Annex to the proposition denitac.pdf (Denise Vella-Chemla, 23.7.2019)

One looks for decomposing an even number n as a sum of 2 prime numbers $p_1 + p_2$.

We can't make reference to $\zeta(-1)$ as we made it in [1]. We can, however, to obtain a minoration for the number of Goldbach decompositions of n, use the cardinal $\left|\mathcal{P}_{\frac{n}{2}}\right|$ of the set of prime numbers that are lesser than or equal to $\frac{n}{2}$ and multiply it by the product $\prod\limits_{p\leq\sqrt{n}}\left(1-\frac{1}{p}\right)$ that counts how many chances the prime number p_1 has not to share its division rest with n in each division by a module p lesser than $\sqrt{n}$ (the fact that p_1 doesn't share its rest in a division by p with n involves that the complementary of p_1 to n (called p_2) is prime too).

The minoration[1] of $\pi(x)$ (the number of prime numbers lesser than x) by $\frac{x}{\log x}$ can be found in [2], page 69, for $x \geq 17$ (Corollary 1, (3.5), of Theorem 2, whose demonstration can be found at paragraph 7 of [2]).

As a consequence, one has $|\mathcal{P}_{\frac{n}{2}}| > \frac{\frac{n}{2}}{\log\left(\frac{n}{2}\right)}$.

The minoration of $\prod\limits_{p\leq\sqrt{n}}\left(1-\frac{1}{p}\right)$ is also provided by [2], page 70 (it's the corollary (3.27) of Theorem 7 whose demonstration is provided at paragraph 8 of [2], with γ the Euler-Mascheroni constant).

$$\text{(3.27)} \qquad \frac{e^{-\gamma}}{\log x}\left(1-\frac{1}{\log^2 x}\right) < \prod_{p\leq x}\left(1-\frac{1}{p}\right) \quad \text{for } 1 < x.$$

Multiplying those expressions together, one obtains that the number of Goldbach decomposants of n must be greater than :

$$\frac{n/2}{\log(n/2)}\frac{e^{-\gamma}}{\log\sqrt{n}}\left(1-\frac{1}{\log^2\sqrt{n}}\right)$$

which is strictly greater than 1 for $n \geq 24$.

Bibliography

[1] http://denisevellachemla.eu/denitac.pdf.

[2] J. B. Rosser et L. Schoenfeld, *Approximate formulas for some functions of prime numbers*, dedicated to Hans Rademacher for his seventieth birthday, Illinois J. Math., Volume 6, Issue 1 (1962), 64-94.

1. This minoration must be distinguished from the Prime Number Theorem, independently proved by Hadamard and by de la Vallée-Poussin, and that provides an asymptotic tendancy for $\pi(x)$.