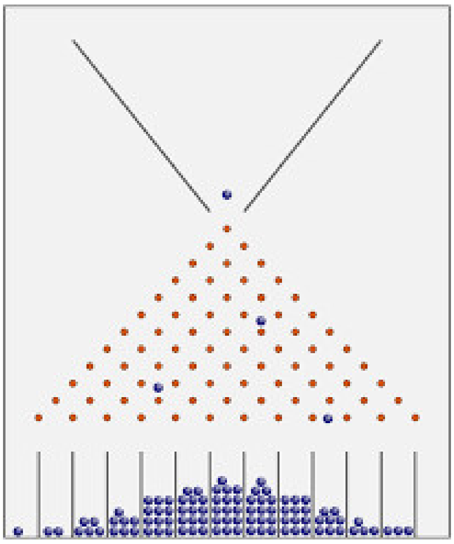
Goldbach decomponents on a Galton board Vella-Chemla, 27.8.18)

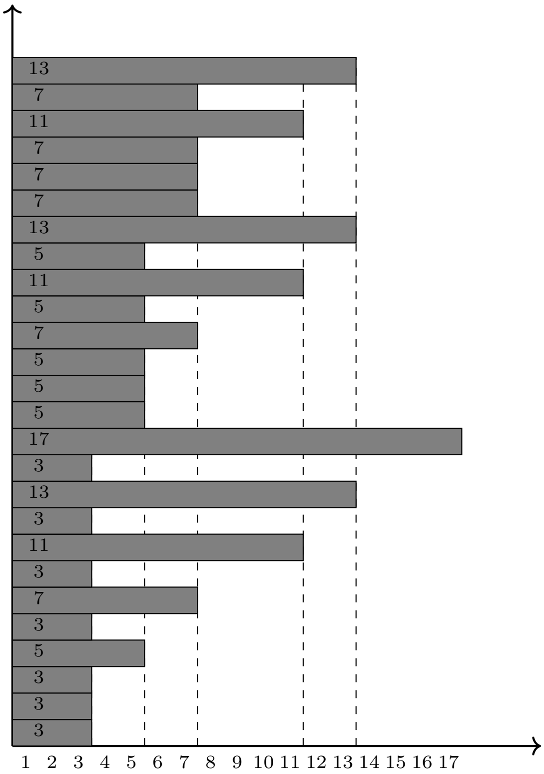
We recently took the full measure concerning the way the hazard is governing Goldbach decompositions (i.e. or decompositions of an even number as a sum of two primes) as well as their number, by studying them accordint to a point of view using 2×2 transition matrices coding kinds of chip exchanges.
Here we'd wish to study, to apprehend even more this hazard, the way quantified Goldbach decomponents (i.e. cut into pieces) fill a sort of "Galton board" ; it's a board in which marbles fall, and divide themselves the more there are according to a Gauss curve (bell curve). This idea came from the often made association in literature between chip-firing game concepts and sandpile ones, a sandpile collapsing grain by grain, grains sliding against each other.



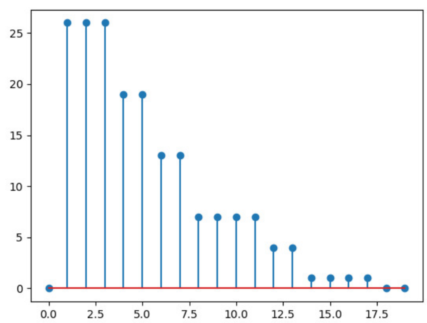
Let us illustrate on the diagram below countings that will be done after by a computer, with Goldbach decompositions of even numbers between 6 and 20. Here are Goldbach decompositions of those numbers :

$$\begin{array}{l}
6 = 3 + 3 \\
8 = 3 + 5 \\
10 = 3 + 7 = 5 + 5 \\
12 = 5 + 7 \\
14 = 3 + 11 = 7 + 7 \\
16 = 3 + 13 = 5 + 11 \\
18 = 5 + 13 = 7 + 11 \\
20 = 3 + 17 = 7 + 13
\end{array}$$

Let us stack them on each other :



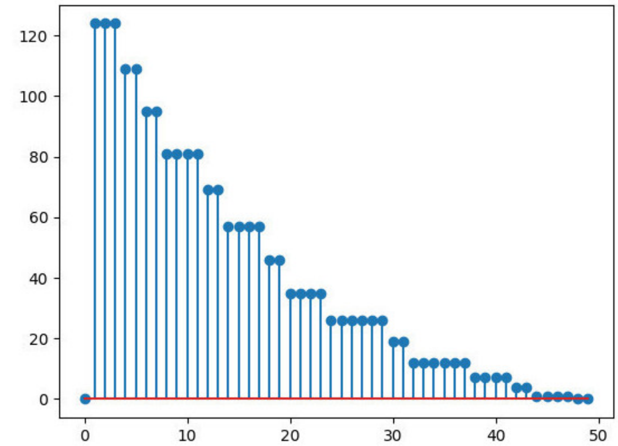
Let us make decompositions'quanta go down as low as possible to count them, one obtains by a program the following diagram :

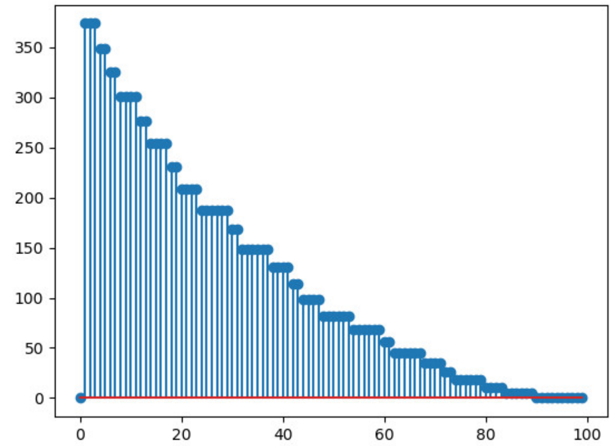


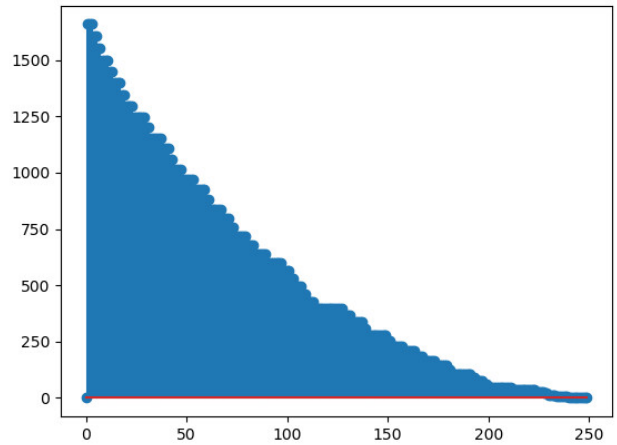
Let us note function values of this diagram in an array and let us explicitate what those values represent (dg acronym means *Goldbach decomponent*) :

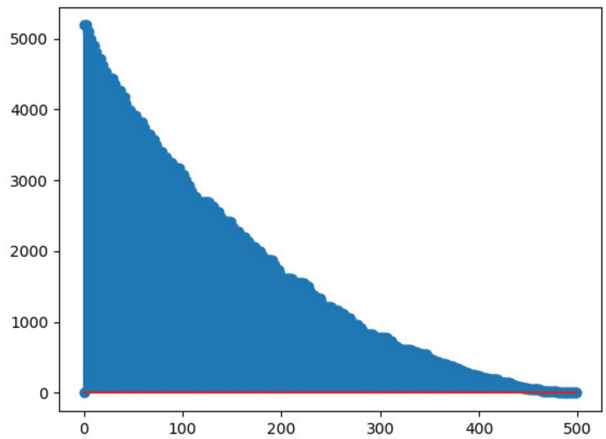
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
26	26	26	19	19	13	13	7	7	7	7	4	4	1	1	1	1
nb. dg ≥ 3			*nb. dg* ≥ 5		*nb. dg* ≥ 7		*nb. dg* ≥ 11				*nb. dg* ≥ 13		*nb. dg* ≥ 17			

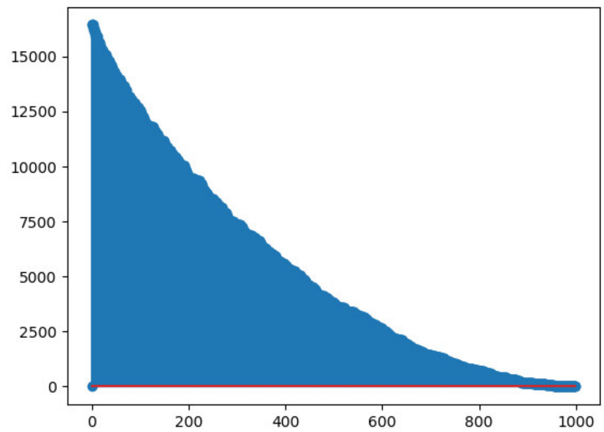
The total amount 26 counts sommants in Goldbach decompositions that are greater than or equal to 3 (we note it only for $f(2)$ but it's equal for $f(1)$ and $f(3)$), the total amount 19 counts sommants in Goldbach decompositions that are greater than or equal to 5 (resp. 13 for the number of sommants ≥ 7, 7 for the number of sommants ≥ 11, 4 for the number of sommants ≥ 13, 1 for the number of sommants ≥ 17).
Our function counts, in all Goldbach decompositions of numbers between 6 and 20, for a number x between two consecutive primes p_k and p_{k+1}, the number of sommants in decompositions that are greater than or equal to p_{k+1}. For instance, 4 sommants are greater than or equal to 13 (great sommants of 16'decompositions, 18'decompositions et 20'decompositions) and thus the function assigns 4 as their image for numbers 12 and 13.
We provide below the general trend that seems to emerge for this function, even if we didn't have the possibility to test it very far (curves until 50, 100, 250, 500, 1000 and 10000).

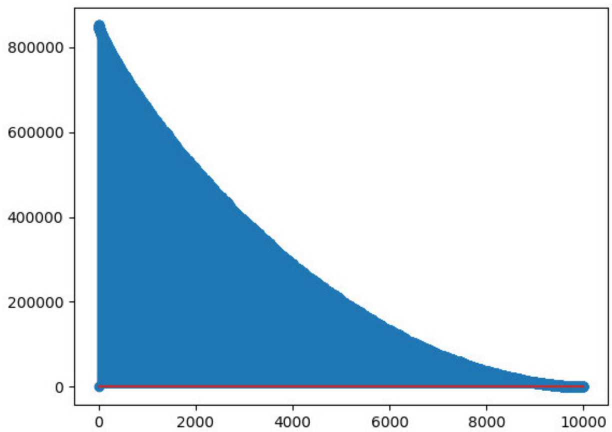












We approximate correctly the first curves $f(x)$ by formulas like :

$$(x - maxx) * (x - maxx)/(2.58498 * sqrt(maxx))$$

for small values of $maxx$ (50, 100, 250) (with 2.58498 Sierpinski'constant [*]) but it doesn't go anymore from

[*]. see wikipedia : Sierpinski' constant is the constant K defined by $K = \lim_{n\to\infty}\left[\sum_{k=1}^{n}\frac{r_2(k)}{k} - \pi\ln n\right]$ where $r_2(k)$ is the number of k's representations as a sum of two squares a^2+b^2 with a and b two natural integers.
Its value is :

$$K = \pi\left(2\ln 2 + 3\ln\pi + 2\gamma - 4\ln\Gamma\left(\tfrac{1}{4}\right)\right) \approx 2.58498,$$

where γ designates Euler-Mascheroni constant and Γ Gamma function.

the diagram corresponding to $maxx = 500$ (look at ordinates for small values, on the left of diagrams).