

Translation¹ of the conversation between
Jean-Pierre Serre
and
Alain Connes
concerning Serre-Grothendieck correspondence

ALAIN CONNES : I propose that our discussion begins precisely in the year 1955. I read what Grothendieck says. He says “The year 1955 marks a crucial turning point in my mathematical work, that of the passage from analysis to geometry.”. And he says “I still remember this striking impression, all subjective, certainly... as if I were leaving arid and bitter steppes.”.

JEAN-PIERRE SERRE : Yes, it’s not nice for what he was doing. So there, we have to tell you that Grothendieck has gone to Nancy and there, on some topics that were quite fashionable, but a little restrictive,

ALAIN CONNES : the topological vector spaces. Yes that’s it, he had almost solved all the problems in the area.

ALAIN CONNES : He had been given 14 problems. It was Dieudonné (or perhaps Schwartz) who gave them...

JEAN-PIERRE SERRE : It was 8 problems, well... I think he had chosen 8.

ALAIN CONNES : They have given him 14 problems to solve.

JEAN-PIERRE SERRE : They didn’t know how to solve them.

ALAIN CONNES : And I think it was the first time that Grothendieck applied his method that Serre described as being... “To solve problems, they must be allowed to dissolve themselves in a rising tide of general theory”. (*They laugh.*)

JEAN-PIERRE SERRE : It was a topic that was a little blocked. One has had the impression that he had solved just about every question. In fact, not

1. Subtitles to be improved : Denise Vella-Chemla, 1.4.2020.

quite true. There were counter-examples to find. There were pretty counter-examples, Banach... There were pretty good examples to find, but he did not find, but he had enough.

ALAIN CONNES : So the question I asked myself because I looked at Grothendieck's thesis, I looked at his second thesis,

JEAN-PIERRE SERRE : What was it, so, the second thesis ?

ALAIN CONNES : It's very, very interesting. Grothendieck's second thesis was Sheaves theory.

JEAN-PIERRE SERRE : Aaahhhh!

ALAIN CONNES : So, this means finally, it's my conjecture, that the moment, he has bifurcated from the analysis, it was certainly what he wanted to do. And he must have wanted to fork. It was at the time of his second thesis, he was asked to exhibit...

JEAN-PIERRE SERRE : So, you have to explain what the second thesis is, because it no longer exists. At the time, when one was passing one's thesis, one had the main thesis done. And then the jury gave you another subject. It was a subject that was given to the person with his agreement, in general. It was going pretty well familiarly. That's often. But not always familiarly. So what? The person in question was talking about half an hour or 20 minutes about this. And it was, it was very good. And I think that Grothendieck has had more or less to choose his subject. In addition, you see, the link with sheaves... What was said at this time is that we realized that his nuclear spaces theory was so good that you could make tensor products, Kunnet... Kunnet's formula worked at such a point that I sold Grothendieck's theory to Bott, one or two years after. I told him "You know, you have a product of varieties, you must act as if it was a tensor product, and that's all! Everything works.

ALAIN CONNES : Yes. Not only that, absolutely, but the philosophical idea, in fact, that are behind the nuclear spaces is that they are finite-dimensional spaces. That is, in fact, that they behave like finite dimensional spaces.

JEAN-PIERRE SERRE : It's a little technical.

ALAIN CONNES : It's a bit technical and we'll see.

JEAN-PIERRE SERRE : But he went very naturally to the theory of sheaves, first to the topology.

ALAIN CONNES : So, the question I wanted to ask you about that, precisely, is when did Grothendieck come into the Bourbaki group?

JEAN-PIERRE SERRE : Oh, it has been a little... I do not know. I am not sure of the date. And it's definitely after that.

ALAIN CONNES : After that I wanted to know...

JEAN-PIERRE SERRE : Here you speak of 55, yes.

ALAIN CONNES : Yes 55.

JEAN-PIERRE SERRE : What is the year he went to Kansas? Because the letter he sent me is on 55.

ALAIN CONNES : the letter on the diplodocus homologicus? He is talking about an annoying essay for Bourbaki.

JEAN-PIERRE SERRE : So it was in 1955 he went to Kansas and I'll say he entered in Bourbaki in 1957, around 1957.

ALAIN CONNES : Alright Alright.

JEAN-PIERRE SERRE : I do not know. He began reporting to Bourbaki. Huge reports, of course. Bourbaki, we are talking about Bourbaki together in 59. In 59, he certainly was there, but it seems to me that's it. I told him about Bourbaki in 58 (*he read letters in Correspondence book*). In 58, I told him "the Bourbaki congress was very nice", so he could have been there. I think those 57 days, Bourbaki in fifties.

ALAIN CONNES : What struck me a lot when I read the correspondence is

really at least at the beginning, and then afterwards, I will talk about something else. But how does Grothendieck manage to gain your trust? In a way, I think there is one point that struck me a lot. This is the moment when he understands your duality through the Ext, that is to say, you tell him it's really hard-working (rupinant in french?), true, but it's the trust between you two.

JEAN-PIERRE SERRE : This trust between us, it had come at least two years before you. I had gone to Nancy. Yes, I taught a little Rational Mechanics in Nancy (*laughing*), but I had it in horror. I was also giving a topology seminar, I discussed somewhat with Grothendieck, I saw very closely his nuclear spaces, you see. It hit me a lot because this idea was very beautiful. It was very, very beautiful this idea concerning tensor products. So we were confident, we had already trusted for a long time.

ALAIN CONNES : But at a mathematical level, there is already. from the beginning of the correspondence, there are several striking points that are, for example, his treatise on homological algebra, that is, the so-called Tohoku.

JEAN-PIERRE SERRE : It didn't interest me, because I considered it as more or less obvious.

ALAIN CONNES : There was the Cartan - Eilenberg.

JEAN-PIERRE SERRE : For me, it was redaction. It was redaction for him too. It was the same. He says "To understand something, I need to write it. And for this, he wrote it, but for me, there was nothing original about it.

ALAIN CONNES : , For me, there was something that struck me a lot there.

JEAN-PIERRE SERRE : There were Axioms on abelian categories.

ALAIN CONNES : Yes, there were Axioms on abelian categories, but not only. It's at the level of examples. Well, of course, the main example was the abelian groups'bundles,

JEAN-PIERRE SERRE : It had not been done, but it was known that it was realisable.

ALAIN CONNES : Of course it worked, okay. But hey, there was the nuance between the Čech's aspect to calculate the cohomology. But in fact, what struck me a lot, I remember, when I read this article in detail, it was another example that looks like nothing, but I'll come back to that later. This is the example of what he called diagrams categories.

JEAN-PIERRE SERRE : Categories of... ?

ALAIN CONNES : ... of diagrams. It goes unnoticed.

JEAN-PIERRE SERRE : effectively.

ALAIN CONNES : But now, I'm going to explain to you what it is and the role it played. I think afterwards, but it's also a conjecture. In fact, what is he doing ? He has a whole chapter on this example. And what does he say ?

JEAN-PIERRE SERRE : It's curious, I didn't remember that.

ALAIN CONNES : He says you take a small category and you take the functors of this small category towards the abelian groups. It's a diagram category for him. And after that, he checks of course that yes, it's an abelian category, everything works, etc.

JEAN-PIERRE SERRE : Yes, indeed. It's an idea that was certainly not in the air.

ALAIN CONNES : If you want, it's this part that interested me a lot as an innovative part, I will come back to this.

JEAN-PIERRE SERRE : But me, it didn't interest me because it was nothing concrete for me. He didn't calculate homotopy groups.

ALAIN CONNES : I agree, of course, of course. But I'll come back to it later. In fact, so, I heard in one of the interviews that you gave that it makes you react when people think they're wrong when they talk about a revolution. about the theory of schemes, I think we agree, it was in the air. In fact, you think this idea goes back to Krull

JEAN-PIERRE SERRE : because Krull was an algebrist. We had the feeling that he guessed the geometry that was behind. And in any case, he built the Krull rings with localization.

ALAIN CONNES : He built the localization that is essential yes,

JEAN-PIERRE SERRE : but he had not jumped the step to go to the projective. He was affine. The geometry when it stays affine, it stays glued.

ALAIN CONNES : Of course, it does not work, it's not interesting enough. What also struck me a lot, when I studied all that, is at what point Grothendieck came to an ideal world. Why? Because Serre and several other people were giving a seminar in Princeton on schemes at the time and they wrote it, Dieudonné helped him to write a little bit.

JEAN-PIERRE SERRE : This is a little after... The beginning, it was purely the correspondence with me, when he was at Kansas, it was for him the net change. It was Kansas, he wrote to me. That's it and it gave the developments.

ALAIN CONNES : Okay.

JEAN-PIERRE SERRE : And then after, he was recruited by Motchane. So I don't know how it works from the point of view of the dates

ALAIN CONNES : It's in 58. It's recruitment by Motchane.

JEAN-PIERRE SERRE : Although Kansas is in 55. There was an intermediary period. He was at the CNRS.

ALAIN CONNES : So maybe, I do not know that.

JEAN-PIERRE SERRE : So, I do not know, but he was stateless. It was not so simple for him.

ALAIN CONNES : At the level of schemas. Well, it's clear. Also, the same, I mean, The matching is ideal at the motives'level. That's great because we

see, in 64, in your correspondence, you talk about motives' metaphysics, etc.

JEAN-PIERRE SERRE : And what the correspondence does not show, is that in fact, a consequence of a lot of remarks he made. I told him "you know, between varieties, if we admit Weil's conjectures, cohomology cuts itself into pieces...". And things simmered in his skull, of course, but then he did something I could never have done myself. He had the idea to define this, with an extraordinary courage

ALAIN CONNES : that characterized him.

JEAN-PIERRE SERRE : because I would have never thought that the algebraic cycles were strong enough to do that. He had the courage to start that. It may be wrong

ALAIN CONNES : Perhaps. It's Hodge's conjecture.

JEAN-PIERRE SERRE : In any case, it was a good start,

ALAIN CONNES : yes, it was a good start. And then, there is another absolutely essential part. And I'll tell you what I know. You will correct me. It's for etale cohomology. So what I've heard told, but I do not know if it's true. You corrected me. What I heard told is that you are the one who gave a seminar, at Chevalley Seminary

JEAN-PIERRE SERRE : Of course, all that is correct, yes.

ALAIN CONNES : in 58. seminar in which you explained that to have locally trivial bundles in terms of algebraic groups, it was necessary to take etale coverings, and that at the exit of your seminar,

JEAN-PIERRE SERRE : it is exactly how... it's absolutely correct. I am still seeing myself, at the blackboard talking to Poincaré and at the end of the talk, Grothendieck telling me "it's going to make the Weil's cohomology." because it was called Weil's cohomology, cohomology that we wanted.

ALAIN CONNES : Okay.

JEAN-PIERRE SERRE : instantaneously because I had presented systematically, I made an H_0 , well the H_0 , it's obvious, I made the H_1 , but I didn't have the intellectual courage to say "it could do an H_2 ", since him, he said instantaneously. And it is perfectly correct. It's a legend that, for one time...

ALAIN CONNES : The current passed, as we say.

JEAN-PIERRE SERRE : This thing made the thing, it triggered. And in some sense, you'll watch to the text I wrote, I wrote it this way, I wrote H_1 . I had the good cohomology in dimension 1, it was my idea. I went out of the idea that the H_1 of Zariski for coverings is ridiculous since we find NOTHING. So you put them by force in the machine and it makes that you obtain a good H_1 but I didn't have the idea. If I had been answered on this, I would have said it was necessary to have new ideas in greater dimensions since him...

ALAIN CONNES : He was convinced it would work for greater dimensions.

JEAN-PIERRE SERRE : He was of an extraordinary optimism.

ALAIN CONNES : Often in the correspondence, we see at what point you gave him counter-examples. (*Laughs*)

JEAN-PIERRE SERRE : But you know, Weil told me one day, it strucked me "It is the optimists who demonstrate theorems.

ALAIN CONNES : Well, but you have not to be too optimistic. But he is surely right.

JEAN-PIERRE SERRE : So this legend is absolutely correct.

ALAIN CONNES : So this is good. Then there is a notion that I would like to address. I think I know what will be your reaction.

JEAN-PIERRE SERRE : So tell it anyway.

ALAIN CONNES : Well, for me, one of the great discoveries of Grothendieck is the notion of topos.

JEAN-PIERRE SERRE : The notion of... ?

ALAIN CONNES : Topos.

JEAN-PIERRE SERRE : I don't even know what it is. I didn't really make the effort to understand exactly because as soon as there were categories in abundance, I stopped, I nearly blocked on those subjects.

ALAIN CONNES : I must admit that I had exactly the same attitude until a few years ago and that finally, I think that it is a notion that one can appreciate only when one meets it independently.

JEAN-PIERRE SERRE : Yes, and I never needed it.

ALAIN CONNES : Okay, so. So, in fact, if I ask you when Grothendieck invented toposes ?

JEAN-PIERRE SERRE : I do not know and I do not care. Not really.

ALAIN CONNES : Yes, yes, yes, that's for sure.

JEAN-PIERRE SERRE : That does not tell me anything. I know one speaks a lot about this notion, it's fashionable, and all that. Okay, but it's exactly what I thought.

JEAN-PIERRE SERRE : It says nothing at all to me.

ALAIN CONNES : There is another distinction. that you make in several interviews, which is a little like the distinction between, precisely, Grothendieck did between functional analysis and algebraic geometry. You make a small distinction between algebraic geometry and the theory of modular forms.

JEAN-PIERRE SERRE : It has a lot more charm to establish that. Here we go in a completely different, they are different directions and it's to say that Grothendieck even seems to be interested in the theories that were logically developing by themselves

ALAIN CONNES : the rising tide of general theories.

JEAN-PIERRE SERRE : However, one of the charms of modular forms and of Langlands program is that they are absolutely not logical at all. This is a brilliant idea that says that two things correspond one to the other.

ALAIN CONNES : Finally, correspond.

JEAN-PIERRE SERRE : However, there is no reason a priori for this correspondance to occur And this, for me, has an irresistible charm. It has an absolutely incomparable charm compared to something that happens from little to little like that.

ALAIN CONNES : I understand this very well.

JEAN-PIERRE SERRE : And that, for me, was incarnated in precise things with modular forms. My conjecture, for example on Galoisian representations, the conjecture on elliptic curves. All of this, for me, took shape. near 1967 ; 67, yes, It was a big year for me.

ALAIN CONNES : Yes it's sure.

JEAN-PIERRE SERRE : For number theory, because that's the year when there was motives theory. I saw right away that the motives were related, you see.

ALAIN CONNES : It's in 64, motives, already in the correspondence.

JEAN-PIERRE SERRE : But for me, it's only in 67 that I see it must be linked... Yes, because it's at this time that there was the Weil's article on elliptic curves.

ALAIN CONNES : Oh yes, this is what we call Taniyama-Weil Conjecture

JEAN-PIERRE SERRE : that was confirming that elliptic curves would have to correspond to modular forms. It's just that until then, it was a kind of vague hope,

ALAIN CONNES : I understand, I agree

JEAN-PIERRE SERRE : until that, it was not concrete. Weil didn't have the notion of conductor but nearly, but I had this notion so I can state more precisely the conjecture. So, she became absolutely convincing. It was incredible.

ALAIN CONNES : Yes it's sure.

JEAN-PIERRE SERRE : I told somewhere after discussion with Weil, I believe, I went home and watched. Oh, I knew there weren't elliptical curves with conductor 1. Oh yes, but there are no modular forms corresponding. Oh there are not also for 8 either. Ah, but it's particular. It was bright, you see? And that's the kind of things absolutely not "Grothendieckian

ALAIN CONNES : Of course, it's totally orthogonal. But that, it shows.

JEAN-PIERRE SERRE : And this makes me a lot more effect, you see.

ALAIN CONNES : Of course, of course.

JEAN-PIERRE SERRE : And at more basic levels, whenever there are matches that are a bit surprising, it touches me. Although Grothendieck, this disgusted him. He does not like it, he does not like it.

ALAIN CONNES : Yes, all that, it comes out perfectly from the correspondence.

JEAN-PIERRE SERRE : It's a bit of a point of view. It's more romantic when you have things with no obvious relationship between them, and finally they are the same. It's just "a marriage was at sky. This is love at first sight.

ALAIN CONNES : That's it. But precisely, we are getting closer to a period where, well, from 68, of this period a little cloudy for Grothendieck, we will talk less about mathematics.

JEAN-PIERRE SERRE : He is starting to leave maths in 68.

ALAIN CONNES : It is nearly in 70 that he leaved the IHES.

JEAN-PIERRE SERRE : I don't remember dates.

ALAIN CONNES : But in any case, he spent two years at College de France. It's you who invite him to College de France for 2 years.

JEAN-PIERRE SERRE : it was a pulpit for foreign scientists that was for a year initially. We renewed the national agreement the year after and we did not want to renew it after.

ALAIN CONNES : Why? Did he make enemies? Do you mean he didn't behave correctly? What happened and...

JEAN-PIERRE SERRE : This was not his place because, listen, he was spending his time, at that moment, saying that we had not to make science, that we had not to do maths, that it was time for ecology. So if he didn't want to make science, he had to go elsewhere. I was not happy anyway that the CNRS would have found a place for him, I feeled it was not good

ALAIN CONNES : it was long after, CNRS.

JEAN-PIERRE SERRE : Not long time after, no.

ALAIN CONNES : It's in 84.

JEAN-PIERRE SERRE : But at that moment. At that moment, he played a pretty naughty game because he didn't want to make maths, but he wanted to get paid, he wanted to have a position, You see, it was... for someone who had principles.

ALAIN CONNES : But was he doing his classes or not? because...

JEAN-PIERRE SERRE : So You know the story of his courses in College, no?

ALAIN CONNES : No, no, no. Absolutely not.

JEAN-PIERRE SERRE : The first year, there was no problem. He did a course.

He did a course, I do not know what.

ALAIN CONNES : Yes, it was Barsotti-Tate or something like that.

JEAN-PIERRE SERRE : No, I think that is in the second year. The second year, yes, he gave us as subject of his course, You know the subjects that a professor must give in June He gave us something about Ecology.

ALAIN CONNES : Hou la! It was at this point? I believed he had given two subjects.

JEAN-PIERRE SERRE : So, then administrator disjoined this proposal from the rest. Besides, we voted yes on everything else. And all the friends had to vote separately on the text of Grothendieck. We voted no by a large majority on the part concerning ecology. I voted no, of course. And then Grothendieck accepted the vote, he didn't have choice. He gave a course on Barsotti-Tate, or something like that, in which he started by "I can not talk to you about Barsotti-Tate without explaining you that... And then four hours, four hours of Ecology (*AC laughs*) and you see that... I believe that Illusie attended and staid... There should not have been many people at this course.

ALAIN CONNES : It was an ecology class, so.

JEAN-PIERRE SERRE : a course of Survive, a course of good feelings and Grothendieck's good feelings. Okay, so you imagine that we didn't want to renew him.

ALAIN CONNES : Of course. But it reminds me, moreover, in the correspondence of the dissensions you had, which was at the moment when Grothendieck had wanted, I think he wrote it, to write a letter to Cartan.

JEAN-PIERRE SERRE : It's different. He had written with Cartan on the Algerian war.

ALAIN CONNES : He wanted to dispense students from ENS of doing military service.

JEAN-PIERRE SERRE : Yes, to dispense mathematicians from military ser-

vice. Yes, I answered that it was difficult when the people skin is involved. I remember that I told him that if some can't be killed, while others can... It is true that some countries actually protected scientifics during war. I think the USSR, for example, protected scientists in those times. They killed them for political reasons, eventually. But they did not want them to be killed in the war anyway.

ALAIN CONNES : He spoke of the United States. Grothendieck's point of view was that if any reason can be taken not to go into the army, we take this reason. It has not been a serious discussion, he wrote this to Cartan and they didn't talk about this anymore after.

ALAIN CONNES : effectively, there are a number of letters actually that have not been answered.

JEAN-PIERRE SERRE : It was the war of Algeria.

ALAIN CONNES : Yes, in 61, I believe. So there are a number of letters where we can see he did not answer. And there was a letter on which I was very, very curious about whether you had seen it after or what. And that's when Dwork demonstrated the rationality of zeta functions. I'm very curious about how he reacted.

JEAN-PIERRE SERRE : Well, he did not care.

ALAIN CONNES : Ah? He didn't care? He did not care. It was outside his scheme, of his program, so he did not care.

ALAIN CONNES : It's incredible because the demonstration of Dwork is magnificent.

JEAN-PIERRE SERRE : Oh yes, I exposed it to Bourbaki and I fest myself. For example, I tried to look at what it was for cohomology with coefficients and I can not remember, but there were difficulties with a group that operates. Okay, but anyway, he was right about his point of view. He wanted it of one way,

ALAIN CONNES : he did not want to turn away from his objective.

JEAN-PIERRE SERRE : It was an accident, we had demonstrated this a little earlier than we should have. I tell you. When did I work on Riemann-Roch, me? about 50.

ALAIN CONNES : Yes, you tell it in the interview of Coliot-Thélène. You are the first who had the idea that it was an Euler characteristic.

JEAN-PIERRE SERRE : Yes, it's me...

ALAIN CONNES : Yes, and this is very important, in fact. You never published that.

JEAN-PIERRE SERRE : No it was not necessary because when I corresponded with Kodaira and Spencer. I saw that they had had the same idea but they didn't have my theorem of duality. So they published their thing and I, I published the theorem of duality. But then, when I thought about that, Riemann-Roch, what was funny for me was that I tried to demonstrate it for curves, you see.

ALAIN CONNES : It's just H_1 and H_0 ...

JEAN-PIERRE SERRE : You joke?! So yes, yes, okay.

JEAN-PIERRE SERRE : But then, I tried to demonstrate something that was known for about 100 years. But I had an idea about demonstrations that could be made. But I didn't wanted them.

ALAIN CONNES : But excuse me, excuse me. When you say you did that, you did that with sheaves, Zariski, and so on.

JEAN-PIERRE SERRE : Yes that's it... Hum, no! At the time, wait, I was Zariski or I was complex analysis? No, I was complex analysis. It was before GAGA. I made FAC and GAGA at the same time. So, this one was complex analytical. Simply, I did not want the existing demonstrations. I did not want because... when I found the one I wanted, and the one I wanted, you must know it, it is. You have a divisor D and what you're showing is that if you know it for D , you know it for $D+P$ where P is a point, you can walk around

like that, and when $D = 0, \dots$

ALAIN CONNES : a sort of recurrence, okay.

JEAN-PIERRE SERRE : I see myself at my work table. When I found this, I wrote it somewhere, I know that 3 minutes after, I had the dimension 2, the theory of surfaces

ALAIN CONNES : you already knew it was an Euler characteristic.

JEAN-PIERRE SERRE : You see how we had to do?! I had to show that if I have a divisor on the surface and if I add something to it, and if I had it for the divisor, I had it for the preceding Riemann-Roch, and a little more. it took me 3 minutes. Dimension 3, I could not, because there were things to demonstrate, that... But that was, typically, it happens often to us... we are not happy with a demonstration because we want a demonstration that does something else. And this one, it was great.

ALAIN CONNES : Of course, and in what year did you do this? Nearly 53.

JEAN-PIERRE SERRE : Well, let's see, my thesis was in 51. I stopped doing homotopy groups almost immediately. In 52, it was the Cartan seminar on Stein varieties. And very quickly, I was raised in Preparing classes, like you in fact, with the idea that it is the projective geometry that is good. The affine geometry, that's joke. However, Cartan, it's affine geometry. Stein varieties are open stuff, and compact things have a certain charm...

ALAIN CONNES : By the way, your correspondence starts with a big mistake of Grothendieck who says that the quotient of a Stein variety by a group...

JEAN-PIERRE SERRE : no, but I think that it's a Grothendieck's typo. he forgot to write finite.

ALAIN CONNES : I agree.

JEAN-PIERRE SERRE : Afterwards, I realized that it was what he meant. But he does not correct. Maybe he believed, anyway. It's for this reason that I left the point of view Cartan.

ALAIN CONNES : but there is still a key point in what you did is the use of Zariski topology, sheaves for Zariski's topology.

JEAN-PIERRE SERRE : This is just after. And this is to speak about me, not to speak about Grothendieck.

ALAIN CONNES : It doesn't matter, it's completely clear. in fact, when you look at a certain distance, I, as a not specialist of this at all, that is, if you want, the influence of Leray...

JEAN-PIERRE SERRE : Oh yes, it's clear I had an influence, I had a big influence on him, it's clear. No, what happened is that on projective space, on sheaves of projective space, complex, you see, I had been able to see that there were modules because I took the sections.

ALAIN CONNES : I saw the proof of your paper on GAGA, in fact.

JEAN-PIERRE SERRE : It was in a Cartan seminar I don't remember, I constructed in this case and I saw a dictionary with the modules. So I said, "It's not possible, it's going to work on any. And that's how I went to FAC and it was written, incredible.

ALAIN CONNES : Yes, yes, that, you said it several times that you did not have to think. The typewriter, an hundred pages paper, as if it existed yet.

ALAIN CONNES : Yes, so now, we are going to approach a more delicate period and that is the end of the correspondence, that is to say that there is a very, very great interruption of correspondence.

JEAN-PIERRE SERRE : Basically, there is no more correspondence in fact.

ALAIN CONNES : Okay, it's true, however, there is the letter that you will write him and that I find very, very relevant if you want. When you received *Crops and sowing*. It was in 86, I think. Then you wrote to him and if you want, I would be sorry for not citing you well, but it is very important.

JEAN-PIERRE SERRE : You made photocopies.

ALAIN CONNES : Of course. So I read you in the text to be sure, “I have received the book *Crops and sowing* you sent me. Thank you very much. I still miss the penultimate booklet from which I only have a few pages isolated.”.

JEAN-PIERRE SERRE : Some hundreds of pages. Yes that’s it.

ALAIN CONNES : “One thing strikes me in the texts that I have had the possibility to see. You are astonished and you are angry by the fact your alumni did not continue the work that you have begun and conducted largely to good. But you do not ask yourself the most obvious question, the one to which every reader expects you to answer : “why you, you have abandoned the work in question ?”.

JEAN-PIERRE SERRE : Of course, it’s obvious. Of course, it’s an obvious question. He writes 600 pages, not answering to this question.

ALAIN CONNES : But what is more interesting is that you have an idea.

JEAN-PIERRE SERRE : Yes, I have an idea... if I know it? I still agree with this idea.

ALAIN CONNES : So you say “I have the impression that despite your so well known energy...

JEAN-PIERRE SERRE : It would have been necessary to know him physically, physically and intellectually, it was the same. He could work almost 24 hours. That, it was amazing. Such a strength. I know nobody with so much strength,...

ALAIN CONNES : That’s it.

JEAN-PIERRE SERRE : Even if there are people who are intellectually very strong like Thompson for instance, or Bombieri is very strong, but Grothendieck was an animal force.

ALAIN CONNES : Okay, so you say “you were just tired of the work that you had done. Especially since there were also SGAs.”. I remember that SGAs, well, SGA3 I think the algebraic groups, SGA4 were Topos. And then you say : “I remember in particular the rather disastrous state of SGA5.

JEAN-PIERRE SERRE : Yes, because well... It was really disastrous, you know. It had been roneo-typed by IHES, but there was too much algebraic commutativity to check. and Illusie who was serious however about a decisive theorem, had said, had written : “I have been unable to verify.”

ALAIN CONNES : And this was terrible. You say : They were reduced to assert without proofs the commutativity, down to the nearest sign to be optimistic and those commutativities were essential for the future.

JEAN-PIERRE SERRE : Well considering the result to which it is applied, the sign, there was no problem because they would have found negative numbers for the number of points. So, obviously, that’s precious to have stuff like that.

ALAIN CONNES : in absolute value...

JEAN-PIERRE SERRE : It’s precious. It means that we can detect an error. But it doesn’t mean that we have a demonstration. Illusie had written downright in the text : “The writer apologizes for not having been able to check the commutativity of the diagram.”

ALAIN CONNES : Okay, but then, what you are saying is really, very, interesting. You say something... I think I should read more because I’m going to stop in the middle. You say “one may wonder, for example, if there is not a deeper explanation than the simple fatigue of having to carry at arm’s length so many thousands of pages. So, you say, you describe it somewhere. We approach math where one does not attack a problem head on, but wraps it up and dissolves it in a rising tide of general theory. All right, that’s the way you work and what you did shows that it works effectively at least for SVT and algebraic geometry.”. And after, you say and I’ll let you talk. “It’s a lot less clear for number theory.

JEAN-PIERRE SERRE : Yes, I think for example, I do not know if I thought at that moment, but exactly, thinking about Langlands theory and about

modular forms. He didn't understand nothing at modular forms. He was extraordinary of incomprehension, sometimes, because when it did not fit within its framework, okay. I told him modular shapes. He told me "But your modular forms, it makes no sense because you see your variety of modules, it is affine and so at infinity, you put artificial conditions." Since me, I had 100 years or 150 years of modular forms behind me. I knew it was a good theory. When we see it, we don't resist. Not him, they were formulas for him. He could not handle with formulas.

ALAIN CONNES : Okay. Okay. Okay. So what are you saying here, actually, you say further than in fact, in number theory, precisely, all mathematics can come in and

JEAN-PIERRE SERRE : and we don't know how it works. This is exciting in number theory. We do not know exactly.

ALAIN CONNES : So now, what I wanted to have was your reaction.

JEAN-PIERRE SERRE : In any case, you can see that I agree with everything that you quoted. I have not changed my mind since that time.

ALAIN CONNES : You have not changed your mind, you have not changed your mind. It's important. There is the answer of Grothendieck in the correspondence. I will not read it, but yes,

JEAN-PIERRE SERRE : There may be things where I told nonsense. in the correspondence, it is possible, but probably I corrected them in the notes because there are notes. So, if I said something that I found silly...

ALAIN CONNES : No, no, no, no, no, I do not think so. I do not think I do not think so. If you want, I think that I, what I tried to do was to slip into the skin of Grothendieck to understand and try to understand how he could, euh, how to say, if you want, I do not say that he has become paranoid, I don't like this word at all. I think there's a word for that, it's a buzz word, it's the word obsidional.

JEAN-PIERRE SERRE : I can give you my own conclusion. Grothendieck made me think at a nuclear power plant and nuclear power plants, you have

to cool them, you have to protect them, and so on. Yes, but as long as he was in the mathematical world okay normal, we served him as a protection. As soon as it became alone, the plant exploded.

ALAIN CONNES : Okay, okay. But then,

JEAN-PIERRE SERRE : It isn't a so kind explanation but...

ALAIN CONNES : But no matter, no matter,
it does not matter but What is there is that the text he wrote,

JEAN-PIERRE SERRE : the texts, he wrote thousands of pages, tens of thousands of pages. What text are you talking about ?

ALAIN CONNES : I am talking about several texts. In fact, I understood. I had to give a talk at Collège de France. I had been asked to give a presentation on the refugees. There was a colloquium on foreign refugees.

JEAN-PIERRE SERRE : On refugees??

ALAIN CONNES : Let me say. It was a general thing, so I had said okay. And then, you know how it is, a month before you start telling yourself "What am I going to talk about?"

JEAN-PIERRE SERRE : You have something to say about refugees, you??

ALAIN CONNES : And then well, I was in very special circumstances because I was taking care of my mother who was very sick. And at night, I read and I read "*The key of dreams*", which is one of the texts of Grothendieck.

JEAN-PIERRE SERRE : I don't know this text... Euh, yes, I know it, I have watched it. because I have been impressed by some of the Grothendieck's dreams that have a level of details in their description. I remember a princess, or something like that, with statements, but I know I never have a dream with such precision, my skull does not manufacture. It's a matter of even greater power of a skull to make mentally with your little neurons, decorations of a lady in her chair. The power of his skull was even visible in his dreams.

ALAIN CONNES : Absolutely. When I stumbled by flipping through it, reading this text on a passage that is absolutely beautiful. And it's the passage on his father, on what happened to his father who was an anarchist when he was in prison, since he stayed more than ten years in prison in Russia. And what happened to him at a given time and that was passed on to Grothendieck by his mother. And Grothendieck describes it in an incredibly accurate way, as you say.

JEAN-PIERRE SERRE : And so, it's in a dream, no ?

ALAIN CONNES : No, no, no. It was in the reality, in reality. he describes what happened to his father, who was promised that he would be released after ten years, where he counted the days etc. And by the time the date arrived, he was not released. He started a hunger strike. And at the end of I do not know, maybe three weeks or a month of strike hunger. There, he had an illumination. Mystic, yes.

JEAN-PIERRE SERRE : This is not surprising,

ALAIN CONNES : It's not very surprising, no, but if you want in this illumination, he forgave his jailers, etc. And the way this is written, it made that... When I read it for the first time to my wife, in a moment before talking about it in College, I had to stop in my reading so it was moving. So, in fact, I read it in College, but it's on that occasion, that I realised the fact that, at the middle of 36 different things, there was in these texts, from time to time, extraordinary things.

JEAN-PIERRE SERRE : Yes, Yes, I think that, yes, I do think so. I find no adjectives. I started by thinking scornful, but that's not quite the case. I was skeptical, in any case on Grothendieck in 30 last years. Yes, but still, I agree. It is clear that there are interesting things.

ALAIN CONNES : we do not talk about them.

JEAN-PIERRE SERRE : No, no, his skull didn't degenerate but it exploded rather than anything else and always intelligently, always intelligently.

ALAIN CONNES : And if you want what this text taught me, this text that I had discovered, what it taught me is that in fact, his father had never managed to do what he wanted to do. His father wanted to be a writer.

JEAN-PIERRE SERRE : True ?

ALAIN CONNES : Yes, his father wanted to write

JEAN-PIERRE SERRE : Oh really ?

ALAIN CONNES : And he had never been able to do it because he was all the time doing things, etc.

JEAN-PIERRE SERRE : It's not possible to be an anarchist and write at the same time.

ALAIN CONNES : And Grothendieck, apparently after a while, decided to write.

JEAN-PIERRE SERRE : Oh that ! To write, for sure he wrote.

ALAIN CONNES : And what was found in his house when he died, we found an incredible number of pages that are that are currently inaccessible.

JEAN-PIERRE SERRE : No, they aren't inaccessible.

ALAIN CONNES : Yes, they are, they are guarded by a lawyer. Because the kids don't agree.

JEAN-PIERRE SERRE : Yes, that's right, but they'll be accessible one day. They're not lost.

ALAIN CONNES : They are not lost exactly. But apparently, that, I find that quite incredible. And, apparently, the main topic, you know, Grothendieck had a mystical evolution, I will say.

JEAN-PIERRE SERRE : yes, a little weird, weird, mystical.

ALAIN CONNES : But apparently, the main subject of these thousands of pages is actually, it's the problem of evil. That is to say, he realized by being mystical, being a religious in a certain way, well, in fact, there was a fundamental problem. Of course, he has tackled this problem and we do not know what's inside.

JEAN-PIERRE SERRE : I don't call them problems, those things. Literary people tend to call it *The problem* with the idea that, above all, one does not try to solve them.

ALAIN CONNES : Of course, yes, but then he, he tried.

JEAN-PIERRE SERRE : But one speaks about it, one speaks, one speaks...

ALAIN CONNES : Me, I'm really very curious about... I do not know if it will be possible, of course. Apparently, it's at least thirty thousand pages.

JEAN-PIERRE SERRE : Thirty thousand ?

ALAIN CONNES : Thirty thousand pages if it's not more, but well classified.

JEAN-PIERRE SERRE : Oh, it's possible with Grothendieck. He wrote at such a rhythm

ALAIN CONNES : Exactly.

JEAN-PIERRE SERRE : And are they typed or are they are handwritten ?

ALAIN CONNES : I do not know, but now, with AI, it's not a problem.

JEAN-PIERRE SERRE : A long time ago, he typed out very fast.

ALAIN CONNES : Yes, of course, but we'll be able to, now, I think it's handwritten, but we'll just have to write a little software, that will transform all that.

JEAN-PIERRE SERRE : because his writing was not terrible. that's not the problem.

JEAN-PIERRE SERRE : We will not take care of that.

ALAIN CONNES : We will not take care of that. So now, I wanted to point you out another fact that I do not know if you know. Of course, you surely knew Paolo Ribenboim.

JEAN-PIERRE SERRE : Yes, he's charming, charming He is from South Africa or he is from Brazil ?

ALAIN CONNES : Brazil, Brazil. He travels between Kingston, Brazil and Paris, he works in number theory. I saw him recently.

JEAN-PIERRE SERRE : He is almost blind.

ALAIN CONNES : Yes, unfortunately, he is almost blind. But if you want, he sees : he uses computer means to see. That is, although being almost blind, it has a machine that greatly amplifies the characters. And so he is still able not to see, but to read what one sends to him, etc. So I talked with him and he taught me something very interesting. He taught me that Grothendieck actually came in the 2000s secretly to Paris.

JEAN-PIERRE SERRE : Really ? I did not know that. But to do what ?

ALAIN CONNES : He came because he absolutely wanted to make an attempt to publish *Crops and sowing*. So, apparently, he came, he came to Paris, he stayed in the apartment of Paolo Ribenboim, who has an apartment in Paris. And he wanted...

JEAN-PIERRE SERRE : with Odile Jacob perhaps ?

ALAIN CONNES : No, I don't think so.

JEAN-PIERRE SERRE : Because initially, it was Odile Jacob.

ALAIN CONNES : Yes, but Odile told me what had happened.

JEAN-PIERRE SERRE : I had been contacted, I had been contacted a long

time ago, well before 2000. On the publication of that, and finally, I hesitated, and I gave unfavorable opinion, because really, he says trully nasty things, on Deligne, Illusie,

ALAIN CONNES : that's right.

JEAN-PIERRE SERRE : and for those people there, who are good people, to see this written, published and to which they can not answer, you see. It was really unpleasant, for Deligne, in particular.

ALAIN CONNES : For sure.

JEAN-PIERRE SERRE : I think maybe it was SMF that was asking the question Perhaps it's a guy from SMF who contacted me? But I hesitated because it's interesting, there is no doubt.

ALAIN CONNES : There is a lot of interesting things. That is to say all the polemical side. well, if we could leave it out, the rest would go to you.

JEAN-PIERRE SERRE : Which length? I do not remember...

ALAIN CONNES : it's not so long...

JEAN-PIERRE SERRE : 600 pages?

ALAIN CONNES : Yes that's it. That's it, a time it's typed, it's between 500 and 600 pages.

JEAN-PIERRE SERRE : Yes it is publishable.

ALAIN CONNES : What he wrote at the beginning, it's to say that he tried to write something which baits the reader, etc. He didn't succeeded in doing that, well, I want to say the non-mathematician reader...

JEAN-PIERRE SERRE : In any case, a day, the whole will be published.

ALAIN CONNES : I even think it's going to be published in the meantime. I do not know if it will be by Hermann or something like that, but I heard

that. I heard that recently. Well,

JEAN-PIERRE SERRE : I was not asked.

ALAIN CONNES : No, no, well, it's better to stay away from that. But OK. But if you want, it's true that I regretted that the passages really interesting, because there are interesting passages, were inaccessible.

JEAN-PIERRE SERRE : So yes, if it's still difficult to extract, it happened to me. I extracted three pages on motives you know. And I copied three or four pages from him, splendid and where he is not telling bad about someone. And I had his permission. At the time.

ALAIN CONNES : And it was not trivial to have his permission, but you know what happened. Nobody knows it. That, I do not know if I should talk about it.

JEAN-PIERRE SERRE : Yes you can speak...

ALAIN CONNES : ... what happened at the 50th anniversary of IHES. It was a little earlier. It was in the month of september.

JEAN-PIERRE SERRE : On what year, the 50th anniversary of IHES ? It was ten years ago, so it was in 2008, the death of Cartan, ten years ago and in the month of September... In the month of September, Grothendieck wrote to the librarian of IHES, to ask for books. I don't know if you know about it.

JEAN-PIERRE SERRE : No, I heard talk about something like that. So they sent him those books in time ?

ALAIN CONNES : No, unfortunately, it was unfortunate that the librarian was on vacation at that time. Therefore, the Grothendieck's letter had no answer, no immediate answer,

JEAN-PIERRE SERRE : But finally he returned from vacation after a while.

ALAIN CONNES : But if you want, he became a little impatient. So he wrote

to the Director.

JEAN-PIERRE SERRE : Yes, well, okay. Indeed, why not ?

ALAIN CONNES : Why not ? It happened that the director was absent at that time...

JEAN-PIERRE SERRE : Aïe ! And that at that moment, the person who answered Grothendieck was not the Director, if you want, Then, there started to be a muddle because finally, the answer was a bit of a generic answer. If you want, saying that good, etc. And there, the tone is mounted.

JEAN-PIERRE SERRE : Telling it was not possible.

ALAIN CONNES : Well, not saying that it was not possible, but let's say that there would be delays, and so on.

JEAN-PIERRE SERRE : Well, it did not say what he wanted to read.

ALAIN CONNES : That's not what he wanted. So, the tone is mounted. Grothendieck wrote a much more... letter, as he was able to do.

JEAN-PIERRE SERRE : Yes, more energetic, yes, already that he was energetic by nature.

ALAIN CONNES : And then the Director was back and the Director tried to answer

JEAN-PIERRE SERRE : Who was the director at that time ?

ALAIN CONNES : It was Jean-Pierre Bourguignon who was head.

JEAN-PIERRE SERRE : Bourguignon who knows how to arrange things.

ALAIN CONNES : Yes, of course, but then he tried to fix things. He tried to fix things in explaining that he was absent, and so on. But the tone was up and Grothendieck phoned to Lafforgue, while Laurent Lafforgue was at home. So Lafforgue is an admirer of Grothendieck absolutely unconditional.

As you know, he has his picture on his desk, etc. And one day, Lafforgue returned home, the phone rings when he hears a voice that says...

JEAN-PIERRE SERRE : but he did not know this voice.

ALAIN CONNES : Of course not. Then he believed it was a hoax?

ALAIN CONNES : No, he was absolutely stunned because he hears on the phone : "It's Alexandre Grothendieck, obviously And then, Grothendieck asked him to pass on to all members of the Scientific Council. At the time, I was a member of the scientific board a copy of these letters and the exchange that had taken place. And that's where I learned that Grothendieck was refugeeed. I did not know where he was. I had no idea where he was.

JEAN-PIERRE SERRE : He was already in the Pyrenees, at that time?

ALAIN CONNES : Of course, he was in the Pyrenees since 90. Here, we were in 2008, we were in 2008 already. Since then, I think 91 or 92. he was already in Pyrenees.

JEAN-PIERRE SERRE : At Lasserre or somewhere like that?

ALAIN CONNES : Well, maybe you're going to laugh. I thought it was not a coincidence. That the place where Grothendieck was refugeeed was called Lasserre because in *Crops and sowing*, there is a whole development on yin and yang, and so on. And on the idea that Grothendieck rightly has a feminine side in his approach to mathematics, that is not completely meaningless. So, the fact that he is a refugee,

JEAN-PIERRE SERRE : It's what is curious, It may be interesting that he considered that he had a feminine spirit. He told me that I, the male spirit, while I, never knew someone as masculine as him. Also incredible.

ALAIN CONNES : In the correspondence, it's clear, he is a Germanic man.

JEAN-PIERRE SERRE : And he considered him, like a feminine character, and me, on the contrary, as a masculine character.

ALAIN CONNES : Absolutely. For him, you, you were the prototype of the masculine.

ALAIN CONNES : And all that is explained in great detail in *Crops and sowing*.

JEAN-PIERRE SERRE : But he was taking this in a serious way. All that, by the way, is a feature of Grothendieck. He took everything in a serious way.

ALAIN CONNES : Yes, by the way, it's a question I wanted to ask because, finally, what stands out a lot of these writings is that we did not not feel like he has a sense of humor.

JEAN-PIERRE SERRE : No no no no. He has a sense of the kind of intellectual duty, to push ideas until their maximum, to deepen them. It is a great honesty intellectual.

ALAIN CONNES : Yes, that's for sure, but not the sense of humor.

JEAN-PIERRE SERRE : No, it's not compatible. I can not imagine myself having heard him laugh. Maybe for other things

ALAIN CONNES : But when you say it's not compatible.

JEAN-PIERRE SERRE : We were not able to joke with him

ALAIN CONNES : Yes, but you say it's not compatible. seeing the interview with Cartan, we can not not being struck by the fact that Cartan has an incredible sense of humor.

JEAN-PIERRE SERRE : Yes, but Cartan could not have done Grothendieck's artwork, with a character as his one. Either me, by the way. Yes, I think there is some... I don't know how to say, it asks for an incredible strength. Not compatible with the need of laughs.

ALAIN CONNES : For sure.

(Laughs from Alain Connes)