Goldbach's conjecture (7, june 1742)

- 271 years old
- **Postulate** : Every even number (n) greater than 2 is the sum of two primes ($n = p + q$).
- p and q are odd ; $3 \leqslant p \leqslant n/2$ and $n/2 \leqslant q \leqslant n - 3$
- $$\begin{aligned} 98 &= 19 + 79 \\ &= 31 + 67 \\ &= 37 + 61 \end{aligned}$$

Booleans

- One represents primality by booleans.
- 0 significates *is prime*, 1 significates *is compound*.
- $23 \to 0$
- $25 \to 1$
-

3	5	7	9	11	13	15	17	19	21	23	25	27	29	...
0	0	0	1	0	0	1	0	0	1	0	1	1	0	...

The space's copy : 4 letters for 4 possibilities

- One represents n's decompositions in sums of two odd numbers by letters.
- $28 = \underset{p}{5} + \underset{p}{23} = prime + prime = a$
- $28 = \underset{c}{9} + \underset{p}{19} = compound + prime = b$
- $28 = \underset{p}{3} + \underset{c}{25} = prime + compound = c$
- $40 = \underset{c}{15} + \underset{c}{25} = compound + compound = d$

40 and 42 words

40	37	35	33	31	29	27	25	23	21
	0	1	1	0	0	1	1	0	1
	0	0	0	1	0	0	1	0	0
	3	5	7	9	11	13	15	17	19
	a	*c*	*c*	*b*	*a*	*c*	*d*	*a*	*c*

42	39	37	35	33	31	29	27	25	23	21
	1	0	1	1	0	0	1	1	0	1
	0	0	0	1	0	0	1	0	0	1
	3	5	7	9	11	13	15	17	19	21
	c	*a*	*c*	*d*	*a*	*a*	*d*	*c*	*a*	*d*

Let us observe diagonal words

6 :	a							
8 :	a							
10 :	a	a						
12 :	c	a						
14 :	a	c	a					
16 :	a	a	c					
18 :	c	a	a	d				
20 :	a	c	a	b				
22 :	a	a	c	b	a			
24 :	c	a	a	d	a			
26 :	a	c	a	b	c	a		
28 :	c	a	c	b	a	c		
30 :	c	c	a	d	a	a	d	
32 :	a	c	c	b	c	a	b	
34 :	a	a	c	d	a	c	b	a

Diagonal words properties

- Diagonal words have their letters either in A_{ab} alphabet or in A_{cd} alphabet.
- Indeed, a diagonal codes decompositions that have the same second sommant and that have as first sommant an odd number from list of successive odd numbers beginning at 3.
- For instance, *aaaba* diagonal, that begins at th *a* first letter of 26's word codes les decompositions $3 + 23, 5 + 23, 7 + 23, 9 + 23, 11 + 23$ and $13 + 23$.

Projection P

$H_{34,3}$: $Z_a = 5$, $Z_c = 2$

6 :	a							
8 :	a							
10 :	a	a						
12 :	c	a						
14 :	a	c	a					
16 :	a	a	c					
18 :	c	a	a	d				
20 :	a	c	a	b				
22 :	a	a	c	b	a			
24 :	c	a	a	d	a			
26 :	a	c	a	b	c	a		
28 :	c	a	c	b	a	c		
30 :	c	c	a	d	a	a	d	
32 :	a	c	c	b	c	a	b	
34 :	a	a	c	d	a	c	b	a

$B_{34,3}$: $Y_a = 5$, $Y_c = 3$

Row 34: $\leftarrow X_a = 4, X_b = 1$
$X_c = 2, X_d = 1$

Example

- Entanglement $Y_a(n), X_a(n), X_b(n)$

$$Y_a(34) = \#\{3+17, 3+19, 3+23, 3+29, 3+31\}$$

$$\begin{aligned} X_a(34) &= \#\{3+31, 5+29, 11+23, 17+17\} \\ X_b(34) &= \#\{15+19\} \end{aligned}$$

- Entanglement $Y_c(n), X_c(n), X_d(n)$

$$Y_c(34) = \#\{3+21, 3+25, 3+27\}$$

$$\begin{aligned} X_c(34) &= \#\{7+27, 13+21\} \\ X_d(34) &= \#\{9+25\} \end{aligned}$$

Variables entanglement properties

- $$Y_a(n) = X_a(n) + X_b(n) \tag{1}$$
- $$Y_c(n) = X_c(n) + X_d(n) \tag{2}$$
- $$Y_a(n) + Y_c(n) = \left\lfloor \frac{n-2}{4} \right\rfloor \tag{3}$$
- $$X_a(n) + X_b(n) + X_c(n) + X_d(n) = \left\lfloor \frac{n-2}{4} \right\rfloor \tag{4}$$
- $$Z_a(n) + Z_c(n) = \left\lfloor \frac{n-4}{4} \right\rfloor \tag{5}$$

Variables entanglement properties

- $$X_a(n) + X_c(n) = Z_a(n) + \delta_{2p}(n) \qquad (6)$$

  with $\delta_{2p}(n)$ that is equal to 1 if n is a prime double
  and that is equal to 0 otherwise.

- $$X_b(n) + X_d(n) = Z_c(n) + \delta_{2c-imp}(n) \qquad (7)$$

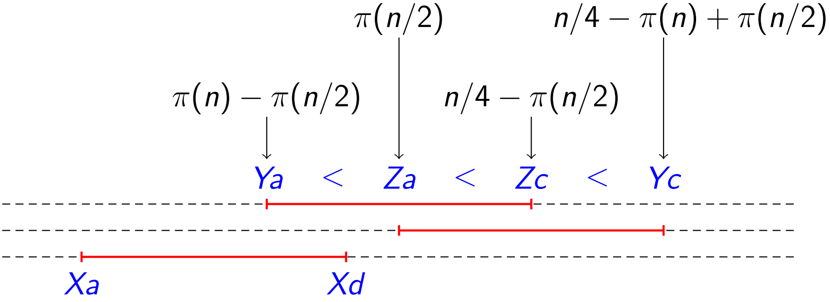
  with $\delta_{2c-imp}(n)$ that is equal to 1 if n is an odd compound number
  and that is equal to 0 otherwise.

- $$Z_c(n) - Y_a(n) = Y_c(n) - Z_a(n) - \delta_{4k+2}(n) \qquad (8)$$

  with $\delta_{4k+2}(n)$ that is equal to 1 if n is an odd double
  and that is equal to 0 otherwise.

- $$Z_c(n) - Y_a(n) = X_d(n) - X_a(n) - \delta_{2c-imp}(n) \qquad (9)$$

Gaps entanglements properties



n	X_a	X_b	X_c	X_d	Y_a	Y_c	A	Z_a	Z_c	B	$\delta_{2p}(n)$	$\delta_{2cimp}(n)$	$\delta_{4k+2}(n)$
14	2	0	1	0	2	1	3	2	0	2	1	0	1
16	2	0	1	0	2	1	3	3	0	3	0	0	0
18	2	0	1	1	2	2	4	3	0	3	0	1	1
20	2	1	1	0	3	1	4	3	1	4	0	0	0
22	3	1	1	0	4	1	5	3	1	4	1	0	1
24	3	0	1	1	3	2	5	4	1	5	0	0	0
26	3	1	2	0	4	2	6	4	1	5	1	0	1
28	2	1	3	0	3	3	6	5	1	6	0	0	0
30	3	0	2	2	3	4	7	5	1	6	0	1	1
32	2	2	3	0	4	3	7	5	2	7	0	0	0
34	4	1	2	1	5	3	8	5	2	7	1	0	1
36	4	0	2	2	4	4	8	6	2	8	0	0	0
38	2	2	5	0	4	5	9	6	2	8	1	0	1
40	3	1	4	1	4	5	9	7	2	9	0	0	0
42	4	0	3	3	4	6	10	7	2	9	0	1	1
44	3	2	4	1	5	5	10	7	3	10	0	0	0
46	4	2	4	1	6	5	11	7	3	10	1	0	1
48	5	0	3	3	5	6	11	8	3	11	0	0	0
50	4	2	4	2	6	6	12	8	3	11	0	1	1
52	3	3	5	1	6	6	12	8	4	12	0	0	0
54	5	1	3	4	6	7	13	8	4	12	0	1	1
56	3	4	5	1	7	6	13	8	5	13	0	0	0
58	4	3	5	2	7	7	14	8	5	13	1	0	1
60	6	0	3	5	6	8	14	9	5	14	0	0	0
62	3	4	7	1	7	8	15	9	5	14	1	0	1
64	5	2	5	3	7	8	15	10	5	15	0	0	0
66	6	1	4	5	7	9	16	10	5	15	0	1	1
68	2	5	8	1	7	9	16	10	6	16	0	0	0
70	5	3	5	4	8	9	17	10	6	16	0	1	1
72	6	2	4	5	8	9	17	10	7	17	0	0	0
74	5	4	6	3	9	9	18	10	7	17	1	0	1
76	5	4	6	3	9	9	18	11	7	18	0	0	0
78	7	2	4	6	9	10	19	11	7	18	0	1	1
80	4	5	7	3	9	10	19	11	8	19	0	0	0

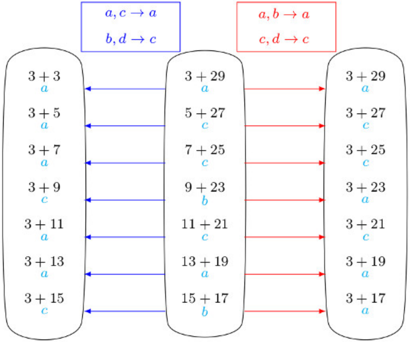
Variables and gaps entanglement (invariants)

n	$X_a(n)$	$X_b(n)$	$X_c(n)$	$X_d(n)$
999 998	4 206	32 754	37 331	175 708
1 000 000	5 402	31 558	36 135	176 904
9 999 998	28 983	287 084	319 529	1 864 403
10 000 000	38 807	277 259	309 705	1 874 228

n	$Y_a(n)$	$Y_c(n)$	$\left\lfloor \frac{n-2}{4} \right\rfloor$	$Z_a(n)$	$Z_c(n)$	$\left\lfloor \frac{n-4}{4} \right\rfloor$	$\delta_{2p}(n)$	$\delta_{2ci}(n)$	$\delta_{4k+2}(n)$
999 998	36 960	213 039	249 999	41 537	208 461	249 998	0	1	1
1 000 000	36 960	213 039	249 999	41 537	208 462	249 999	0	1	0
9 999 998	316 067	2 183 932	2 499 999	348 511	2 151 487	2 499 998	1	0	1
10 000 000	316 066	2 183 933	2 499 999	348 512	2 151 487	2 499 999	0	1	0

Bijections

- $n = 32$



$Z_a = X_a + X_c = 5$

$Z_c = X_b + X_d = 2$

$Z_a + Z_c = \left\lfloor \frac{n-4}{4} \right\rfloor = 7$

$Y_a = X_a + X_b = 4$

$Y_c = X_c + X_d = 3$

$Y_a + Y_c = \left\lfloor \frac{n-2}{4} \right\rfloor = 7$

$X_a = 2$

$X_b = 2$

$X_c = 3$

$X_d = 0$

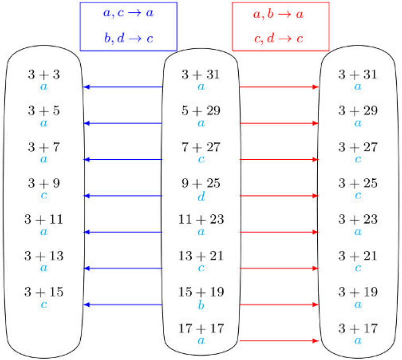
Bijections

- $n = 34$

$Z_a = X_a + X_c = 5$

$Z_c = X_b + X_d = 2$

$Z_a + Z_c = \left\lfloor \frac{n-4}{4} \right\rfloor = 7$



$Y_a = X_a + X_b = 5$

$Y_c = X_c + X_d = 3$

$Y_a + Y_c = \left\lfloor \frac{n-2}{4} \right\rfloor = 8$

$X_a = 4$

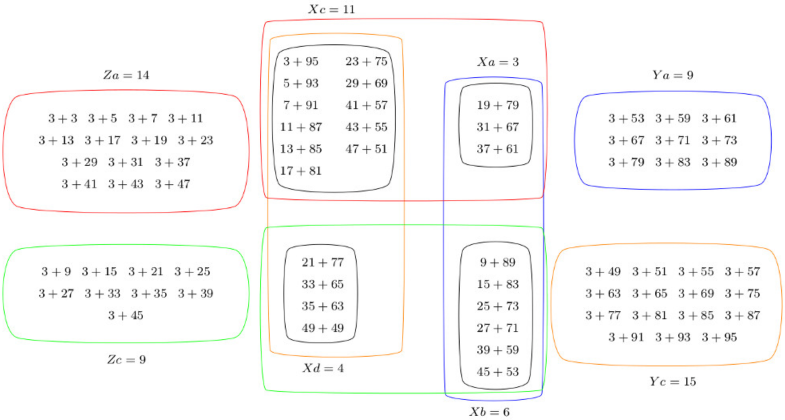
$X_b = 1$

$X_c = 2$

$X_d = 1$

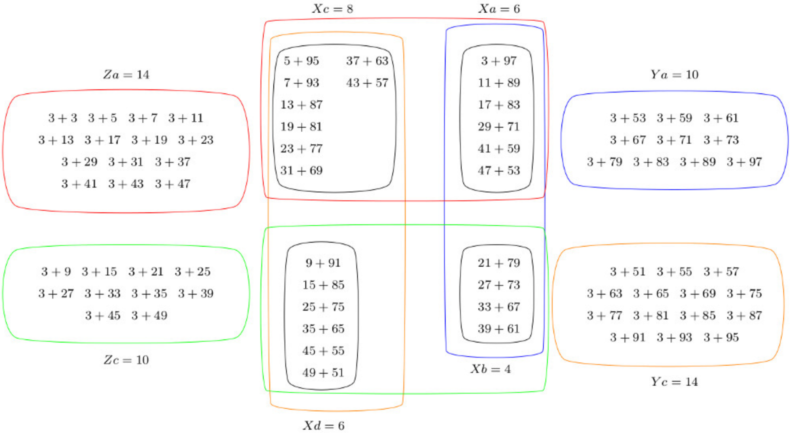
Bijections : double counting visualization

- $n = 98$



Bijections : double counting visualization

- $n = 100$



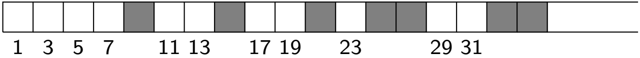
Laisant's strips

- **Charles-Ange Laisant** : Sur un procédé expérimental de vérification de la conjecture de Goldbach, Bulletin de la SMF, 25, 1897.

- *"This famous empirical theorem : every even number is the sum of two primes, whose demonstration seems to overpass actual possibilities, has generated a certain amount of works and contestations. Lionnet tried to establish the proposition should probably be false. M. Georg Cantor verified it numerically until 1000, giving for each even number all decompositions in two primes, and he noticed that this decompositions number is always growing in average, even if it presents big irregularities."*

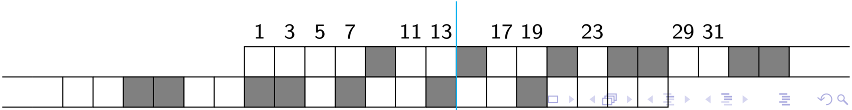
Laisant's strips

- *"Let us show a process that would permit to make without computing the experimental verification we mentioned, and to have, for each even number, only inspecting a figure, all the decompositions. Let us suppose that on a strip constituted by pasted squares, representing odd successive numbers, we had constructed the Erathosthene's sieve, shading compound numbers, until any limit 2n."*



Laisant's strips

- *"If we constructed two similar strips, and if we put the second one behind the first one returning it and making correspond 1 cell with 2n cell, it is evident that if Goldbach theorem was true for 2n, there would be somewhere two white cells corresponding to each other ; and all the couples of white cells will give diverse decompositions. We will even have them reading only the half of the figure, because of the symmetry around the middle. In this way, verification concerning 28 even number will give figure above and will show that we have* $28 = 5 + 23 = 11 + 17$*."*
- *"We understand that, strips being constructed in advance, a single shift permits to pass from one number to another, verifications are very rapid."*



Summary

- $X_a : p + p$
- $X_b : c + p$
- $X_c : p + c$
- $X_d : c + c$
- $Z_a : 3 + p, \quad (p < n/2)$
- $Z_c : 3 + c, \quad (c < n/2)$
- $Y_a : 3 + p, \quad (p \geqslant n/2)$
- $Y_c : 3 + c, \quad (c \geqslant n/2)$
- $\{3 + p_k\} \quad Y_a = X_a + X_b \quad \{p + p_i\} \cup \{c + p_j\} \quad (p_i, p_j, p_k \geqslant n/2)$
- $\{3 + c_k\} \quad Y_c = X_c + X_d \quad \{p + c_i\} \cup \{c + c_j\} \quad (c_i, c_j, c_k \geqslant n/2)$
- $\{3 + p_k\} \quad Z_a = X_a + X_c \quad \{p_i + p\} \cup \{p_j + c\} \quad (p_i, p_j, p_k < n/2)$
- $\{3 + c_k\} \quad Z_c = X_b + X_d \quad \{c_i + p\} \cup \{c_j + c\} \quad (c_i, c_j, c_k < n/2)$

Summary

- $Y_a + Y_c = X_a + X_b + X_c + X_d = \left\lfloor \frac{n-2}{4} \right\rfloor$

- $Z_a + Z_c = \left\lfloor \frac{n-4}{4} \right\rfloor$

- $\left\lfloor \frac{n-4}{4} \right\rfloor = Z_a + Z_c \simeq Y_a + Y_c = \left\lfloor \frac{n-2}{4} \right\rfloor$

- $Z_c - Y_a \simeq X_d - X_a$
  by definition because $Z_c - Y_a$ corresponds to
  $$\{c+p\} \cup \{c+c\} \setminus \{c+p\} \cup \{p+p\}$$

Conclusion

- We used a 4 letters language to represent n decompositions as two odd numbers sums.
- We use a lexical theory of numbers, according to which numbers are words.
- We have always to well observe letters order in words.