

What follows is the edited text of an extensive interview with Connes, held in July 2020.

Early life in the South of France

ALLYN JACKSON : *You were born in Draguignan, which is in the south of France. Can you tell me about your childhood growing up there ?*

ALAIN CONNES : Let me tell you how my grandparents got there. My grandparents on my mother's side were both born in Constantine, Algeria. They came to France in 1918, after my mother was born ; she was born in Tangiers, Morocco. When they arrived in France they looked in a reference book for the citythat was the healthiest in France. And they found Draguignan !

My grandfather was an engineer, but he retired quite early in his life. They then bought an estate in Draguignan. My parents met in Draguignan in 1944, when the Americans came to the south of France at the end of the war, in what we call the *débarquement*. There were perhaps 1000 glider planes, people jumping out with parachutes - coming to Draguignan !

My mother died last year at age 101. She and my father both died (he also lived until age 101) in the house in Draguignan where I was born. It is heartbreaking because now we are selling the house, which is where all of my subconscious is mapped. It's not just a house, it's an estate with a big garden, a small wood, vineyards, and very big, old cedar trees. I am attached to that place, because of the quietness. It's a place thathas serenity built-in.

When I was eight years old, my father decided that education in Draguignan was not good enough for us. So he took a very dangerous job in Marseille, as the leader of a police squad focused on the trafficking of alcohol. From time to time, he would disappear during the night when he was arresting the bandits.

ALLYN JACKSON : *Was alcohol illegal at the time ?*

ALAIN CONNES : Alcohol was not illegal, but it was very much taxed, so there was a lot of illegal trafficking. Fortunately, he left this job just at the beginning of the Mafia's arrival in 1966. So he didn't have to deal with the Mafia, but it was still quite dangerous. He had a car with a racing engine and officers on motorbikes with him. It was never a simple thing to arrest the people, because they had to arrest them while they were trafficking. We were all in stress whenever he would disappear like that during the night.

ALLYN JACKSON : *Your father was French, is that right ?*

ALAIN CONNES : Yes. His parents were from the south-west of France.

ALLYN JACKSON : *What kind of work did your father do when you lived in Draguignan*

ALAIN CONNES : At that time, my father was employed by the tax office in a job also related to alcohol, overseeing the production of wine in the south. His own father had been killed in the first World War when my fatherwas one year old, so he never knew his own father. He wanted to give us an education as a real father. That was quite important. I have two brothers. My older brother created his own software company, and he is quite rich. My younger brother is a doctor and a wellknown specialist for Crohn's disease ; he is retired now.

We had a very tough education. For instance, during holidays we would get additional work from my father, and when school was in session he would add to the work assigned by the school - his additional contribution !

ALLYN JACKSON : *How did you feel about that ?*

ALAIN CONNES : It was terrible ! We would find all possible means to escape from that, of course

ALLYN JACKSON : *Do you think it ultimately helped you ?*

ALAIN CONNES : It's difficult to say. We three brothers were quite different. My older brother would fight with my father. I was not fighting. I was sort of cool, taking it easy. It's not clear at all that it's a good recipe, but it influenced us a lot. For instance, once I came home, and my father asked about the results of a math competition I had been in. I said, "I was second." Then he slammed me in the face, because I was not first.

ALLYN JACKSON : *That's pretty heavy pressure.*

ALAIN CONNES : There was pressure. But because we were three brothers and were united, we would find all possible ways to escape. For instance, we knew where my father hid the translations of the Latin assignments he gave us. That made life a lot easier ! He would ask us to recite the lessons twice : on the day when we were given the lessons and a second time on the day before we were to recite them in school. We would recite them to our mother because then it was possible to look at the book while my mother was cooking ! So we found all kinds of tricks to escape. But it was a very tough education. There was really a fear in all of us, during our whole childhood.

One thing I should add is that I really loved my maternal grandmother. She brought in the soothing, emotional side, which my mother also brought in. My grandmother was a pianist.

ALLYN JACKSON : *Your grandmother was a pianist and your grandfather an engineer. So they were well off when they were in Algeria.*

ALAIN CONNES : Yes, they were very well off when they arrived in France, but they lost everything because they bought exactly the wrong type of bonds. They lost their whole fortune, everything.

ALLYN JACKSON : *But they had their home, the estate in Draguignan...*

ALAIN CONNES : Yes, they had their home, which was extremely helpful of course.

From lycée to Ecole Normale

ALLYN JACKSON : *When you moved to Marseille when you were eight years old, what kind of school did your father put you in*

ALAIN CONNES : We were put in the Lycée St Charles, which was very close to our home. It was a standard type of school, but good, with good teachers. At that time itwas completely egalitarian. If you got into the school,you would have a good education

ALLYN JACKSON : *Were you interested in mathematics at that time ?*

ALAIN CONNES : Yes, but I was not interested in competition. I was really interested in my own thinking, and this went on until I was in preparatory school for entering the Ecole Normale. At that time I had already developed a theory that I liked very much. If the problems that the teacher assigned were related to my own thinking, then I could do very well. But if they were not related, then they didn't appeal to me.

I had one teacher when I was in preparatory school who was very good and who was interested in what I was developing. This was very nice, to feel that there was somebody who cared about my idea.

ALLYN JACKSON : *What was this idea ?*

ALAIN CONNES : Later when I went to the Ecole Normale I discovered that it was already known.

The idea was to replace differentiation by finite difference operations. I had developed a whole system for this. What was really important was that it was my own. It was not something that I had picked up in books.

When I entered Ecole Normale, I was much more preoccupied by trying to find a girlfriend than working. This is the truth! Also, at that age I had a very bad complex about my physical appearance. For instance, I would walk only on the right-hand side of the street because I hated my right profile and didn't want people to see it.

ALLYN JACKSON : *But your right side looks fine.*

ALAIN CONNES : It looks fine now! It was strange, this complex. Also, when I would try to work, I would sing and be disturbed by my own singing. That was peculiar, but that's how I was!

ALLYN JACKSON : *This was a stage of growing up.*

ALAIN CONNES : Exactly. I was not so much obsessed by mathematics. I was really interested in my own doings, not so much in passing exams or things like that.

ALLYN JACKSON : *Did your brothers also do the preparatory classes and go to the Ecole Normale ?*

ALAIN CONNES : Yes. Two years before me, my older brother had entered the Ecole Normale. So he opened the way. My younger brother did his studies in medicine, because there was a tradition in the family of my mother of medical doctors. She was a medical doctor, and her grandfather also.

ALLYN JACKSON : *Did you learn to play an instrument ?*

ALAIN CONNES : When I was five years old I began piano lessons, and I really loved it. But when we moved to Marseille, we could not have a piano in the house. My father told me I had to choose between music and studies. So I dropped the piano then. I of course always regretted immensely to have done that. When I was twenty, I started again to play the piano, but of course I had missed the most important years for learning. I have done a lot of work to recover from that, but I never recovered to the point that I would have been at. But okay - this is life.

ALLYN JACKSON : *You can't do everything.*

ALAIN CONNES : You can't do everything. I now see very well that I have a part of the brain that is musical. In fact I just wrote a paper for the *Journal of Mathematics and Music*. But I know that the part of the brain that is occupied by music is sort of competing with the part that is occupied by mathematics. Of course, they are extremely close. This might sound strange, but often I learn a lot in mathematics by studying scores of music.

ALLYN JACKSON : *How does that happen ?*

ALAIN CONNES : In mathematics, you might in some cases have the impression that you have reached the highest level of sophistication. But then you study a great musical score, and you find that the composer has a level of sophistication that is about twice the level of sophistication of the best mathematics. This is what I have in mind. There are composers, especially of the Romantic

period, who have reached a level of musical precision that I always find comforting and a source of energy to do mathematics. So I use musical scores as a source of sophistication, but I also like to improvise and to let things out.

ALLYN JACKSON : *And the singing when you were at the Ecole Normale ?*

ALAIN CONNES : That was poor singing, Corsican songs ! It was just for fun. I had a happy temperament, especially coming from the south of France and finding myself in Paris, where people were much more intellectual.

ALLYN JACKSON : *Was it a big culture shock to go to Paris ?*

ALAIN CONNES : Oh, yes. Without being disparaging, it's true that the south, in particular Marseille, doesn't have at all the kind of intellectual background that Paris has. I was amazed when I was in Paris because it was very acceptable to be totally immersed in intellectual stuff. In Marseille you had to be well dressed. People were judged on their physical appearance much more than in Paris. In Paris you could see in the streets people who were not dressed correctly and looked like tramps. They didn't care, it was not important. In Marseille, it was important. I don't know if it has changed by now, but this was how it was at the time.

ALLYN JACKSON : *Was your father satisfied when you got into the Ecole Normale ?*

ALAIN CONNES : Oh, sure. My mother would have preferred us to go to Ecole Polytechnique because they had a beautiful uniform ! Otherwise, my parents were very satisfied.

Freedom to think and grow

ALLYN JACKSON : *It was 1966 when you went to the Ecole Normale. What was that like ?*

ALAIN CONNES : We had a marvelous promotion of young guys, and many became excellent mathematicians. In that specific year, at this specific time at the Ecole Normale, we had no pressure. We had the opportunity to stop doing routine work of preparatory school and to try to think. I have beautiful memories from that year. A friend of mine would ask me a problem, and then for the whole weekend I would think only about this problem. That was great. We were genuinely interested in problems in mathematics. This was our daily bread. But we were not working on assignments ; we would not follow classes. We had some minor exams to pass at the end of the year, but we were free to think about mathematics.

Nowadays people in Ecole Normale are much more treated like children. They have to pass exams and do this and that. They are not given this fundamental blessing, which is time to think and develop on their own. All the friends I made then did extremely well exactly because we were treated in a way that allowed us to grow.

This is the time when I learned that if for instance you have a very complicated calculation to do, the best way is to put things in your head first, and then take a walk. No paper, no pencil. When you take a walk, your mind will learn to build a mental picture. To construct this mental picture, to make it exist - this is the most difficult part of mathematics. In order to do it, you have to be

fighting with a problem for a while - not reading a book, not believing a result is true because somebody says it is. No, this doesn't matter. What really matters is you fight with it by yourself, alone. Then gradually the mental picture will exist in your mind.

ALLYN JACKSON : *What is this mental picture ? It is a geometric picture ?*

ALAIN CONNES : I don't know how it is materialized in the brain, but it is something that, when you think aboutit, lights up and sends you signals. Even more striking is that it will continue to send you signals even when you are not thinking about it. It's exactly like when you leave your home and five minutes later you say, "Oh shit, I forgot to turn off the stove." These things exist in the brain, and they send you signals. Similarly in music, you can have something that exists in your mind, a tune or a theme. This is something amazing and very hard to define.

ALLYN JACKSON : *With music you can go over a piece in your mind as the piece proceeds in time. Is it like that with the mental picture of mathematics ?*

ALAIN CONNES : It depends whether it's algebra or geometry. If the problem is geometric and there is a solution, it will appear in one stroke, with no time dependence. It will be one shot. But not in algebra. Algebra is much more time-dependent and evolving. In algebra, when you are doing computations, there is a definite analogy with the time dependence in music, which is extremely striking.

Noncommutativity generates time

ALAIN CONNES : In fact, it goes much further. One of the things to which I contributed in 1972 was the fact that, when you take a noncommutative algebra, you have a canonical time evolution.

Two Japanese mathematicians, [Minoru] Tomita and [Masamichi] Takesaki, had discovered that if you have a state on a certain kind of algebra, then there is a time evolution. What I discovered by doing extremely complicated calculations over many months is that this time evolution is in fact independent of the state, when you look at it in the right way, meaning that you forget about the trivial automorphisms. The proof when I wrote it down was incredibly simple, but it came from doing lots of calculations. The fruit was extremely simple, but the preparation was extremely complicated.

The outcome of this still fascinates me now : the noncommutativity, which was discovered by people in quantum mechanics, in fact is a generator of time. I am still thinking about the fact that the passage of time, or the way we feel that time is going on and we cannot stop it, is in fact exactly the consequence of the noncommutativity of quantum mechanics, or more explicitly of the inherent randomness of quantum mechanics.

Something Heisenberg discovered, which is absolutely amazing, is that when you repeat certain microscopic experiments, the results will never be the same. You send a photon through a very small slit and look where it lands on a target. If you repeat the experiment, you will never be able to predict where the photon will land. One can use this fact to concoct random numbers, and, unlike generating random numbers by computer, one could create a security system that would be perfectly safe. Even if an attacker knew all the devices you are using, the attacker would never be able to reproduce it. This is the most striking fact of quantum mechanics. The philosophical issue

that has fascinated me for all these years, is that I believe it is precisely this type of randomness that is at the origin of the passing of time.

I wrote a book with my wife and with my teacher, Jacques Dixmier, *Le Théâtre quantique.*[1] The purpose of the book is to explain this idea, which is much more a philosophical issue than a mathematical issue.

ALLYN JACKSON : *Why does the randomness in quantum mechanics produce time ?*

ALAIN CONNES : The noncommutativity is at the origin of this randomness. What Heisenberg discovered is that if you try very hard to know the position of the photon, together with its momentum, you cannot do it. This is prevented by the fact that position and momentum do not commute. Why does this noncommutativity generate time ? In the relevant equation, even though two things do not commute, you can still interchange their order, so you replace AB by BA. This changes the meaning, just as "melon" is not the same thing as "lemon," even though the letters are the same. But there is a price to pay to interchange the order : When you permute A and B, and you make the A pass on the other side, you have to make it evolve with time. And the time in which it has to evolve is in fact the purely imaginary number i. This is what is behind the scenes.

Heisenberg made his discovery at a time when he was sick with hay fever in the springtime. He was sent to an island called Heligoland, which is in the North Sea. He stayed there for a few weeks, doing his own computations. One night, at I think 4 in the morning, he had in front of his eyes the whole scene. And he was scared, because what he saw was quantum mechanics, later called matrix mechanics. He had discovered noncommutativity of physical quantities.

In the noncommutative world there is something thatis totally original that does not exist in the commutative world, where "melon" would be the same thing as "lemon", and that is this God-given time evolution. It makes things much more interesting than if they were static. When you pass to the commutative, you lose a lot of information that, if you keep it, will allow you to compress the external world in a much simpler manner.

Finding your own garden

ALLYN JACKSON : *I would like to return to your early days at the Ecole Normale. When you went there in 1966, the IHES [Institut des Hautes Études Scientifiques, founded in 1958] was going strong. Alexander Grothendieck and his school were there. Were you a part of that at all ?*

ALAIN CONNES : No. At that time, the way I perceived the development around Grothendieck was : I have only oneway to be myself, which is to stay as far apart as I can from this group. But I have to add that now I have read Grothendieck's book *Récoltes et Semailles,*[2] and I have of course read many of his papers. I have come to love those developments. I am also now involved in trying to have some texts of Grothendieck published and revived.

When you begin to do mathematics, you have to have your own garden, even if it is remote from

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1. *Le Théâtre quantique*, by Alain Connes, Danye Chéreau, and Jacques Dixmier. Odile Jacob, 2013.
2. *Récoltes et Semailles : Réflexions et témoignage sur un passé de mathématicien* was written by Alexander Grothendieck between 1983 and 1986. He printed and distributed copies himself but never published it.

the very fashionable things. And you have to begin to exist there. It doesn't matter if it's a small garden. What matters is that it's yours. What matters is that you have been thinking a lot about it and you like it, and you take it as a starting point. This is the way I felt.

ALLYN JACKSON : *At the time a lot of mathematics was very much dominated by Grothendieck and his students.*

ALAIN CONNES : Not only that, but I heard some people saying, "Why are you doing mathematics ? It will all be done by these people."

ALLYN JACKSON : *That there was no mathematics outside what they were doing at IHES ?*

ALAIN CONNES : Yes. Much later Grothendieck understood that this was the wrong attitude. The title of his book, *Récoltes et Semailles* ["reaping and sowing"], in a way can be understood as saying that he comprehended that, by being too forceful, he had had a negative effect. Fortunately mathematics is such an immense subject that there is room for everybody. Still, sociologically, when you are a beginner, it's very hard.

ALLYN JACKSON : *You were in Paris during the big upheavals in 1968.*

ALAIN CONNES : Yes. In 1968, my older brother Bernard was fighting on the barricades. On my side, not at all. I was having a love affair, so I didn't care. I didn't get involved at all. I was distant. I was not involved politically. I didn't want to be.

ALLYN JACKSON : *What were you thinking about mathematically at this time ?*

ALAIN CONNES : When I was at the Ecole Normale, I developed something quite special about zeros of polynomials in the complex plane. I was asked by [Charles] Pisot, a number theorist, to talk in his seminar about what I did. It was a rather original approach, but it was about a marginal topic. I wrote a *Comptes-Rendus* note about this. [3]

I was also participating in the seminar of [Gustave] Choquet. Choquet was a very bright, very witty mathematician. His seminar was very enjoyable. He decided I should learn physics, so he sent me to a summer school in physics in Les Houches in 1970. I was there with my future wife. This was the first time I learned about operator algebras. That was great. I met a lot of people. Then the year after, some people I had met invited me to a meeting in Seattle.

A story of serendipity

ALAIN CONNES : This is how I started to work on the Tomita-Takesaki theory. It is a story of serendipity. Before I went to Seattle I got married with my wife, in 1971. Neither of us had traveled to the US before. I decided to accept the invitation just because I wanted to visit the US ! I didn't look at all at the topic of the conference. We flew to New York to visit my brother, who was in Princeton at the time. This was in July, and it was so warm that the only place that was sort of acceptable was the bookstore. We spent a lot of time in the bookstore. We were going to travel

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3. Alain Connes, "Ordres faibles et localisation des zéros de polynômes," *Comptes-Rendus de l'Académie des Sciences de Paris Série A-B* **269** (1969), A373-A376.

by train from Montreal to Vancouver and then to Seattle. We had something like five days in the train, with the Great Plains to be crossed - rather boring. So I said, Why don't I buy a math book to read during the trip? I hesitated between several books that looked quite interesting. Finally I bought a small book of lecture notes. [4]

When we were on the train, I opened the book, and it looked fascinating. Finally we arrive in Seattle, I go to the conference, and I look at the program. Oh myGod - the author of the book, Takesaki, is one of the lecturers! This is a sign! I decided to go to no lectures except his lectures and to study this stuff.

We were in Seattle for a few weeks and had a wonderful time. When we came back I looked for who in France was doing this kind of math and found it was Jacques Dixmier. I decided that in September I should go to the seminar of Dixmier.

He opened the seminar by bringing in several papers and asking who wanted to speak about which paper. I raised my hand and took one paper, [5] just randomly. It was on a totally different topic from the Tomita-Takesaki theory. I went back home by train, and in the train I found that what the authors, [Huzihiro] Araki and [Edward James] Woods, were doing was in fact deeply related to the Tomita-Takesaki theory.

The same day I wrote a letter to Dixmier, and soon after that I had an appointment with him. The only thing he told me is : "*Foncez !*" "Go, go, go - go fast." I wrote immediately a *Comptes-Rendus* note [6] to explain that the invariants of Araki and Woods could be computed using the Tomita-Takesaki theory. That was the beginning of my work

ALLYN JACKSON : *Dixmier understood exactly that you were onto something*

ALAIN CONNES : He understood completely. And of course he has been my friend since then.

ALLYN JACKSON : *But this is all quite random, isn't it, that you picked up that book in Princeton and found Takesaki in Seattle?*

ALAIN CONNES : Yes, it was totally random. Some people said things that were not so nice ; they said I was lucky. But serendipity is not being lucky. It's transforming what you are given into luck. As you say, there is a definite element of randomness, and then one has to do an enormous amount of work. But somehow, it's work that is guided by the idea that there is something there. In mathematics this matters more than anything else, the gut feeling that there is something. It's not at the level of rational thinking ; it's at the level of intuition. It is something that is hard to transmit to somebody else but that inhabits you and allows you to go forward. And Dixmier perceived this completely.

ALLYN JACKSON : *He is 96 years old now.*

4. Masamichi Takesaki, *Tomita's theory of modular Hilbert algebras and its applications, Lecture Notes in Mathematics*, vol. 128, Springer-Verlag, 1970. https://link.springer.com/book/10.1007/BFb0065832.

5. Huzihiro Araki and E. J. Woods, "A classification of factors," *Publications of the Research Institute for Mathematical Sciences Series A* **4** (1968/1969), 51-130. https://www.ems-ph.org/journals/show$_a$bstract.php?issn = $0034 - 5318 vol = 4 iss = 1 rank = 4$.

6. Alain Connes, "Calcul des deux invariants d'Araki et Woods par la théorie de Tomita et Takesaki," *Comptes-Rendus de l'Académie des Sciences de Paris Série A-B* **274** (1972), A175-A177.

ALAIN CONNES : Yes, and recently we wrote, with my collaborator [Caterina] Consani, a very technical paper. He is the only one I know who really understood what we are doing there! He is an amazing man. At 96, he had comments that were perfect.

From factors to foliations

ALLYN JACKSON : *In 1973 you finished your thesis, under Dixmier's direction. Can you tell me conceptually what you did in your thesis?*

ALAIN CONNES : I did two fundamental things. The first was to show that this time evolution was in fact independent of the state, which gives many invariants of von Neumann algebras, of factors. Factors were introduced by von Neumann to explore non trivial factorizations of the Hilbert space in quantum mechanics. The second was the main thing, which was to reduce the Type III factors, which were the ones von Neumann had left out completely, to the Type II and automorphisms.

ALLYN JACKSON : *At the time that you started working on this, the Type III factors were not well understood?*

ALAIN CONNES : They were not at all understood. What I proved in my thesis is that, first of all, they are classified in Type III$_\lambda$, where lambda is between 0 and 1. Then I gave a complete reduction, except for the Type III$_1$ to Type II and automorphisms. Much later, Takesaki did the case of Type III$_1$.

After I did this work in June 1972, I went for holidays with my wife. I was not worrying at all about priority. Dixmier had to call me during the holiday and tell me that I should publish something, because otherwise it would be lost. I was kind of naive.

ALLYN JACKSON : *Somebody else was working on the samething?*

ALAIN CONNES : Sure. There was a group of people in Kingston, Ontario, working later on the same thing. But I was the first to discover the most important results.[7] This problem of priority would recur many times in my career. But we don't work to have our name on something. We work for the pleasure of discovery. And this pleasure is something that no one can take away from us. I remember I made the discovery when I was visiting Erling Størmer in Norway, during those long days in June when the sun doesn't actually set. I have wonderful memories from this time.

Also in my thesis I found that there are factors that are hyperfinite but are not infinite tensor products. This was a result I announced in July of the same year, 1972, and that used the whole power of my theory. It was not just an abstract result. It had many consequences that were surprising to people at the time.

In 1976 I was admitted to IHES as a visitor. I was a well known specialist in my area, but the area was not as well known as those of the people at IHES. So at that time I felt like a stranger. I felt that what I was doing was very nice, but people did not know about it.

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7. Alain Connes, "Une classification des facteurs de type III," Comptes-Rendus de l'Académie des Sciences, Série A-B **275** (1972), A523-A525.

Then I met a fantastic person, Dennis Sullivan, who was at IHES at the time. He has this incredible Socratic power. He would sit with a newcomer and ask, What are you doing in mathematics? The newcomer would think, This guy is an idiot, he is asking such simple questions. You think you know everything and he knows nothing. But after a while you realize, my God, this is something that I did not understand in my own work!

With Sullivan explaining a lot of things to me, I found that, while the subject I had been working on was not familiar to so many people, there was a way to fabricate factors in a well known geometric context, the context of foliations. So I made contact with differential geometry. I discovered that their familiar objects, foliations, immediately gives rise to factors, and the most exotic factors were appearing from the most natural foliations.

An example is the Anosov foliation, a well studied foliation that comes from the geodesic flow on a Riemann surface. It turns out that the Anosov foliation gives rise exactly to the hyperfinite Type III factor, which is a very difficult and exotic factor in the classification.

This occurred between 1976 and 1978, when Sullivan and I were discussing a lot together.

ALLYN JACKSON : *This was also around the time that Vaughan Jones became a student of [André] Haefliger in Geneva. Haefliger was a major figure in foliations at that time. Did you have contact with Haefliger because of the connection of your work to foliations?*

ALAIN CONNES : No, only marginally. The way of thinking of Sullivan was much closer to my own way. I don't like to read papers, and neither does Sullivan. He has a way of communicating that is oral but is also gestural. This fitted me perfectly. He would explain notions that, if I were trying to learn them from books, it would have taken forever, and I wouldn't have got it. But he would just make some gestures and explain something, and I got it.

You see there the enormous influence of institutions like IHES. Just giving talks is not the same. You have to live with these people, you have to be around, you have to have leisure time, time for lunch, time for tea. And progress occurs by accident. You could never plan it.

ALLYN JACKSON : *Going back to Vaughan Jones - he was essentially your PhD student, even though he was a student in Geneva and Haefliger was formally his adviser.*

ALAIN CONNES : That's true. Vaughan is a very good friend. He picked up on something I had done when I was in Kingston in 1975, about automorphisms of finite factors, and then he developed a beautiful general theory of subfactors. In the 1980s he made a magnificent discovery, the discovery of the connection with knot theory. That was fantastic.

It's a strange story in a way, because after Vaughan discovered his new knot invariant, which came from factors, it was recast in a different manner by [Edward] Witten, under the influence of [Michael] Atiyah also. I had to put my foot in the door so that Vaughan Jones would get the Fields Medal. His discovery was dressed up in terms of functional integrals and things of this type, while the real input, the real strength of the discovery, was from his own work on subfactors. I was a little bit put off by this.

ALLYN JACKSON : *What do you mean you had to put your foot in the door about his Fields Medal ?*

ALAIN CONNES : What I am saying is that the trend of that time was to put more emphasis on the functional integral aspect of the knot theory, than on the true origin of the invariant, which was coming from the factors. Of course, when you write things that are more geometric, it's easier to understand. On the other hand, it's absolutely amazing that the theory of factors, which looks rather exotic, turns out to be related to knot theory, which is very concrete, very basic. And Vaughan discovered a real invariant in knot theory. This is an amazing discovery. I don't know many discoveries that can compete with it. You need an openness of mind to do that. He was in Switzerland with people who were geometers, but I don't know the role that this might have played in his discovery. One would have to ask him. [8]

Complementary ideas

ALLYN JACKSON : *You talked about how important it is to have your own ideas, "your own garden," as you put it. But what is amazing in mathematics is that you go anywhere in the world, and other mathematicians have those same ideas, and you can talk to them about them.*

ALAIN CONNES : Well, the ideas are not exactly the same. Yes, we can communicate, but what is really interesting is to meet mathematicians with complementary ideas.

In 1978 I spent a year at the IAS [Institute for Ad vanced Study] in Princeton, and I met the person who would become my greatest collaborator, the representation theorist Henri Moscovici. The stay in Princeton was important to me because I met him there. Otherwise, I felt the IAS was a rather strange place. There was a huge cafeteria, where people would sit at different tables. I didn't find it very congenial, except for meeting my collaborator Henri.

ALLYN JACKSON : *Really ? Many mathematicians talk about the great atmosphere in the IAS dining hall and how they love to sit at the math table.*

ALAIN CONNES : Somehow, there was a huge contrast with IHES. At IHES the cafeteria is small, and people are forced to be together, whereas in Princeton you could easily sit alone at a table and be ignored.

Fortunately I met Henri Moscovici at this time, and we did for many years a lot of work together. Otherwise I would have been quite isolated, I think. He had many ideas that I was missing and knew things I did not know. This also happened in 1980 when I met Paul Baum. It was an encounter with somebody who didn't have the same way of thinking as I had. It was complementary. I met Paul Baum at a conference in Kingston, at the time when I had discovered noncommutative geometry. Before I went to Kingston I wrote a *Comptes-Rendus* note [9] about an idea that came from foliations. The point was that foliations not only have measure theory, which I found gives rise to factors that are exotic, but they also have differential geometry. I realized this differential geometry could be brought to bear in the noncommutative framework. In the *Comptes-Rendus* note I just mentioned, I had done the full noncommutative differential geometry for noncommutative tori.

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8. Two months after this interview took place, Vaughan Jones died at the age of 67.

9. Alain Connes, "C^*-algèbres et géométrie différentielle," *Comptes-Rendus de l'Académie des Sciences de Paris Série A-B* **290** (1980), n° 13, A599-A604.

Then I met Paul Baum, and he had exactly what I was missing. I had constructed, using geometric transversals, modules on the algebras of foliations, which meant that I had constructed elements of K-theory. But I didn't know how to construct them in general. Baum had exactly the idea, in a completely different topic, that would lead to the construction of general elements of K-theory. We met, and poof, there was a spark.

ALLYN JACKSON : *What was the other topic that he was looking at ?*

ALAIN CONNES : He was working on a geometric realization of what are called the K-homology cycles. K-homology was developed first in the Hilbert space language by Atiyah, and then by [Aleksandr] Mischenko, [Gennadi] Kasparov, and many others. The point of Paul Baum was to make it geometric. He had defined a geometric object that he was using only for Riemannian manifolds. What I saw immediately was that this worked for foliations as well.

Noncommutative geometry and physics

ALLYN JACKSON : *What is the main idea of noncommutative geometry ?*

ALAIN CONNES : There are spaces, like the space of leaves of foliations or the space of Penrose tilings, that, when you try to view them as ordinary spaces, are intractable. They become tractable provided you generalize the idea of Descartes of using coordinates, to situations where the coordinates no longer commute. Once you accept the use of noncommutative algebras as algebras of coordinates, then you discover that you can treat spaces that, with the ordinary tools, would be totally intractable.

At the end of the 1970s and in the 1980s, I began to develop geometry - full geometry, including differential geometry and de Rham theory, which gave cyclic cohomology - so that all the tools that we normally have would be available in this generalized, noncommutative setup. The beauty here comes from the fact that you are not just generalizing something ; these new spaces have totally new features. One of them is this God-given time evolution. The ordinary spaces are static, while these new spaces have the great property that they are dynamical and have this time evolution.

When you discover something truly original, you can be sure that people will be against you and will try to dismiss it. This is a fact of life. If you do something in the ordinary way, everybody will be happy and can understand it. But as soon as you do something that people cannot understand because they are not in the right framework, then you can be sure there will be a lot of opposition.

ALLYN JACKSON : *What is behind the opposition ? Is it just a technical barrier ?*

ALAIN CONNES : No, it's not a technical barrier. Mathematics is evolved and complicated as it is. You do not want to introduce something new. This is the reaction of people.

It's normal of course to be confronted with skepticism. For instance, with Henri Moscovici we solved the Novikov conjecture for hyperbolic groups using our work.[10] This was a problem that was

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10. Alain Connes and Henri Moscovici, "Conjecture de Novikov et groupes hyperboliques," *Comptes-Rendus des Séances de l'Académie des Sciences, Série I. Mathématique* **307** :9 (1988), 475-480.

known independently of the new technique. The new technique has to make its merits on problems that were posed before. Otherwise people will not accept it and will say, "We knew this before", or "Why are you interested in this?"

In the middle of the 1980s I discovered something I found very surprising. When you have these new spaces available, then you can rethink ordinary space time. What I found is that spacetime has a fine structure, which is not the ordinary continuum and which is just a little bit more complicated; it is noncommutative. When you take this fine structure into account, you find that pure gravity will give you the Standard Model coupled to gravity. At the time I wrote just one paper about this. [11] The idea came to full fruition in the 1990s, in my work with Ali Chamseddine.

What is mysterious and strange in the Standard Model is what is called the Higgs sector, though it is actually due to three people, [Robert] Brout, [François] Englert, and [Peter] Higgs. This sector was called the "toilet" of the Standard Model : it is something you really need in your house but you would not show it off to your guests. This sector is very strange. It gives masses to all particles, but it is due to a scalar field, so it is a field with spin zero. This comes out of the blue.

Now, from the point of view of noncommutative geometry, the mental picture is incredibly neat. If you think of spacetime as like a sheet of paper, it is two-sided. When you differentiate a function on this space, you can differentiate it on its restriction to the upper side of the sheet, or you can differentiate it on its restriction to the bottom side. But you can also differentiate it by taking the finite difference across the two sides - the differences of the values of the function on the two sides of the paper. That gives you a field of spin zero, the Higgs field. This tells you that, provided you refine the geometry of spacetime, you will understand why the Standard Model looks so complicated, even though it is just pure gravity.

Allyn Jackson : *So you have the piece of paper with the two sides - where is the noncommutative aspect ?*

Alain Connes : The noncommutative aspect comes from the fact that when you look at that finite difference, it be comes noncommutative differential geometry. There is also a slight amount of noncommutativity in the algebra of functions, and it is this amount of noncommutativity that actually generates the gauge fields of the strong force and the electroweak force. The development of these ideas came to a culmination in 2014, in a paper [12] with Chamseddine and [Viatcheslav] Mukhanov, where we really understood the noncom mutativity that one had to include in order to get the full story.

The starting point for that was in the 1980s, when de veloping noncommutative geometry occupied a lot of my time. I had some very explicit examples, like spacetime, which were motivating the general theory, and of course foliations. Another example is Penrose tilings. In the late 1980s I went to a conference held in a castle near Munich, Schloss Ringberg. [Roger] Pen rose gave an intriguing talk about Penrose tilings. These are tilings of the plane that are not periodic. They were discovered by logicians. The initial tilings used many different tiles, but Penrose simplified them to only two tiles, which is quite remarkable. You can tile the plane in many different ways with these two tiles.

11. Alain Connes, "Interprétation géométrique du modèle standard de la physique des particules et structure fine de l'espace-temps," *Comptes-Rendus de l'Académie des Sciences. Série Générale. La Vie des Sciences* **10** :3 (1993), 223-234.

12. Ali H. Chamseddine, Alain Connes, and Viatcheslav Mukhanov, "Geometry and the quantum : basics," *Journal of High Energy Physics* (2014) nº 12, 098. https://link.springer.com/article/10.1007/JHEP12(2014)098.

Penrose showed an amazing property of these tilings. If you have two tilings that are not the same, then you can take a portion of one of the tilings, and you can find that portion infinitely many times within the other tiling. He put up two transparencies showing this, and he said, "There is something quantum behind that." When I came back from the conference, I realized immediately that the space of Penrose tilings was a noncommutative space.

ALLYN JACKSON : *What does that mean, that "there is something quantum" about this space?*

ALAIN CONNES : What it means is that the algebra of functions will be noncommutative, so it will be a Hilbert space story. But here is what it means at the level of Cantor and of set theory. If you view the collection of Penrose tilings as a set, it has the cardinality of the continuum. But the claim that I make - and this is a characteristic property of noncommutative spaces - is that you cannot put it effectively in bijection with the real numbers. In fact, you cannot inject it effectively in the real numbers. If I have two different real numbers and look at their decimal expansions, they will be different at some point. But this is not the case for Penrose tilings, because if I look at them locally, I cannot distinguish between two of them.

When I wrote my book *Noncommutative Geometry* [13] in 1994, I put Penrose tilings at the beginning, because it is an example that is very striking. Penrose had the right intuition. The space has a topology, a noncommutative topology. And the Golden Ratio comes out by a miracle from the algebra.

ALLYN JACKSON : *How does it come out?*

ALAIN CONNES : When you have a noncommutative algebra, you have its K-theory, which was first invented by Grothendieck and was adapted by Atiyah to the topological framework and which makes sense in the noncommutative case. You can compute the K-theory and also map it to the real numbers by the trace, if there is a trace on the algebra. For the case of the Penrose tilings, when you map its K-theory to the real numbers, you get the Golden Ratio, just by a miracle.

In the new noncommutative world, there are things that were easy to adapt, like K-theory, which was almost built for noncommutative situations. Then there are things that were much harder to adapt to the noncommutative world, and this is what I did with cyclic cohomology at the beginning of the 1980s.

Connecting to the Riemann Hypothesis

ALLYN JACKSON : *Your work made connections to the Riemann Hypothesis. How did that come about?*

ALAIN CONNES : In the 1990s I collaborated with Jean-Benoît Bost on a system of quantum statistical mechanics that had a very striking property called spontaneous symmetry breaking. This can be explained quite simply. Imagine you are sitting at a round table with several people. On each side of each person there is a bread plate. As soon as one of the people decides to take the plate on the left, it is clear that all others will have to take the one on the left. But that first person could

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13. Alain Connes, *Noncommutative Geometry*, Academic Press, 1994. This is a much-expanded English-language edition of the French edition, *Géométrie noncommutative*, published by InterEdtions in 1990.

have picked the plate on the right, and then everybody else would have to do the same. That's called spontaneous symmetry breaking.

With Jean-Benoît Bost, we found a system with spontaneous symmetry breaking. [14] Its partition function was the Riemann zeta function, which was bizarre. It came out of the blue. Because of that paper with Bost, I got invited in 1996 to a conference in Seattle in honor of Atle Selberg, who had made a lot of discoveries about the Riemann zeta function. I went to the conference, and there was quite an interesting crowd of people, including Paul Cohen for instance and several physicists. I gave a talk about the work with Bost. Afterward Selberg came to me and said, "You know, it's unclear that your work will truly be related to the zeros of the Riemann zeta function."

ALLYN JACKSON : *Why did Selberg think that ?*

ALAIN CONNES : In my talk, the function was just appearing as a function ; the zeros did not have any meaning for my talk. So of course for him, it was not clear at all that there would be any relation.

I came back from Seattle, and instead of trying to adapt to the local time - as you know there is nine hours of jet lag - I kept living on Seattle time, more or less. I could do this thanks to the understanding of my wife ! I was not working. I was reading *The Right Stuff*, which recounts the story of Apollo 13. After one week, I suddenly realized that there is a space that pops out extremely naturally from the system we had with Jean-Benoît Bost, which was a noncommutative space and from which the zeros of zeta were appearing completely naturally. The terms of the Riemann- Weil explicit formula also came in extremely naturally.

A problem much stressed by some physicists in Seattle is that, when you try to realize the zeros of zeta as a spectrum - which everyone was trying to do - it is problematic because of a persistent minus sign in some of the terms. This minus sign prevents the naive expression of the zeros as a spectrum. When I found the terms of the Riemann-Weil explicit formula were appearing naturally, I understood that one shouldn't look for an emission spectrum but an absorption spectrum. Let me explain the difference between the two, because it is crucial in my work.

When you pass sunlight through a prism, the light decomposes into various wavelengths, or frequencies, and this gives you a beautiful image of a rainbow. After Newton, a German optician named [Joseph von] Fraunhofer, who lived at the beginning of the 19th century, studied this. He decomposed sunlight through a prism and then looked at it almost at the level of a microscope. What he found is that, in the beautiful rainbow, there are some dark lines. In fact, one dark line had been discovered before Fraunhofer, the line of sodium. But he discovered that there are in fact more than 500 dark lines. They are called absorption lines.

When light goes through the atmosphere of the sun, the chemicals in the atmosphere make transitions by absorbing photons at certain wavelengths. The corresponding wavelengths of light do not reach the earth, and thus you have the dark absorption lines. Around 1860, [Gustav] Kirchhoff and [Robert] Bunsen discovered that the dark lines of absorption spectra coincide with bright lines of emission spectra, which appear when elements are heated.

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14. Jean-Benoît Bost and Alain Connes, "Hecke algebras, type III factors and phase transitions with spontaneous symmetry breaking in number theory," *Selecta Mathematica* (N.S.) **1** :3 (1995), 411-457. https://link.springer.com/article/10.1007/BF01589495.

In mathematics, it is very easy to describe an emission spectrum and much harder to describe an absorption spectrum. My idea was that the spectrum giving the zeros of the Riemann zeta function was an absorption spectrum, which explained the minus sign : the absorption is like taking the negative of a picture, and it is this negative that gives you the minus sign. So I did a calculation at that time, in 1996, and it gave me the right spectrum. I was quite excited and wrote a *Comptes-Rendus* note [15] explaining this.

At that time I had the hope that this would give some insight on the zeros of zeta. But it was an absorption spectrum, so it is much more difficult to handle than an emission spectrum. That started a long story, which has continued up to now, in work with my collaborator Caterina Consani. The ideas remain very powerful. In a very recent paper [16] that we put on the arXiv, these ideas allowed us to make more progress. The outcome is that the space of primes could be seen coming out of a noncommutative space that was extremely natural.

ALLYN JACKSON : *Why are the absorption lines so much more difficult to deal with than the emission lines ?*

ALAIN CONNES : When you have an absorption line, it's a single line, so normally it would not be seen. Unless the line had some thickness, some width, you would not see it. Mathematically speaking, the zeros of zeta don't have a thickness. You have to artificially make them a little bit thick in order to be able to see them. And that's very hard. It is exactly this technical point that is now being treated much better but that took enormous time to understand.

This is a development on which I have been working, parallel to the physics development of the Standard Model, since the 1990s. These two developments for me are crucial. If noncommutative geometry were just dealing with very strange spaces, I don't think it would be very convincing to people. But for two fundamental spaces - for spacetime, the space where we live, and for the space of primes - it can bring something new and embody the intuition behind them.

Understanding renormalization

ALAIN CONNES : Another development also played a key role. Since the 1970s, I have been fascinated by a technique in physics called renormalization.

In the 1930s, when Dirac created quantum field theory, he quantized the electromagnetic field in a way that is truly mind-blowing. Einstein had this incredible intuition that somehow frequencies had to be quantized, so their energy was not arbitrary but had to be integral multiples of $h\nu$. This was like an *Ansatz*, a prescription. Dirac was able to make this a mathematical fact. His idea was to use exactly the noncommutativity to force a certain quantity to be an integer.

Once Dirac had done that, he tried to apply the same technique in order to deal with quantum fields of more complicated systems. But then nothing worked. The expressions he wrote down didn't make any sense. In the late 1940s, more precise measurements of what is known as the Lamb

15. Alain Connes, "Formule de trace en géométrie non-commutative et hypothèse de Riemann," *Comptes-Rendus de l'Académie des Sciences de Paris Série I Mathématique* **323** :12 (1996), 12, 1231-1236.

16. Alain Connes and Caterina Consani, "Weil positivity and trace formula, the archimedean place,", submitted to arXiv on 25 June 2020. https://arxiv.org/abs/2006.13771.

shift were obtained. Efforts to explain the measurements from the physics failed because the quantities that should have explained it were meaningless because they were given by divergent integrals.

It was a terrible time for physics. Then brave people like [Sin-Itiro] Tomonaga and [Julian] Schwinger and [Richard] Feynman came to the fore. They manipulated the infinities in order to extract finite quantities and compared those quantities to measurements. For what is called the anomalous moment of the electron, the precision of the agreement was the precision of the width of a hair in proportion to the distance from Paris to New York. Nobody could deny that they had stumbled onto something great. On the other hand, if you put a mathematician to look at what they were doing, you would hear the mathematician screaming!

ALLYN JACKSON: *Because of the way the physicists were manipulating the infinities?*

ALAIN CONNES: Right. They were manipulating the infinities in a way that was totally understandable. What they were doing is called renormalization. Starting in the 1970s, I became fascinated with this.

At the end of the 1990s, I was working with my collaborator Henri Moscovici. We were working on the cyclic cohomology adapted to a certain Hopf algebra. There was a visitor in IHES, Dirk Kreimer, who is really a hard-core physicist. He had a wonderful new idea that, when one is manipulating Feynman graphs, there is behind the scenes something like a Hopf algebra. We started working together with Dirk; that was in 1998. We put together all the mathematics that was required behind this Hopf algebra.

Then there was a moment of revelation that came to me in September of 2000. Normally if you make a discovery, you are up for one hour, and then you come back to earth. But then I was up for a week. The discovery was that this incredibly complicated technique that physicists are using for renormalization is in fact a well known technique in mathematics called the Birkhoff decomposition. It was pioneered by [G. D.] Birkhoff, and Grothendieck also used it to prove a very important theorem about vector bundles on the sphere.

After that, when I collaborated with Matilde Marcolli, we found that behind renormalization was not only the Birkhoff decomposition, but also an even more fundamental problem, the Riemann-Hilbert problem. We worked for several years on that. But in a sense, I had stopped thinking about renormalization, because in my mind it's resolved.

ALLYN JACKSON: *The mathematicians are no longer screaming?*

ALAIN CONNES: Right, now they understand. But it takes a lot of time because mathematicians don't know what renormalization is, and physicists don't know what the Riemann-Hilbert problem is!

Overcoming prejudice against algebraic geometry

ALAIN CONNES: I would like to continue the story of the number theory side, because the physics and the number theory were constantly intertwined in my mind.

Before I met Katia Consani and started a very long collaboration with her, I had a lot of prejudice against algebraic geometry. I was stupid, because when you have prejudice against something, very often it's just ignorance. I was ignorant. When Katia Consani and I started working together, I learned about concepts invented by Grothendieck, like the concept of scheme.

I talked before about the noncommutative space I discovered that embodies the prime numbers. If you looked at it as a noncommutative space, you could feel that fundamental understanding of this space was missing, in particular its relation to algebraic geometry and to other fundamental points of view in mathematics.

In 2014, we discovered with Katia Consani that there is a topos - in the sense of Grothendieck - that is the topos of arithmetic. This topos immediately gave rise to the same space as the noncommutative space that I had found in 1996. So this meant that this space, rather than being arbitrary or being constructed for the purpose of doing something, was in fact an absolutely fundamental space. We wrote a *Comptes-Rendus* note [17] about this.

Let me explain the notion of topos, because it is equally as important as noncommutative geometry and very much connected with it. Around 1958, Grothendieck discovered a new notion of geometric space, which he called topos. Normally when you do geometry, you put onto the stage the space you are studying. The main actor is the space, and you talk about the points of the space, you talk about its topology, its differential geometry, and so on. The idea of Grothendieck is that there is another way to comprehend the space. This other way is not to put it onto the stage but to hide it behind the stage, in what we call in French la coulisse. I am not sure of the term in English.

ALLYN JACKSON : *In the context of the theater, that would be "backstage."*

ALAIN CONNES : Yes, like in a theater, "backstage." The space in question will be in the backstage. You will never see the space. Its role will be that, while you are doing mathematics with the ordinary characters - the integers, the real numbers, the spaces you are used to work with - all of these things will actually depend on a parameter that is in the backstage. What is in the backstage will govern a randomness that will be inherited by the usual characters of mathematics that you are working with.

This is a fantastic idea. Technically speaking, what does it mean ? Instead of looking at the space, you look at what are called the sheaves of sets over the space. Then when you do set theory, you can do any mathematics you want, you can look at sheaves of groups or sheaves of topological spaces. You can recover the space and its topology, which is in the backstage, just by looking at set theory "with parameter". What is a point of the space ? A point of the space is a way to suppress the randomness in the events that are occur ring on the stage. Then when you use a point to look at what is going on on the stage, it is as if it is no longer random.

What I find incredibly revealing, is that when you compute the points of a topos, even of a very simple topos, you get in general a noncommutative space ! This is what makes the connection between the point of view of Grothendieck, of topos, and the point of view of noncommutative geometry. They are deeply interconnected.

ALLYN JACKSON : *You never got to discuss this with Grothendieck.*

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17. Alain Connes and Caterina Consani. "The arithmetic site," *Comptes-Rendus de l'Académie des Sciences de Paris,* **352** :12 (2014), 971-975. https://www.sciencedirect.com/science/article/pii/S1631073X14001666?via%3Dihub.

Alain Connes : No, unfortunately. By the early 1990s he had disappeared to a place in the Pyrénées. Even his family didn't know where he was. He stayed there until his death in 2014. It would have been very difficult to discuss with him. He became a mystic over the years. During the time when he was secluded in the Pyrénées, he wrote an enormous amount, tens of thousands of pages. The main topic was the problem of evil. I have read many of his writings, including an unpublished text called *La Clef des Songes* [The Key to Dreams], in which he tells the story of his father.

I never met Grothendieck, but I think that I know him so well, from his writings. In several of them, he complains that people don't understand what a topos is. It shows how common it is that mathematicians say "This is not mathematics", or "This is not serious", just because they don't understand. He suffered a lot from that. The concept of topos is an amazing discovery that gives a completely new way of thinking about mathematics. But unless you do an epsilon of progress in some definite topic that already exists and that is well paved, people don't pay attention.

I am not exempt of prejudices myself, so I understand perfectly why people would have these prejudices.

Allyn Jackson : *You had a prejudice about the idea of topos, and about algebraic geometry.*

Alain Connes : Yes. Before I understood what a topos was, I would say, "This is bullshit !" You really understand something only when you use it on some other purpose, and that depends on the occasion you get. In the work with Katia Consani, we finally understood that there is a topos, and it is extremely natural. This opened up a point of view that is totally different.

Allyn Jackson : *You made a link to new things, and you just jumped into them. You have done that several times in your life. What allows you to do that ? Is that confidence ?*

Alain Connes : No, no, no. I am not really motivated by confidence. Nor by curiosity. What I would say is, it's more anxiety. I spend much more time being anxious than being confident or being curious. My mind sort of constantly worries. It's not confidence - okay, I have of course some self-confidence, but it's not a kind of overreaching confidence, by no means. I knew only one person who had overreaching confidence, that was Michael Atiyah. I really liked him a lot. He could jump to other topics. But I am not like him. I am much more motivated by the fact that when I do not understand something, it makes me suffer. It puts me into a state of misery. I am feeling bad until I understand. That's exactly the motivating force.

This is also why I like very much to collaborate because then you share this uneasiness. You are not alone ! And I love to collaborate with people who are more confident than I am, exactly because of my problem.

Grothendieck wrote something in *Récoltes et Semailles* that I like to quote. He said that to fear the error is the same as to fear the truth. But if one is ready to confront the error, then this fear becomes a blessing. One crosses this difficult time and comes out with much more.

A grand unified theory from gravity

ALLYN JACKSON : *You talked before about looking at spacetime from the noncommutative point of view, which gives the Standard Model coupled with gravity. That sounds like a "grand unified theory" of the fundamental forces. Is this what you are saying ?*

ALAIN CONNES : Let me explain that in some detail. In the mid-1980s, I had realized that you could get the Higgs sector of the Standard Model from the geometric picture. But I did not have something that would unify gravity with the other forces ; that only came in 1996, when I began to work with Ali Chamseddine. We realized that if one takes a spectral point of view of geometry, then there is a natural manner of defining what is called in physics an "action", for that geometry. This will measure how suitable the geometry is. This action turned out to be spectral and depends only upon the line element, upon its spectrum.

To explain this I have to make a digression, but first, to answer your question : it is a unification. You consider pure gravity on a geometric space, and when you compute what you get from pure gravity, you not only get the ordinary gravitation field, but you also get the bosonic fields of the Standard Model and the fermionic fields. So you do get the full picture out of pure gravity. Some people have tried for instance to obtain gravity from gauge fields, but what I am saying is quite different. What I am saying is that once you introduce some fine structure in the geometry of space time, then pure gravity will give you not only the ordinary gravitational force, but also the other forces of nature, which are the electroweak and the strong force. So it's not a unification from gauge fields, but it's a unification from gravity.

Now let me enter the digression. Why is it natural to view a space spectrally and to define the action from a spectral invariant ? This goes back to the story of the measurement of length. It starts sometime before the French Revolution. At that time France had no unification of the meter of length. If you were for instance in the trade of linen, you would need a unit of length to measure pieces of linen to sell. So every town or village displayed at its entrance a unit of length. But the units were often different, which made things extremely complicated. People started to say they needed a way to unify the measurement of length. Scientists in France, and also in England, thought a lot about this. They decided that the best idea would be to take the largest available object, which is the earth, and then define the unit of length as some proportion of the circumference of the earth. They decided one meter would be one-forty millionth of the circumference of the earth.

To measure the circumference, you measure an angle that can be defined by looking at stars and then concretely measure the distance between the two points that limit the angle. The angle chosen was between Dunkerque in the north of France and Barcelona in the north of Spain, which are more or less on the same longitude. In 1792 two French astronomers were put in charge of measuring this distance. They did triangulations, which means that they would put a telescope on top of a hill and make some measurements. Of course when they did that in Spain, which was in a war with France, it was quite difficult to explain that they were not spies ! There are a lot of interesting stories about what happened to them.

They eventually obtained a reasonably precise measurement. The measurement was used to cast a platinum bar to represent one meter, and the bar was deposited in the Pavillon de Breteuil, which is near Paris. This was considered the universal unit of length in the metric system. Of course, this was not very practical, because if you are in a foreign country and you want to measure your bed,

you have to travel to the Pavillon de Breteuil to know how long a meter is! So replicas were made and distributed.

That was fine until the beginning of the 20th century. By 1925, people had better ways to measure length using spectroscopy, by comparing a given length with a wavelength of a known atomic transition. They then realized that the famous unit of length deposited near Paris didn't have a constant length.

ALLYN JACKSON : *Because it was made of metal?*

ALAIN CONNES : Exactly, because metal contracts and expands. They decided that the apparatus that allowed people to see that the length of the platinum bar was changing was a better device to define the unit of length than the bar itself. For some time they used krypton, which was not very satisfactory, because krypton is very rare. Eventually they switched to cesium. Today one can buy in a store a cheap instrument based on the cesium transition that will give you measurements of length with a precision of 10 decimals.

When I defined noncommutative geometry, I defined it as a spectral triple. The shift from the classical standpoint, which was the Riemann standpoint on geometry, to the new standpoint that I had defined, which is spectral, is exactly parallel to the shift that occurred in physics between the definition of the unit of length by means of the platinum bar, and the definition by means of comparison with wavelengths of a fixed chemical. It's very striking.

The action principle, which we defined with Chamseddine, then allows you not only to recover gravity but to find gravity coupled with matter. This action is simply measuring the spectrum of the line element. The formula for distance in noncommutative geometry will use the fact that the line element does not commute with the coordinates in the space.

On the conceptual level, what this means is that a paradigm of noncommutative geometry, that of a spectral triple, in fact is very closely related to physics. It has a big advantage in the quest to unify gravity with other forces, which is that it is both quantum and geometric. After the quantum was discovered by Heisenberg, von Neumann understood that the correct stage on which to develop quantum mechanics was Hilbert space. So the geometry I am talking about is the geometry that is on that stage, in Hilbert space. Hence it's congenial to quantum from the start.

The influence of quantum entanglement

ALLYN JACKSON : *Some complexity theorists recently proved a result[18] involving quantum computing and entanglement, and they thereby resolved something called Tsirelson's problem. This in turn resolved the Connes Embedding Conjecture. Can you tell me what this conjecture is, and how you see this new work?*

ALAIN CONNES : First of all, it's not a conjecture, it's a problem. When I was working in Kingston in 1975, at some point I stumbled on a certain property of a factor. I saw immediately that this property was less restrictive than being hyperfinite. I was proving something about hyperfinite factors

18. Zhengfeng Ji, Anand Natarajan, Thomas Vidick, John Wright, and Henry Yuen, "MIP* = RE," *submitted to arXiv on 13 January 2020. https : //arxiv.org/abs/*2001.04383.

and had found examples of factors that were not hyperfinite, but which did fulfill this property. The property is that the factor is not hyperfinite but resembles a hyperfinite factor as much as possible. Technically, it means that the factor can be embedded in an ultraproduct of hyperfinite factors. The question is whether every factor of what is called Type II_1 has this property. What I had noticed at the time was that all the books and papers I knew about were verifying this property for the associated factor. So I wrote about this in three lines in my paper. And - this is the honest truth ! - I never thought more about it.

Then these few lines that I had written were picked up by a number of different people. [Eberhard] Kirchberg proved that this problem is equivalent to something he was working on. It was used by [Dan Virgil] Voiculescu in defining his new notion of entropy and by other people in quantum theory like [Boris S.] Tsirelson.

I don't know how much the complexity theory paper has been checked. Apparently it's quite long. What it is saying, roughly, is that there are things that cannot be approximated at all by something that is finite-dimensional. It's something quite weird. It would probably be quite significant if they have really found an example of this. Whether the example would have relevance for physics, I have no idea. I always had the feeling, or the belief, that nature is really finite-dimensional in a sense ; even though we are approximating it by something continuous, everything is essentially finite-dimensional.

As I said, I never thought myself about the problem. I am really the worst person to ask !

ALLYN JACKSON : *The complexity theory result has to do with quantum entanglement. Does this interest you ?*

ALAIN CONNES : Entanglement is something I find extremely interesting and important, but for a different reason. The book we wrote with my wife and Dixmier contains a provocative phrase. In French, it's "*l'aléa de quantique est le tic-tac de l'horloge divine.*" In English this could be "the vagary of the quantum is the tick-tock of the divine clock."

The reason time is passing, and passing in a way that we don't control at all, is precisely the lack of reproducibility of the quantum. When you send a photon to a target, you will never be able to reproduce the result. It's something totally random and uncontrollable. I was able, to some extent, to develop a theory that would have time springing from this quantum randomness, as I explained earlier. But if you have a time at point A and a time at point B, and quantum randomness at A and quantum randomness at B, there would be no link at all. Right ? No, that's not right, because if at A and B you are measuring the quantum randomness from entangled states, then you will get results that are correlated. The quantum randomness is correlated by quantum entanglement.

One would need a mind like Einstein's to invent a notion of time that would spring from the quantum and that would make us at peace with entanglement and tell us that entanglement is just the harmony or the correlation between the various randomness at various points. I believe the person who might be closest to that is [Anton] Zeilinger, a Swiss physicist who has done experi ments on entanglement for very distant points, points that are more than 100 kilometers apart. [19] I heard

19. X.-S. Ma, T. Herbst, T. Scheidl, D. Wang, S. Kropatschek, W. Naylor, B. Wittmann, A. Mech, J. Kofler, E. Anisimova, V. Makarov, T. Jennewein, R. Ursin, and A. Zeilinger, "Quantum teleportation over 143 kilometresusing active feed-forward," *Nature* **489** (2012), 269-273. https://www.nature.com/articles/nature11472.

a talk by him in which he said that they are looking for things that cannot be entangled in their experiments. The mathematics of the time evolution, plus understanding of entanglement - there is enough stuff there to create a completely new point of view on time.

What is extremely troubling about entanglement is that if you have two entangled states and you make an observation on one, it immediately acts on the other and gives you a correlated result. Einstein was upset about this and called it "spooky action at a distance." Spooky is the right word. Alain Aspect has made measurements showing that the effect is much faster than the speed of light. This seems to contradict the principle of relativity. But in fact it does not, because you cannot transmit the information. Suppose you have two correlated states, so that if you find a plus in one, the other will be minus, or if you find minus, then other will be plus. The answer, plus or minus, is not my own choice; it is a result of the experiment. The other guy had minus or plus, but he will get no information from me. It is not a way to transmit information, so it doesn't contradict the principle of relativity.

Still, it's troublesome. If I am in a different frame, it will not be that something was first done at point A and then point B reacted; it will be that it was done first at point B and then A reacted. When you have space-like events seen from different frames of reference, one can be before and the other can be after, or vice versa. It's your own choice. This means that the notion of causality, or the notion of time, is totally up set by the phenomenon of entanglement. I interpret it as meaning that there is something more primitive than the passing of time, which is quantum randomness.

The gift of the Riemann Hypothesis

Allyn Jackson: *How do you see the prospects for proving the Riemann Hypothesis?*

Alain Connes: I worked a lot on that and made some progress recently with Katia Consani. But until you are done, you cannot say anything.

But there is a mental picture of the problem that is comforting. The problem is like an infinite pole, and you want to show the pole is vertical. This is the mental picture. With Katia, it is as if we are building foundations that are tighter and tighter and tighter. There is an infinite set of stairs, but each step you take on the stairs is actually blocking the pole, to make it more and more vertical. What we have done recently is to cross the first step of the stair.

The stairs are infinite. But the beauty is that, because of an idea of André Weil, you only have to consider finitely many primes at a time to solve the problem. If you stick to that idea, then you are sure not to fall into the black hole of proving results that are equivalent tothe Riemann Hypothesis. What I found in 1996 was something that is difficult when you apply it to the infinitude of primes, but that, when you apply it to only finitely many primes, gives you exactly the Hilbert space and the quantized calculus framework that is apparently very suitable to attack the problem.

One can be hopeful, but until you are done, forget it - you cannot say anything. I like to work on this problem because it is a test of myself that I cannot escape from. It's not like you build a new theory, and then you can think you are the greatest. In mathematics, there is no better way to progress than to be confronted with a problem that you cannot solve. If you work on a problem that you can solve, it means that it's not the right problem. Fighting with a very hard problem is a much

better way to build a mental picture than when you are working on an easy problem. When the mind is blocked, it has much more power to build and to conceive. I see a problem like that as a gift.

With my wife and my teacher, Jacques Dixmier, we wrote a second novel called *Le Spectre d'Atacama*.[20] It recounts the story of a mathematician who is confronted with the Riemann Hypothesis. He realizes that beings from outer space are sending spectra to the earth and these spectra contain the zeros of the Riemann zeta function. I recently published in the *Journal of Mathematics and Music* a paper[21] that came from an issue we were exposed to while writing the book.

The mathematician and the other scientists had to be sure that the being communicating with them from outer space was an intelligent being and not a machine. So they had to devise a Turing Test that would make them completely sure. Such a test is possible, and it is related to André Weil's work on the Riemann Hypothesis. He discovered that when you work with function fields, all zeros of the analogue of zeta are on the critical line, and they are periodic - they repeat periodically.

What we realized when we were writing the book is the link with what the composer [Olivier] Messiaen had invented with his *rythmes non-rétrogradables*. The rhythmic patterns of Messaien have exactly the same property as the periodic patterns that you find from the zeros of Weil's analogue of the Riemann zeta function.

Weil's rhythmic patterns are associated to each prime. What the characters in the book did was to send into outer space the patterns associated to the primes, but they would omit one prime. If the beings receiving the message were really intelligent, they would answer by sending the pattern for the missing prime.

ALLYN JACKSON : *Is your wife a writer ?*

ALAIN CONNES : My wife is retired now, but she was a teacher of Latin and Greek in high school. She is a literary person. We are very complementary. She knows so much that I don't know.

ALLYN JACKSON : *Dixmier wrote science fiction when he was younger, right ?*

ALAIN CONNES : Yes, he wrote science fiction and also police stories. The way our first novel started is that in the summer of 2012 Dixmier sent us a postcard. The postcard said, "I have the title of the book. You write it, and I will proofread it !" Of course we laughed.

Then my wife and I made a trip to Venice and were visiting a small museum with an exhibit of some very striking modern art, including a sculpture by [Maurizio] Cattelan. The sculpture showed nine dead bodies, full size, aligned next to each other. I saw the sculpture, and just then a prize had been given to nine people![22] This immediately kicked something in my mind, and I started writing a draft of the book that Dixmier had suggested.

When my wife saw the draft she said, "No, this is awful !" Then she took over. She managed to

20. Alain Connes, Danye Chéreau, and Jacques Dixmier, *Le Spectre d'Atacama*, Odile Jacob, 2018.

21. Alain Connes, "Motivic Rhythms," to appear in *Journal of Mathematics and Music*. https://arxiv.org/pdf/1812.09946.pdf.

22. The Breakthrough Prize in Fundamental Physics was awarded for the first time in 2012, to nine individuals : Nima Arkani-Hamed, Alan Guth, Alexei Kitaev, Maxim Kontsevich, Andrei Linde, Juan Maldacena, Nathan Seiberg, Ashoke Sen, and Edward Witten.

salvage the idea, though what she wrote was totally different. She started writing about a young woman physicist who is asked to become the head of CERN. So that was the starting point of the book. Amazingly, a few years after the book appeared, an Italian woman, Fabiola Gianotti, was nominated as the head of CERN!

I always like these motivations that are very odd, very strange, like a kick, to start something.

ALLYN JACKSON : *You seem to have a really fun life.*

ALAIN CONNES : Sure. I try at least! But as I said, it's not so much fun because of the persistent anxiety.

ALLYN JACKSON : *But as you said in connection with Grothendieck, the anxiety pushes you towards the truth.*

ALAIN CONNES : Exactly.