Great interview with mathematician Alain Connes

Nicolas Martin : We often find in great scientists this common point, this line of escape or this beautiful escape to the world of the arts. Poet, painter, musician or novelist, in this case for Alain Connes, Fields medallist and Gold medal from CNRS, mathematician at the origin of noncommutative geometry, a branch of mathematics that aims to embrace GUT, the Grand Unified Theory, the theory of everything that would reconcile general relativity and quantum mechanics. Novelist, therefore, musician too, but above all and forever, obsessive researcher. Alain Connes is our great guest for the hour to come. Welcome to *La méthode scientifique.*

Thank you and hello Alain Connes.

Alain Connes : Hello.

Nicolas Martin : A thousand thanks for accepting our invitation, so I'm going to a quick summary presentation that I will leave you to complete for our listeners who don't know you yet. You are therefore a mathematician in this paradise for researchers that is the IHÉS, the Institute of Advanced Scientific Studies in Bures sur Yvette.

Alain Connes : I'm first at the Collège de France, let's not forget, the Collège of France.

Nicolas Martin : I'm coming, also Professor emeritus at the Collège de France, holder of the Chair in Analysis and Geometry, member of the Académie des sciences française. But other Academies of Sciences, including the Academy of Sciences in the United States, but also in Denmark and Norway. You got the Fields medal which is, I repeat, the greatest distinction mathematical in 1982 for your work on operator algebras.

You could say that you have somehow revolutionized algebra by founding noncommutative geometry, you will talk about it again and the CNRS has awarded you his Gold medal in 2004 for solving the mathematical problems raised by quantum physics and relativity. And you just published your second novel after *"Le théâtre quantique"*, *"Le Spectre d'Atacama"*, co-written with your wife Danye Chéreau and your former thesis director Jacques Dixmier. It's up to Odile Jacob editions. What

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should one add to this description, Alain Connes ?

Alain Connes : Let's say that if you want, indeed, it's a scientific journey, we can now look back on it. And I will start by saying, if you want, that each mathematician is a special case. So I mean there are no generalities to make and in fact, the course that I followed, I put a lot of time to find my way, that is to say, initially, if you want, I had started by doing logic with non-standard analysis, with Gustave Choquet. I had done some number theory too, and it's finally with Jacques Dixmier that I found my way. And so, in fact, the journey begins with it, with the operator algebras and in fact, with, if you will, what von Neumann had understood from the discoveries of quantum mechanics, it was von Neumann, who had formalized the quantum mechanics and therefore the formalism that he had developed, if you will, hasn't changed since, you could say that this framework he created, the framework of Hilbert space, vectors space in Hilbert space, states, etc., is something that has never been questioned since the 1930s. But it has been shot further, with a collaborator called Murray, and basically, if you will, von Neumann asked himself the question of when could we define a subsystem of a quantum system. That is to say that when we take a system quantum, normally, it involves all operators in Hilbert space. No, that's a bit technical, but von Neumann had asked himself the question of knowing when do we have a subsystem ? And at the start, you would think that simply when you have a subsystem, Hilbert Space is factorized into the product of two subsystems, but von Neumann had thought much deeper, trying to understand at the algebraic level, so, we come back to algebra, at the algebraic level, how this factorization manifested itself. And with Murray, they had an extraordinary surprise, that is to say that they found that beyond the very simple factorizations of the Hilbert space in tensor product, as we call this in mathematics, there were algebraic factorizations, which gave the notion of factor, that is to say that in the language of operator algebras, there is an essential notion that we must come to understand from the start as coming from an essential problem of quantum mechanics, which is to know when we can characterize a subsystem. So what ? The wonder that happened is this, the creation of factors by von Neumann. Dieudonné called them von Neumann algebra, since they were due to von Neumann, Dixmier worked a lot on it. And when I arrived, I was lucky to arrive at a good time. It was at the time when a Japanese mathematician named Tomita has, maybe 5 or 6 years ago, find a very, very interesting theory. And I was lucky to discover that fact, the evolution over time that was associated with each state, normally, in an algebra, after doing very, very, very complicated calculations during months and months, I finally discovered that this evolution over time, it was unique, it was actually independent of the state, modulo the so-called interior automorphisms, it is something invisible. So, in fact, I understood at that point that, if you will, these von Neumann factors when they were of a fairly exotic type called type III, they spawned their own time. And because they generate their own time, it has created a lot of invariants that allowed to completely unlock the classification of these factors. These factors appeared to be intractable before, and in my thesis, under Jacques Dixmier, in fact, I showed how we could, if you will, reduce these factors to much simpler things thanks to this evolution in the weather and how they had, if you will, all kinds of invariants, like the periods, etc., etc.

Nicolas Martin : It is noncommutativity, that is what must be remembered.

Alain Connes : What you have to remember, at an abstract level, at the conceptual level what must be remembered is that noncommutativity was discovered by a physicist, by Heisenberg. So, that was a discovery, almost, how to say after... almost from experience, that is to say that Heisenberg has based himself on the laws of spectroscopy. That is to say, we observe spectra. These spectra have very specific properties and Heisenberg understood, from what we call the Ritz-Rydberg principle of composition, that in fact, if you want the algebra which was underlying quantum mechanics, he understood that in 1925 and it's a fundamental discovery, was a noncommutative algebra. I don't recall the anecdote, of course, which was that he was on the Helligoland island, that he was alone. He could finally work quietly because he had no more lessons to give, because he was sent there, because he had a cold hay. He lived with an old lady, he could work as long as he wanted. He was doing calculations, very complicated calculations. And then one night at 4 a.m. he understood.

Nicolas Martin : Eureka ! It exists, therefore !

Alain Connes : It exists. And he had before his eyes, he says, an absolutely wonderful landscape, which was almost scary of novelty. It was the landscape of quantum mechanics and it was the landscape of the noncommutative. What Heisenberg understood was that when we work with a microscopic system, a very small system, we no longer have the right to swap letters when doing physics calculations, you know, when we write $e = mc^2$, we could write $e = c^2$ times m, it would be kif-kif, it would be the same. Well.

Nicolas Martin : That is commutative.

Alain Connes : That is commutative. But what Heisenberg understood is that when we work precisely, for example, with position and momemt, well, it is the speed multiplied by the mass of a microscopic particle, at that time, we have to be careful, just as we are careful when we write, you see. When we write, obviously, we don't have the right to swap letters, since if we swap them, that makes an anagram, we can get anything starting from something. The first book we wrote with Danye Chéreau and Jacques Dixmier,...

Nicolas Martin : *Le théâtre quantique...*

Alain Connes : Yes, *Le théâtre quantique*, we cite anagrams, so, I mean, for example, *L'horloge des anges ici-bas* and *Le boson scalaire de Higgs*. We see that by swapping the letters, we can change the meaning completely.

Nicolas Martin : It is the happiness of our colleague Etienne Klein.

Alain Connes : Absolutely. Etienne Klein is a great specialist in anagrams.

Nicolas Martin : Alain Connes, that makes Nicolas Anne, you see, it's about near my math level.

Alain Connes : No, but I received an email from someone. I did absolutely not understand what he meant. I thought he had gone mad, but there was the anagram of my name five times. Okay, which is easy to find, I mean... So going back to Heisenberg, if you will, he made this extraordinary discovery, which is that when we work with a microscopic system, what we calls the observables, the natural variables of the system no longer switch between themselves. What does it mean ? It means that when we take what we call in physics the space of the system phases, it is a space which no longer corresponds to the description that Descartes made and which was at the source of all the algebraic geometry, what is called algebraic geometry, i.e. on the one hand, there is geometry, and on the other side, there are the coordinates of space like Descartes defined them, but these coordinates usually switch. Heisenberg's discovery is that when we take the space of the phases of the physical, we can no longer assume that the coordinates commute. And what is the root of noncommutative geometry is exactly that. That is to say, there are spaces which are in fact natural spaces, they are not pathological spaces or whatever. There are natural spaces in which, precisely, the coordinates do not more commute. So actually, if you will, what made the theory interesting, what made the theory really interesting, because generalizing algebraic geometry in cases where the coordinates no longer commute, it seems a tedious task, and which does not hold big surprises. But what motivated me, if you will, to develop noncommutative geometry, this is precisely the work I had done in my thesis under the supervision of Jacques Dixmier, and which had shown that a noncommutative space, i.e. a noncommutative algebra, generates its own time. And then, if you want, it's so new, compared to the ordinary geometry ... What does that mean ? It means that ordinary geometry is commutative, it is static, it does not move, while noncommutative geometry automatically generates its own time. And this time will allow us to do things that we would have no idea of doing otherwise. In particular, it allows to do the thermodynamics of a noncommutative space. It allows for example to have a noncommutative space, for example, and to cool it. So this is completely unexpected, if you will, it is something that is completely new. And that's what, of course, motivated me for years and years, for practically all of my scientific journey, exploring these spaces, exploring geometry for these spaces which are completely new.

Nicolas Martin : And this notion of time, by the way, that you explored also with Carlo Rovelli, whom we received here very recently. We go and put the link back on Twitter feed. I would like to question, Alain Connes, something that I often do with great scientists who follow each other at this micro, that's the question of vocation. You describe yourself at the start of your career that you come telling us brilliantly and this anecdote, that you ended up to tell us, of Heisenberg, finally, despite having said you would not. One describes you as a young mathematician with exceptional talent. What is the feeling... What is your conception of this vocation, of this attraction for mathematics ? When is it born ? How does it germinate ? Is there a vocation or is there not a vocation, is it a mishap ?

Alain Connes : Well, I think it's something that comes about rather slowly, that is to say that, if you want in my studies, indeed, quickly enough, I spent much more time trying to develop my own ideas, and to create my own ground than to be at school and to take classes, etc. So it happened actually happened very early and I remember, for example *(small laugh)*. I remember that when I was a child, I think it was in course of Seconde or Première : I had a math teacher and he said in the class that there was no formula which gave the number of prime numbers smaller than n. So obviously it's not true, I mean. I think what he had in mind is that there is no simple formula ; in fact, by the way, there is a simple formula, but it is not very, very useful. I can give it to you, so we'll see.

Nicolas Martin : Give it to us.

Alain Connes : So it's not a formula for $\pi(n)$, the number of prime numbers smaller than n. It is a formula for $n - 2\pi(n) - 2$. But anyway, whatever. And then it's only true for n greater than 13. Okay ? But still, it's very simple. It is the integer part of the sum from 1 to n of cosine of $\pi\Gamma(k)$ over k. Okay, we can't say it's very complicated. Okay. So, the next day, I came back, I came back and I gave my teacher a formula that was much more complicated than that. But from that moment on, I had taken a step which is an essential step for the young mathematician and this step essential is to believe in yourself, that is, not to give credit to authority. And that is extremely important. And I think math are a subject in which it is possible. It would be much more difficult in chemistry, in history, etc. Because there, knowledge plays an absolutely essential role...

Nicolas Martin : The observation ?

Alain Connes : Not only, but the accumulation of knowledge, while in mathematics, we can very well find ourselves face to face with a problem. The problem is very easy to apply and a priori, there is no reason why if we find a solution, it is not fair. So math is very special in this sense where there is not, if you want a priori, a kind of bead of knowledge, of knowledge that prevents a young person who is starting to understand something no one else understood. This is extremely important.

Nicolas Martin : Neither God nor master in mathematics.

Alain Connes : Yes, in a way. I'm going to say, and in fact, one of essential conditions in the path of a mathematician is to arrive at questioning yourself, that is to say if you want, if, from the moment one believes that one is stronger than the others, etc. There, it is the beginning of the decline. I think it is absolutely essential to never think that you have acquired enough knowledge, etc. or I think that it is essential not to believe that the path we are following is necessarily the right one. I think that this is one of the essential subjects of our book with Danye Chéreau and Jacques Dixmier. So, in *Le Spectre d'Atacama*, we describe the journey of a mathematician who is called Armand, Armand Lafforêt. And we just highlight this quality, that essential property of doubt. Why ? Because nothing says the way in which we are engaged when we want to solve a problem be the right one. It is necessary to constantly question yourself. We must constantly ask ourselves the question of know if, of course, if we reach the end, so much the better. But when the problem is very difficult and the problem we are talking about in the book is an extremely difficult problem, in these cases, actually, there is no other way out than constantly doubting and being able to question yourself.

Nicolas Martin : On precisely this relationship to your work, on this relentlessness to solve the problems, you tackled Riemann's conjecture, the Riemann's hypothesis which is the eighth of 23 mathematical problems for the 20^{th} century of Hilbert, that has until now not been proven, and I speak under your control. You talk, Alain Connes, about that and on your work in general, of the mathematician's obsession, there is something obsessive in this work ?

Alain Connes : Yes, in fact, I was interested in this problem completely by chance. You have to know that. What does this want to say ? That is to say that as it is one of the big problems, my starting principle, it was the opposite, that is to say it is always to remain marginal, to remain a little ambush and never take an interest in a problem like this one. Except that what happened was in 1996, and I was invited to a conference that was for the 70th anniversary of Atle Selberg. So Atle Selberg is a very, very great Norwegian mathematician who worked, for him, tremendously valiant on Riemann's hypothesis and found great things. And on the occasion of this meeting which took place in Seattle, in 96, there was a day... Well, I did my conference because I found with a collaborator, with Jean-Benoît Bost, we had found a system of quantum mechanics which was related to Riemann's zeta function, but it appeared to be related to it in a very peripheral manner, that is to say the zeta function appeared as what is called the partition function but it was peripheral. Now, what happened is that I gave my talk. And then, at the end of my conference, Atle Selberg came to see me and he said to me "It is not so clear that what you do is related to the Riemann hypothesis."

Nicolas Martin translates this sentence in french.

Alain Connes : Exactly. Then afterwards, there was, we had a meeting, etc. Then when I got home, I was really thoughtful, for a week. At the time, there was no email. I was not looking at my emails. I could be completely disconnected. I was jet lagged for about 8 days. And then, after 8 days, I realized that in fact, the system that we had defined with Jean-Benoît Bost gave exactly the space that people had sought to about that. So hey, then I said "gave *the* space". Nothing says it is still the right one. Nevertheless what it showed immediately, it showed that a formula which is essential in this theory called the explicit Riemann-Weil formula appeared completely naturally from the geometry that we had defined. So, if you want, I wrote a note to the Accounts. And then, thread by needle, I was caught in this kind of situation in which one does not control more, because it's true that if you want, as soon as someone is interested in this problem, basically, I'm kidding of course, but basically, if you want, the other mathematicians wish you that you fall over and above all, that you does not resolve it, and for a good reason.

Nicolas Martin : The mathematical world is even more anarchic than we have ever imagined it before starting this show.

Alain Connes : In fact, it's more complicated than that because in fact, how to say, it is very interesting, the sociology of the mathematical environment. But this assumption, Riemann's hypothesis, you have to understand in fact that without this being obvious, it is behind countless very fruitful developments in mathematics of the 20^{th} century. It started with the theory of almost-periodical Bohr's functions. It continued with everything André Weil did and then, well sure, Hasse, Artin, etc. on geometry with finite characteristic, what Deligne did, what Grothendieck did. So if you want, there is a huge influence from this conjecture on the development of mathematics. And in any case, what would sadden me terribly is if it were resolved anecdotally. And in fact, I recently, for example I made money from a number theory journal which quite often receives articles which claim to demonstrate this assumption. In fact, they send it to me and they pay me when I find the error. Why ? Because therefore, I mean. It's very, very complicated. It's an extremely complicated, extremely interesting, extremely mentally interesting problem, because in fact, what is likely is that it will be demonstrated only when the surrounding landscape will be fully revealed. It's a little like a mountain peak. But before we really understand what's behind, apparently, there is no way to cut, if you will, there is no shortcut and if there was one, it would be a bit catastrophic because it would mean that the magnificent landscape that we have to discover about this guess, well, it wouldn't have been released.

Nicolas Martin : And at 4 :22 p.m., we continue our interview with Alain Connes, who has just published a novel, his second, *Le Spectre d'Atacama*, co-written with Danye Chéreau and Jacques Dixmier, since we were talking about the vocation in your mathematical career, Alain Connes, where does this will come from, this desire for romantic writing, for fiction ?

Alain Connes : Ah then that, it's a desire for freedom. In fact, that means that we have discussed a lot together, the three authors. But mathematical work is work in which, of course, imagination plays a significant role, it is obvious. But this imagination is terribly corseted. That is to say, there is a mathematical reality. I use the computer a lot, a lot. And this mathematical reality, it is undeniable. That is to say that if we have an idea of a formula, etc., we can try to check it out, and if it works, it works and if it doesn't work, it doesn't work. So imagination plays a big role. Of course, it has the role especially, I would say, of the creation of mental images, that is to say that when I say imagination, it's a lot more, the fact that by looking for a problem, even if we cannot solve it, by done, when we can't solve it, it's better because it means it's a problem that allows us to improve ourselves, at that time, we create mental images. When you see someone on the subway reading a music score, if you're not a musician, that doesn't mean anything to you. If you see a mathematician who reads a sheet of math, it doesn't mean anything to you neither. And on the other hand, a mathematician is fine with this speak right away. It will speak to him right away because he has mental images. And these mental images, they wake up as soon as he sees the corresponding formulas. So this is extremely important. Unfortunately, it is very hard. This is very, very difficult, that is to say, well, we can have an idea. And then, after a moment, when we try to write a demonstration, no, there is something that doesn't stick, etc., so if you want, we come up against a reality which is extremely resistant. On the other hand, in the romantic writing, which is this pleasure that we all had the three, these two times, but especially the second, because we spent a long time to write this second book, in this romantic writing right there, imagination can unfold. And in fact, what strikes me when I look at this book, it's especially with current developments, what strikes me is the infinite freedom enjoyed by the hero, who is Armand.

Nicolas Martin : In which it is difficult not to recognize you.

Alain Connes : Yes, but in fact, it's not true, there are a few ingredients.

Nicolas Martin : He is a mathematician, he works at IHÉS...

Alain Connes : Yes, of course, but in fact, no, no, no, no. In fact, he's a character from a novel and a character from a novel who enjoys a freedom which, unfortunately, will be more and more difficult to have. For example, well, he's going to Chile. Afterwards, he decides to take a boat, he goes to the Staten Island, etc. So what ? We say to ourselves that right now, he would have had a Facebook account, that people would have glimpses that he is no longer responding, that he is no longer there. They would have gone and look for him. So this notion of fundamental freedom, this magnificent freedom is present in the book. It shows, if you will, how essential it is to maturation of an idea, etc., precisely when the mathematician is obsessed with an idea. But what scares me is how much it risks disappearing. You know, I always tell this anecdote which had struck me so much, which was at the time of an unexpected visit by the President of the United States to the Princeton Institute and the Director of the Institute was showing him around the offices since hey, he wanted to show... At one point, they knocked on the door of a mathematician and they entered the office. They had found the mathematician lying on his table, sleeping. So here, when now, they would have entered, what would they have seen ? They would have seen the mathematician, in front of his computer, answering 36 solicitations, in general completely without interest. He should have written a report or do etc. But if you want, this notion of doing nothing, this notion of leaving your mind completely freewheeling and being able, precisely, at some point, to wake up and tell you there it is, there is something, etc., well, this notion it is, unfortunately, very, very threatened by technology, by the fact that we are more and more regimented, more and more corseted, more and more labeled. So, I'm going to say, reading this book, I think, gives pleasure in that sense, that is to say we see this pure state being threatened, disappearing, unfortunately.

Nicolas Martin : We will come back... because you are speaking, in fact, in this book *Le Spectre d'Atacama* of Artificial Intelligence on which you don't have, will we say, a very mild look. But before that, maybe, a word, there are a lot of things that come to mind, but maybe, for the rest of our interview, for listeners, a word on what *Le Spectre d'Atacama* says, three people, you said it, Armand, a mathematician, Charlotte, a physicist, and Ali, a computer scientist, these are the same characters as those of your first novel, moreover.

Alain Connes : Of course, they are the same characters. In fact, the novel is the story of a spectrum, which is received by the Alma Observatory...

Nicolas Martin : So it's not a ghost.

Alain Connes : It's not a ghost ; a spectrum, you know, that's something that has both physical and mathematical meaning. The first way in which the spectra appeared, they were called absorption spectra. And it was Fraunhofer, a physicist who was optician rather, a German, who had this idea awesome which was to look at the spectrum of the sun which had been for example disclosed by Newton. That is to say, we pass the rays of the sun through a prism and we of course gets the colors of the rainbow. But he had this extraordinary idea of looking at this spectrum, for example, with a microscope, and he noticed that there were black lines. So of course, at first, we can think that those black lines are caused by the lens that's dirty or something like that. In fact, it was not. These black stripes, he had listed about 500 and this is the first spectrum from which we got the physical trace of a chemical element. And this is called an absorption spectrum. What does that mean ? It means if you want that these black lines, in fact, come from the signature of certain chemical bodies which are contained in the solar corona, it means that the light which comes from the sun is absorbed by these chemical bodies. And since they have a chemical signature, it allows us to know what is the composition of the solar corona. So after, we realized that they were not only absorption spectra, but they were also emission spectra.

For example, when you take sodium, heat it, and pass the light matter coming out of sodium through a prism, this time, we get brilliant lines on a black background and these bright lines on a black background matched exactly to some of the black lines, on the light background, in the spectrum of the sun. But there was also something great that happened. Because among the spectra that we could recognize, there was one which was completely mysterious, which did not correspond to a chemical body on Earth. And the physicians and chemists had this great idea to say "Oh ! It's a chemical body we don't know". And they called it helium like the Sun, well sure. So the wonder of wonders is that there was an eruption of Vesuvius, I don't know more in which year and that we could observe in the lava of Vesuvius exactly the helium spectrum. So the circle was complete. These are the spectra in physics. At the beginning of the 20^{th} century, mathematicians and physicists understood how to calculate these spectra of physics, from mathematics, and from a notion of spectrum which comes from mathematics and which is central in the quantum mechanics von Neumann's formalism. So an operator's spectrum exists, it's its variability, it's its vital space, if you will. And in fact, then the starting point of the book is Armand, who is a mathematician, who is a bit obsessed with a problem, etc. And then one day he receives a message from Rodrigo, who is a friend of him, who is an astronomer at the Alma observatory and who said to him "Come and see me, come and see me, there is something extremely mysterious, in Chile, in the Atacama Desert". And finally Armand, well, his friend is no longer available because he had a cerebral stroke, eventually he recovers the spectrum, so he gets a very, very bizarre, very bizarre spectrum. And then after, he will embark on this spectrum in all kinds of adventures which are a species of escape from the how to say, the hubbub of the modern world. He tries to escape the hubbub of the modern world to try to focus, to understand what this spectrum is. So he will go from adventure to adventure. Like that, he'll...

If you want, his physical journey is a metaphor for his intellectual journey, of course. Okay ? And then after, what is absolutely incredible is that it will find itself over his adventures, another of the characters in the first book, which is Charlotte and Char- monkfish, had also had an experience. She had really lived in her flesh, if you want, Charlotte, physicist, who is a physicist at CERN and in the first book, she had had an experience in her own flesh. And it's an experience that looked crazy in the first book I think a lot of people who read the first book considered it a completely crazy experience... Quantum theater, published by Odile Jacob. And so, in fact, for this experiment, we understood what was behind, in the second book, and we have, how to say, we explained what had happened to him because in actually, it's actually very funny because Armand had fled, and when he learns the experience of which Charlotte is a survivor...

Nicolas Martin : A quantum life experience

Alain Connes : A quantum life experience, when he learns this new velle, in fact, it is juxtaposed with an article from Le Monde on a representation of The Sleeping Beauty. And what's behind it, because there are a lot of things hidden in the book, you have to read it several times, there are a lot of things that you have to understand. What you have to understand is that Charlotte's experience, it was exactly the experience of Sleeping Beauty. That is to say ? She had been pierced by a needle and she was woken up by a Prince Charming, in this case, Prince Charming, is Florimont. It's a computer. And we learn in the book that while in the first book, we said "she is risen : she is dead. She died since she was pierced in CERN, in one of CERN's. His brain was completely taken over by computers, etc. And then she is resurrected by the computer. So we didn't understand. In fact, in the second book, we understand that she never died because death is brain death. And when his brain was recovered by the computer, then we might ask "but why did the computer want to resuscitate it ?". In fact, it is itself that has risen. And it is herself who has risen by adding something, it added a little more in the brain. And in the second book, precisely, what is absolutely amazing is that it added something to the brain. So it allows it to work a lot better when it is near Florimont, it is not by chance. That's the name of the resuscitating computer, in the ballet, it is Florimont who plays this role. And so, in fact, what happens is that as it is, in fact, it is it a little trans-human, that is to say that she knows how to add something to the brain, which makes it work better when it's close to the computer. So what is very amazing, is that it allows him to decanulate, as we say in math- ticks, i.e. to understand another spectrum, which is also sent alternately by the Alma Observatory. But it is Armand who finds the meaning of the first spectrum.

Nicolas Martin : Don't tell us everything because... there is a lot of future tures, indeed, to be read on different levels, and then, there is something also very important in this novel whose cover you will have to explain to us also, because it remains surprising, and this other element is the place of music and especially this music.

Extract from the Quartet for the end of Messiaen's time.

Nicolas Martin : Here it is, an extract from the Quartet for the end of Oli's time to see Messiaen. Why, Alain Connes, in a few words, because we will hear your co-author, Jacques Dixmier, on this subject, why the importance of Olivier Messiaen ?

Alain Connes : So why Olivier Messiaen in particular ? Because fact, if you like, time, as I said, played a permanent role in my evolution, in my mathematician journey, and about zeta, so the function Riemann's zeta, while most other researchers on the problem, seek zeta zeros, i.e. the spectrum, like an energy spectrum or a spectrum of frequencies, I realized that in my approach, it appears as a spectrum of time, a spectrum of lengths. And then, when we look at what corresponds to zeta, but which has already been understood through work... by André Weil, if you like, it's an analogous, but simpler case. Well, in this case, we get times, just like in the case of zeta. And these times verify an extremely particular property as attack time in a melody and this extremely peculiar property is a property which had been highlighted by Messiaen, under the name of non-demoteable rhythms, and it comes down to palindromy. And this property is an essential property of the corresponding zeta function. So I thought it was an amazing opportunity to connect, precisely, zeros of zeta function, for the case analogous to that of André Weil with Messiaen.

Nicolas Martin : Don't say too much since we are going to hear precisely your co-author and ex-thesis director on this subject on the link between mathematics and music, since you went to meet Jacques Dixmier, Céline Loozen.

Céline Loozen : Yes, hello Nicolas, hello Alain Connes, hello everyone. I went to see your former thesis director, Jacques Dixmier, to understand take a little bit of the link between math and music because in the writing of your novel, you drew ideas from Messiaen who inspired you on the question of rhythm and time. And you discovered a direct relationship between the concepts developed by Messiaen and mathematics. That then gave you the idea of composing music yourself, from prime numbers to create what are called non-demoteable rhythms.

Jacques Dixmier : The aspects of music in question here are related elementary. We were specially inspired by the Treaty of Messiaen, Treaty music and ornithology, I think so. And since there was a lot of talk about spectrum in the beginning of the story, it's related, in the story, it's related to observations then it's related to earthquake waves. Finally, the waves, eigenvalues often intervene. So it wasn't so surprising that the music intervenes since it is a question of frequencies, all the same, the high sound, that means the frequency of the air's vibration.

Céline Loosen : What is the relationship between prime numbers and the com- musical arithmetic position which is highlighted through one of the pieces by Olivier Messiaen.

Jacques Dixmier : So that's at the end of the book, actually, where we use prime numbers to make rhythms. To find certain rhythms, we then use a mathematical theory which it is extremely elaborate. It's called the theory of algebraic curves on finite fields and the action of Frobenius' automorphism (JD laughs.) It's related to that. The diagrams that we find towards the end, with the different rhythms, are related to this problem.

Céline Loosen : And we find the idea of ââthe spectrum which is omnipresent in through history.

Jacques Dixmier : Yes, the idea of ââspectrum intervenes in the book, to many aspects. This is not surprising, given the work of Alain Connes. For him, the spectrum of an operator is something fundamental. But then, what amused us is that it can intervene in a story and not in a brief...

Céline Loosen : In particular, it is about space, space to a range musical. And what is found in the music of Messiaen who is quoted all throughout the book ?

Jacques Dixmier : Well, he talks about rhythm.

Céline Loosen : But non-demoteable rhythms. What does it mean ?
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Jacques Dixmier : That means that we can turn them back in time and that they are identical to themselves. These are rhythms, therefore non-demoteable, it is a rather particular relation to time which are obtained by the method of curves on the finished bodies that are in the book. There are non-demoteable rhythms. You can find the designs. Here, there, for example, the right represents time and small vertical bars give the attack times of the notes. The fact that it is non-demotion, you can see it very well. For example, take the rhythm that is there and if you go back and forth, look. You have two, two notes close together here too. There, it is therefore rhythms, not ups teurs. You see that if you read backwards...

Céline Loosen : Is it like a palindrome ?

Jacques Dixmier : Yes, it is also the term he uses. But there is more net, look. It starts here or there. And then, you have two very close notes, two very close notes, etc. So you can read it backwards.

Céline Loosen : There is a form of symmetry.

Jacques Dixmier : Well, a mathematician would rather talk about sy- metrics. Moreover, when we speak of Frobenius automorphism of the curves on the finite bodies, we speak of the symmetry of its spectrum. Yes, but then, so you see the numbers that are there, these are prime numbers. Finally, these are between 43 and 83. And in particular, therefore, they give rise. This is the point of view of the book. We explain how to each prime number, we can associate a rhythm. That's it.

Céline Loosen : And what does this have to do with physical space ?

Jacques Dixmier : In a way, there is none. Except we're talking about spectrum in both cases. In physical space, there are... Well, for example, you have heard of gravitational waves, which we have just highlighted and experienced rimentally. Well, like all waves, they have wavelengths, and frequencies. So it's just starting out. So we still know almost nothing, but we will measure vibrations of the entire universe. We will be able to do that in a few years. There will surely be operator spectra that we can analyze ma- thematically. We will verify experimentally and this will be the analog on the scale of the universe, vibrations of a drum, vibrations, in history, of the Atacama desert, when there is an earthquake, guitar notes, it all comes out of a fairly general scheme.

Back to the initial interview.

Nicolas Martin : This is Jacques Dixmier, co-author with you, Alain Connes, of the Atacama spectrum. A word on this analysis, this interpretation in any case of your use of music in the novel ?

Alain Connes : Yes, so let's say that indeed, there are two aspects. First, if you will, when we look at the analogous case of the zeta function, but which had been resolved, him, by André Weil, as Jacques Dixmier explains, what will it give ? It will give rhythms, it will give attack times, but who have this particular property of palindromy, of symmetry and which Messiaen calls non-demoteable rhythms. But what does he have in mind ? He has in mind that if we downgrade them, we will get the same. So it will not give anything new. That's the idea. So, in fact, so there, these are rhythms. But I had to compose for each prime number of pitches which would then be played by these rhythms. So that's what we're going to hear. And to compose these heights, I harnessed myself to the task, if you want to associate a melody with each first, but in a way purely mathematical. That's what we're going to listen to.

Nicolas Martin : That's what we're going to listen to. We can recall, of course, to our listeners who really do not have the mathematical fiber, that a number which is not divisible only by itself and by 1. Here.

Courte écoute musicale.

Nicolas Martin : What did we just hear, Alain Connes ?

Alain Connes : So what we just heard is very surprising, that we just heard a melody that is different for each of the prime numbers between 7 and 67 and how it was constructed, this melody, it was constructed from purely mathematical, that is to say what we did : we took the spectrum guitar. When you look at the frets on a guitar, actually, these frets, they are not spaced equally at all. And when we look at what it is- mathematically signifies, these are the powers of a number and this number... so they are spaced exactly like the powers of a number. It is q power n , let's say. And that number is the twelfth root of two, but it's practically also the nineteenth root of three. And that's what is behind the music. Okay, so, in fact, that what we did to define this melody associated with each of the prime numbers between 7 and 67, it's to look at the prime number to do its development in what's called a fraction continues. But taking in relation to these powers of q , that is to say in trying to write it as a power of q and at that point we get automa- a melody which is palindromic here. And the way we heard it there, we heard it so that, whenever there was a prime number, there was a corresponding melody. It was different for each of the numbers first since we know that their development in continuous fractions are different. And now we're going to hear this melody play for each prime number which was played equally. We will hear him play by a rhythm which is a Messiaen rhythm, but which is associated with a zeta function, as explained quait Jacques Dixmier, which is associated with a curve. So if you want the difference fundamental, it's going to be that the zeta function is going to give you a way of playing that is going to be different, in terms of rhythm, in terms of note attacks. But otherwise, the melody will be exactly the same. Here, so if you want, what we heard, we saw that there, there was, your ta tata (accelerations, very short notes) , so the way of playing is complete very different. So what is quite extraordinary is that we did the calculations for 6 different curves, and we see that each curve, so, it has, how say, her personality and she plays in a way... but in a way that is consistent. So he's a kind of interpreter. So what we're saying here is that when we get to perceive, speak, speak, speak by Louis, by hearing, we can perceive by hearing, by hearing, we can perceive something that is normally very, very difficult to understand, which is precisely, well, these proper values ââof the Frobenius which do this analog of the Riemann zeta function, but which are perceived this time from rhythmically, which are perceived as times, okay, so that's a... Where does that come from in the book ? It plays a crucial role in the book because that basically one of the messages in the book is that if there is a way to communicate with extraterrestrials, with extraterrestrial intelligence, mathematics is an extraordinary tool for that. And in fact, therefore, there there, the Alma Observatory has received two spectra which are sent alternately. This is what we learn at the end of a moment in the book and the fact that we receive these two spectra and that we have understood thanks to Charlotte, for the first spectrum, thanks to Armand, for the second, what these two spectra mean, well, there is a revelation, it is necessarily, that comes to us from intelligent beings. So intelligent beings, in what sense ? intelligent, in what meaning ? There it is the pinnacle of intelligence. This is something that was found by Bernhard Riemann, who was a mathematician of the 19$^{\text{th}}$ century. And it's probably the pinnacle of intelligence. It is to have understood that what governs the pre- miers is music. What governs prime numbers are what we call precisely these zeros of the zeta function. They are the ones who govern the hazard that is in the prime numbers and the Riemann conjecture we were talking about earlier, just- ment, it is extremely significant for the following reason. Is that what she says, basically, in the corresponding spectrum which is the Atacama Spectrum, which is the cover of the book. In this spectrum, what Riemann's conjecture says is that it there will be no, there will not be, it will always be extremely precise lines, there will will not have what is called resonances, that is to say that there will be no place where, at instead of having a precise attack time, there is a diffuse attack time. That's what the guess says. What she says about prime numbers is that fact, although they look completely random, they are governed by a random, but a hazard which is perfectly controlled, if you will, and which is perfectly... Yes ?

Nicolas Martin : I would like, because we have just a short time left, a small minute, just a word, all the same, Alain Connes, to conclude, on this temptation that I want to call holistic temptation : you are a mathematician, novelist, musician, the will to understand, to integrate this mathematical language as a universal language, what do you think ?

Alain Connes : If you want, that's what I think is the following : is that one of the great discoveries of the human spirit is to understand, good. The human mind, in the 19$^{\text{th}}$ century, I spoke of Bernard Riemann, but talking about Galois : Galois was able, without having a computer, without having means of calculation, etc. to understand how to completely capture the rational relationships between the roots of an equation, by associating it with another equation and solving this one tautologically, practically. But he said at the time : "Let's jump on foot attached on the calculations." I had to make, at the Academy, a presentation on Galois for the two hundredth anniversary of his birth, I showed, since we can now do the calculations with the computer I showed for a fifth degree equation very simple, what the calculations would give, in this case : we can see that Galois, that was absolutely impossible for him. Nevertheless, he understood perfectly, conceived all of which was behind and the message of the book, a message that is very, very important is that nowadays, we tend today to let go of the temptation to do without understanding as opposed to understanding without making Galois and as opposed to the creation of concepts which is the prerogative of mathematics.

Nicolas Martin : It will be the last word, Alain Connes, since it is 4 :52 pm, I remember the title of your book : *The Specter of Atacama*, co-written with Danye Chéreau and Jacques Dixmier, and published by Odile Jacob editions.