

Alexander Grothendieck, creator refugee in himself

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So, Alexander was a math giant, a French mathematician who died two years ago, in November 2014. And in fact, if you will, when I was asked to make a presentation, I gladly accepted, with the main motivation that of restoring a fragment of truth in front of a book which was written on Grothendieck, that I will not quote, by a non-mathematician, fascinated by the character, but whose the judgment on Grothendieck's writings, in particular on *Crops and sowing*, that he thinks he can summarize in one sentence, appeared to me as an insult to the memory of the great scientist.

I gave the title Alexander Grothendieck, creator refugee in himself. What I had in mind when giving this title, it was his journey, from his childhood of refugee, his prodigious creativity, both mathematical and literary, and then of this second half of his life, which led him in the past 25 years, to take refuge in himself in a small village in the Pyrenees, that of Lasserre, where he wrote thirty-five thousand pages.

The correspondence between Jean-Pierre Serre and Alexander Grothendieck, who was published in the form of a magnificent volume, shows how their ideas upset algebraic geometry. They bear witness of a deep friendship and of what was Bourbaki's spirit in those years. Unlimited dedication to beauty of math, completely free of all individualism. After an episode of discouragement due to the death of his mother in 1957, Grothendieck had a period of radiant creativity, which resulted in particular in the concept of topos. This notion was implicitly present in an article which, at the outset, was, in quotation marks, an "annoying draft" intended for Bourbaki and which, in fact, when it was published, made famous the newspaper in which it was published, at point that we designate the article simply by the name of Tohoku (the newspaper is called *Tohoku Maths Journal*). Were there already... so I'm going to talk to you a little bit of math, but it won't last very long. So there were already the categories of diagrams and that of sheaves of sets, but Grothendieck had not still released the new principle which makes it possible to include these two examples as special cases of the same concept, that of topos. So let's listen to him. actually, I will spend most of my talk quoting Grothendieck.

"The point of view and the language of the sheaves introduced by Leray led us to look at spaces and varieties of all kinds in a new light.

Lecture given at the Collège de France within the framework of the conference Migrations, refugees, exiles, October 13, 2016.
<https://www.youtube.com/watch?v=FkBtSRyv6l4>

However, he did not touch the very notion of space, contenting himself with making us understand these traditional spaces already familiar to all more clearly with new eyes. However, it turned out that this notion of space is inadequate for report on the most essential topological invariants that express the abstract algebraic varieties shape. For the expected marriages of the number and greatness, it was like a decidedly narrow bed where only one of the future spouses, namely the bride, could at the very least find nesting as well as wrong, never have both at the same time.

The new principle, which remained to be found, to consume marriage put by auspicious fairies, it was none other than this spacious bed that was lacking future spouses without anyone having noticed it before. This double places bed appeared as if by magic wand with the idea of the topos. This is the theme of the topos and not that of the diagrams which is this bed, or this deep river where come to marry geometry and algebra, topology and arithmetic, logic mathematics and category theory, the world of the continuous and the one of discontinuous or discrete structures.

If the theme of diagrams is like the heart of new geometry (it was an invention of Grothendieck too), the theme of the topos is the envelope or the abode. It is ... (therefore, it's always Grothendieck who speaks, in heard) ... what I designed larger to grasp with finesse, using the same language, rich in geometric resonances, a common essence to the most distant situations from each others, coming from such or such region, from the vast universe of mathematical things."

So if you want, at that time, Grothendieck made an extraordinary discovery. He unveiled a concept of incomparable reach, both by wealth infinite spaces it allows to cover, but also and above all because in fact, if you want, it shows how ... how ... what is the true nature of a geometric space, which should be used simply as a parameter space for a variable set. And then, one of the great, one of the wonderful discoveries, precisely, of the notion of topos, is that when you work in a topos, it's exactly like if we worked in set theory, except that we can no longer apply the excluded middle rule.

We cannot say, we cannot reason by the absurd, but all intuitionist reasoning continues to work. *"So we have a wonderful example of a concept from pure mathematics, but whose scope, if only by its relations with logic, is no longer limited to this field of science."* For example, if you will, the notion of truth becomes a much more subtle notion in a topos.

And I mean, it's a notion that should be much better known than it is. So I come, I pass, now, so it's over for the mathematical part, I move on to Grothendieck's relationship with the world of mathematicians, which is in fact one of the main themes of *Crops and Sowing*. Let's hear it, at new. This is what Grothendieck says :

“It seems to me that the time has come to express myself about my relationship with the world of mathematicians. This is a very different thing from my relationship to mathematics. It existed and was strong from a young age, long before I even doubt that there existed a world and an environment of mathematicians, a whole complex world, with its learned societies, its periodicals, its meetings, colloquia, congresses, its prima donna and its hardworkers, its power structure, its gray eminences and the no less gray mass of cutters and laborers, in need of theses or articles. And of those also rarer, which are rich in means and in ideas and come up against closed doors, desperate to find the support of one of these powerful men, in a hurry and feared, and who have this magic power : have an article published. I discovered the existence of a mathematical world when landing in Paris in 1948 (Grothendieck was born in 1928) at the age of 20, with in my meager suitcase, a license in science of the University of Montpellier and a manuscript with tight lines, written on both sides borderless, the paper was expensive, representing three years of solitary reflection on this which, I learned later, was then well known as the theory of measurement or the integral of Lebesgue.

I had juggled with the sets that I called measurable, without having met besides of sets which are not, and with convergence almost everywhere, but I did not know what a topological space was. I hadn't heard of it yet, in a mathematical context at least, strange or barbaric words, like group, body, ring, module, complex, homology and so on, which suddenly, without crying out, surging over me all at the same time. The shock was severe. If I survived to this shock and continued to do math and even make it my job, is that in these distant times, the mathematical world hardly looked like what it is since then.

It is also possible that I had the chance to land in a more welcoming corner than another in this unsuspected world. I had a vague recommendation from one of my teachers at the faculty of Montpellier, who had been a student of Cartan. Like Élie Cartan was then already offside, his son, Henri Cartan, was the first congener that I had the time to meet. I had no idea, then, how happy it was augurs well. I was greeted by him with this kindly courtesy which distinguished him, well known from the generations of normaliens who had this chance to do their very first weapons with him.

He must not have realized, moreover, the whole extent of my ignorance, judging by the advice he gave me then to guide my studies. What in any case, his kindness, obviously, was addressed to the person, not to the luggage or to possible donations, or to a reputation or notoriety. In the year that followed, I was the host of a Cartan seminar at the school to which I clung firmly. I also assisted to the Cartan seminar, as a dumbfounded witness to the discussions between him and Serre, with big hits from spectral sequences... Grothendieck writes in parentheses (Brrrrr!) ... and drawings called diagrams, full of arrows covering the whole painting. It was the heroic epoch of the sheaves theory and a whole arsenal whose meaning was totally eluding me, while I

somehow forced me to swallow up the definitions given, and to check the demonstrations. On Bourbaki seminar days, bringing together a small twenty participants and listeners, we saw disembark, like a group of friends a little noisy, the members of this famous Bourbaki gang.

They were all familiar, spoke the same language which almost escaped me, also smoked a lot and laughed happily. Only were missing the boxes of beer to complete the ambiance. It was replaced by chalk and sponge. At the time, I went to see Mr. Leray at the Collège de France to ask him, if I remember, what his course would cover. I don't remember either the explanations could give me, nor if I understood anything there, only that there too, I felt a warm welcome, addressing the first stranger to come. That's it, and nothing else, surely, that made me go to this course and hold on to it bravely, as at the Cartan seminar, when the meaning of what Leray exposed there almost completely escaped me. The strange thing is that in this world I was in newcomer and whose language I hardly understood and spoken even less, I did not feel like a stranger when I had little opportunity to speak and to cause, with one of these happy guys, I still felt accepted, I would even say almost one of their own. I don't remember a single occasion when I was treated with condescension by one of these men, nor occasion where my thirst for knowledge, and later, again, my joy of discovering, was rejected by a self-importance or by a disdain. If it had not been so, I would not have become a mathematician, as they say, I would have chosen another profession where I could give my measure without having to face contempt.

Whereas objectively, I was a stranger to this world, just as I was a stranger in France, a bond, however, united me to these men from another background, another culture, another destiny, a common passion. I doubt that in this year crucial where I discovered the world of mathematicians, one of them perceived in me the same passion that inhabited them. For them, I had to be one among a mass course and seminar listeners, taking notes and obviously not well in the shot.

If perhaps, I distinguished myself in some way from the other listeners, it is that I was not afraid to ask questions, which most often had to denote above all my phenomenal ignorance, both of language and of mathematical things. The responses could be brief, even astonished. Never the amazed howl I was so did not meet with a rebuff, a rebate in my place, nor in the middle without the Bourbaki group's manners, nor in the more austere context of Leray's lessons at the Collège de France.

During these years, since I had landed in Paris with a letter for Élie Cartan in my pocket, I never felt like I was in front of a clan, of a closed, even hostile world. If I have known, well known this contraction inner face of contempt, it's not in this world, not at that time, at least. Respect for people was part of the air I breathed. There was not to deserve respect, to prove yourself before being accepted and treated

with some amenity. Strange thing, maybe, it was enough to be a person, to have a human face.”

So Grothendieck continues. You should know that Grothendieck left deliberately the mathematical world around 1970. This is what he called the great turning point. *“It was only after the great turning point of 1970, the first awakening should I say, that I realized that this cozy and friendly microcosm represented only a very small portion of the mathematical world and that the traits I liked to lend to this world, which I continued to ignore, which I had never thought as presenting an interest for me, were fictional traits.*

During these 22 years, so between 48 when he arrived in Paris and 70, this microcosm itself had changed its face in a surrounding world which also changed. Me too, surely, over the years and without suspecting it, I had changed like the world around me. I don't know if my friends and colleagues were more aware than I of this change in the surrounding world, in their own microcosm, and in themselves. I cannot say either how it did that this strange change probably came insidiously, discreetly.

The notorious man was feared, myself I was feared, if not by my pupils or my friends, or by those who knew me personally, at least by those who only knew me through notoriety and who did not feel themselves protected by comparable notoriety. I learned about the fear in the mathematical world that the day after I wake up, there are almost fifteen years.” (When he wrote *Crops and sowing* and the meaning of *Crops and sowing* is exactly that : he reaped what he sowed. It was in 85, fifteen years later.) *“During the fifteen years that had preceded, gradually and without suspecting it, (That was before 70.) I had entered the role of the big boss in the Who is who mathematical world. Without suspecting it too, I was a prisoner of this role which isolated me from all except a few peers and a few students. It is only once that I came out of this role that at least part of the fear that surrounded it felt, tongues untied, which had been mute before me for years. The testimony they brought to me was not only that of fear, it was also that of contempt, especially the contempt of people in place towards others, a contempt that arouses and fuels fear. I had little experience of fear, but that of contempt, in times when the person and the life of a person do not weighed not heavy. He no longer had to forget the time of contempt and lo and behold, it remembered itself to my memory. Maybe he never stopped, when I was satisfied just to change the world, as it seemed to me, to look elsewhere, or just pretending to see nothing, hear nothing, apart from passion-endless mathematical discussions. In those days, I finally accepted to learn that contempt was rampant all around me, in the world I had chosen as mine, with whom I identified myself, who had my deposit and who had pampered me.”*

So if you want, that's a summary of what's said in the main topic of *Crops and*

sowing, of course, which is Grothendieck's relationship to the mathematic. I pass to an absolutely essential text, another text by Grothendieck called *The key of dreams*. And when I prepared this talk, I was rereading *The Key of dreams* and I noticed one thing. I understood in fact that without knowing it and without wanting it, I had, by giving my title, left ajar the possibility of a completely different interpretation which actually touches the heart of the book which is *The Key of Dreams* and where the word creator seems in a sense that I let you guess, as I read his testimony. And I'm going to read you Grothendieck's testimony, we've heard a lot about Grothendieck's childhood, etc. But of course, it's much better to hear what he has to say about it himself.

I'm going to read you Grothendieck's testimony about his childhood, which is in *The key of dreams*. *"I lived the first five years of my life with my parents and with my sister in Berlin. It's Grothendieck who is speaking, of course. "My parents were atheists. For them, religions were archaic survivals and churches and other religious institutions, instruments of exploitation and of domination of men. Religion and Church were destined to be swept away without return by the world revolution that would end social inequality and all forms of cruelty and injustice, and would ensure the free development of all men.*

However, since my parents were both from religious families, this gave them a certain tolerance for religious beliefs and practices with others or towards people of religion. They were for them people like the others, but who happened to have that kind of anachronism, as others also had theirs. My father was from a Pious Jewish family in a small town in Ukraine, Novo Zubkov. He even had a grandfather who was a rabbi.

Religion, however, should not have had much control over him, even in his childhood. Very early on, he felt solidarity with peasants and ordinary people, more than with his middle class family. At the age of 14, he took off to join an anarchist group who crisscrossed the country preaching revolution, sharing land and property, and the freedom of men. Enough to make a general heart beat red and bold. It was in Czarist Russia in 1904. And until the end of his life again, and upside down, he saw himself as ... his name was Sacha Piotr, that was his name in the movement ... anarchist and revolutionary whose mission was to prepare the world revolution for the emancipation of all peoples. For two years he shares the hectic life of the group he had joined then surrounded by ... (so, it was in 1906) ... surrounded by the police, and after a hard fight, he was took prisoner with all his comrades. All are condemned to death and all, except him, are executed. For three weeks he waits day after day to be taken to the peloton.

He was finally pardoned because of his young age and his sentence commuted to that of life sentence. He stays in prison for eleven years, from the age of 16 to the

age of 27 years old, with hectic episodes of escapes, revolts, hunger strikes. He was liberated by the revolution in 1917, then participated very actively in the revolution, especially in Ukraine, where he fought at the head of an autonomous group of well-armed anarchist fighters in contact with Makhno, the head of the Ukrainian army of peasants. Sentenced to death by the Bolsheviks and after their stranglehold on the country, he left the country clandestinely in 1921 to land first in Paris, just like Makhno. During the past four years of intense militant and fighting activity, he also has a fairly tumultuous love life from which a child came, my half brother Dodek.” (It’s of course Grothendieck who is speaking.) “In emigration, first in Paris, then in Berlin, then again in France, he makes a living as best as he can, as a traveling photographer who assures him his material independence. In 1924, on the occasion of a trip to Berlin, he met there the one who was to become my mother. Love at first sight on both sides. They remained indissolubly attached to each other, for better and especially for worse, living in common-law until the death of my father in 1942, in deportation to Auschwitz.

I am the only child resulting from this union in 1928. My sister of 4 years my elder, was from a previous marriage. My mother was born in 1900 in Hamburg, thousand well-off Protestant who had experienced an inexorable social decline throughout of her childhood and adolescence. Like my father, she had an exceptionally strong personality. She begins to emerge from the moral authority of her parents at the age of 14. At 17, she goes through a religious crisis and emerges from the naive and problem-free faith of his childhood, which gave her no answer to questions from her own life and the spectacle of the world.

She spoke to me of it as a painful and necessary wrenching. As well my mother and my father had remarkable literary gifts. My father even had an imperative vocation there, which he felt inseparable from his revolutionary commitment. From the few fragments he left, I have no doubt that he had the makings of the great writer.” In fact, if you think about it, you will see that Grothendieck realized what his father had not had time to do, that is to say this writing. “*And for many years after the abrupt end of a huge epic, he carried within him the work to be accomplished, a fresco rich in faith and hope and pain and laughter and tears and shed blood, thick and vast like his own untamed life ; and alive like a song of freedom.*

It was up to him to embody this work which was becoming dense and heavy, and which grew and demanded to be born. It would be his voice, his message, what he had to say to men, what no one else knew and could not say. If he had been faithful to himself, that child who wanted to be born would not have asked for it in vain. While he scattered to the four winds, he knew it well deep down and that if he left his life and his force to be nibbled by the petty things in the lives of migrants is that it was connivance. And my mother also had blessed gifts that predestined her for great things. But they chose to neutralize each other in a past confrontation endless, both selling

their birthright for the satisfaction of a flamboyant married life with great love, superhuman dimensions and neither one nor the other, until their death, will take care to update the nature and the true ones springs. After the advent of Hitler in 1933, my parents emigrated to France, land of asylum and freedom for a few more years, leaving my sister on one side in Berlin, me on the other in Blankenese near Hamburg” (So Grothendieck spent six years of his early childhood alone, without his parents.) “and without much regard to their cumbersome offspring until 1939. I joined them in Paris in 1939, the situation for me, in Nazi Germany becoming more and more dangerous, a few months before the outbreak of the world war, it was time. We are interned as unwanted strangers, father in winter 1939, my mother with me in early 1940.

I stayed two years in the concentration camp, then I was welcomed in 1942 in a Swiss children’s home in Chambon-sur-Lignon, in the protestant Cévennes region, where many Jews are hiding, hunted like us by the deportation. The same year, my father was deported from the Vernet camp to an unknown destination. It is years later that my mother and I will have official notification of his death at Auschwitz. My mother stayed at the camp until January 1944.

She died in December 1957 from contracted pulmonary tuberculosis at the camp.” So, I think it’s better that I skip a little passage. I will come back afterwards, possibly because I want to read you, probably the most important texts that I collected in *The key of dreams*, and which, I hope, will give you the meaning, the second meaning of the title. Again, Grothendieck speaks, and he will tell an episode that happened to his father.

So that’s what Grothendieck is telling us. *“In recent months, a such density by the action of God in me, I sometimes thought of an event in my father’s life, which took place long before I was born and which I had rarely had the opportunity to think to. They never spoke to me about it elsewhere, nor to a soul who long live, moreover, except to my mother, in the weeks of tumultuous passion which have followed their meeting in 1924. She was the one who told me about it and years after his death. This is an experience he had in prison, in his eighth year in captivity, therefore around the year 1914. It was after a year of solitary confinement that he had worth an escape attempt, during a transfer from one prison to another. It was surely the hardest year of his life and which would have destroyed or broken or extinguished more of one, total loneliness, with nothing to read, write or occupy, in an isolated cell in the middle of a deserted floor, cut off even from the sounds of the living, except the immutable and haunting daily scenario, three times a day, the brief appearance of the guardian wearing the pittance and in the evening, a flash appearance from the director coming in person inspect the hard head of the prison. Every day stretched, like a purgatory without end. There were 365 to pass, before he was again attached to the world of alive, with books, a pencil. He counted them on those days, those eternities he had to cross, but at the end of the 365th, he barely could grasp that it was indeed the end of*

his endless ordeal.

And for the next three days, nothing. At the end of the third, to his deasks “The year has passed, now ... When will I have books?”, a terse “Wait” from the director. Three days later, the same! We were playing with him, who was delivered to their mercy. But the revolt brooded, ulcerated, in the man pushed to the limit. The next day, hardly pronounced the same impassive answer “Wait”, the heavy spittoon in copper with sharp edges almost smashed the head of the imprudent tormentor.

Throwing himself to one side just in time, he felt the breath in his temples, before the projectile crashes on the opposite wall of the corridor and it hurriedly rejects behind him the heavy clad door. It is a miracle for me that my father was not hanged on the spot. Perhaps a scruples of conscience from the director who “feared God” and who felt confusedly by the very death that had grazed him so closely, that he had gone too far. Still, the young rebel is beaten like plaster, it was the slightest of things, then thrown into irons, into a stinking dungeon, into the dark total, for an indefinite period. Every third day, we open the shutters, and during the day relay the moist night. However, the revolt is not broken. Total hunger strike, without eating or drinking, despite the young body that stubbornly wants to live, the ulcerated soul, eaten away by the impossible revolt and humiliation of helplessness, and swollen flesh protruding into glass beads around the iron rings on the wrists and ankles.

It was the days when he reached the bottom of human misery, aware of it, even, that of the body, that of the soul. It’s at the end of the sixth dungeon day, day with open shutters that took place the incredible thing, which was the most precious secret and the best kept in his life, in the ten years that followed. It was a sudden wave of light of unspeakable intensity, in two successive movements, which fills its cell and penetrates and fills it, like deep water, which soothes and erases all pain and like a burning fire burning with love, boundless love for all the living, all distinctions of friends and enemies swept away, erased.

I don’t remember that my mother had a ready-made name to name this someone else’s experience. I will now call it an illumination, exceptional and fleeting condition, close to what the testimonies of certain sacred texts and many mystics. But this experience takes place here outside of any context that is commonly called religious. It was over ten years surely that my father had detached himself from the grip of a religion in order not to never come back to it.

It is safe for me, even without having precise details on this subject, that this event has profoundly transformed his perception of things and his whole inner attitude in the days and weeks at least that followed. Days of very hard trials, surely. But I have good reason to believe that neither then nor later did he attempts to locate what had

happened to him, in his vision of the world and of himself. It was not for him the beginning of a deep and long inner work breath, which would have made fruitful and multiply the extraordinary gift that had been given to him done and entrusted. He must have reserved a very separate hut for her, like a jewel that we hold in a closed box, taking care not to put it in contact with the rest of his life. However, I have no doubt that this incredible grace, which had changed in an instant the excess of misery in an unspeakable splendor, was intended not to be kept thus locked up, but to irrigate and fertilize his whole subsequent life.

It was an extraordinary chance which was offered to him, and which he did not seize. A bread which he ate only once in the mouth and to which he no longer touched. Ten years later, the way he opened up to my mother, in the intoxication of his first love with a woman who was going to tie him, feet and fists, it was good like a jewel unusual and very precious, which he would have given her the first. And when she has me spoken, more than twenty years later, I knew that she had really appreciated, and appreciated again, this tribute thrown at her feet, and which she had greeted with emurgently and as a brilliant testimony of total communion with adored man, and of an intimacy which has nothing more to seal. And myself, hearing it, young man of 17 or 18, read it with eagerness moved quite similar. I also saw the jewel that also makes the shine for me again and again from this prestigious and incomparable hero father, at the same time as that of my mother who, alone among all the mortals, had been judged worthy to take part in it. So the bread given by God as inexhaustible nourishment of a soul, which, perhaps, would grow and would nourish other souls still, ended up becoming a family adornment, coming enhance the splendor of an expensive myth and fuel a common vanity.”

Here. I'm done, thank you.