

Bourbaki : 1945-75
Dialogue organised
by the Hugot Foundation
from Collège de France,
between Jean-Pierre Serre, Pierre Cartier,
Jacques Dixmier and Alain Connes
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ALAIN CONNES : So we are here to talk about Bourbaki, but of Bourbaki in the period known as the golden age of Bourbaki and, in discussing with Jacques, we set this golden age, say over 30 years, between 1945 and 1975. I don't want to have precise limits, of course. So my role will be simply the role of moderator in the discussion and I will insist above all of course on personal testimonies and not on the generalities that one can find in literature, etc. What really interests me is having personal testimonials and I'll just start saying that what forces admiration, when we look at this period, it is above all, finally first, it is the erasure of ego in this task...

JEAN-PIERRE SERRE : The erasure of... ?

ALAIN CONNES : Ego, individuality.

JEAN-PIERRE SERRE : Ah !

ALAIN CONNES : The erasure of ego, that is to say the fact that from the start, it was agreed that none of the Bourbaki members would attach his name to what was produced and in fact, I think it's not at all foreign, this fact that there is this erasure of ego, with what it has generated, well sure, we will talk about it, at the mathematical level, the extraordinary successes that it engendered, but also, it engendered a spirit of brotherhood, I would say, in the math community, and this spirit of brotherhood is visible, not only of course inside Bourbaki but in fact I think it overflowed with Bourbaki per se and of course it was due... you can't say that Bourbaki took power, but we can say that there was such a panoply of talents in Bourbaki that in fact

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I mean they were the model of a whole generation of mathematicians, not only French, but also abroad since there were foreigners who participated in Bourbaki. Therefore in fact this is my starting point and the way I want to use to get you involved is to go around the table on fairly specific, but, as I said, I want subjects that don't have their answer already in the literature, or in what is found on the internet, etc., but who will have completely personalized answers. So, simply to see if the round works and then to heat up, the first subject, this is a fairly simple subject, it's the next question, "Did you know Bourbaki before being recruited? How have you been recruited? And what are your memories of the first Bourbaki congress?". That's it. So I think... I don't know in what order we can proceed (*pointing his hand to Jean-Pierre Serre*).

JEAN-PIERRE SERRE : I think we could answer in order in which we joined Bourbaki, you know? Now there, I was taken first.

ALAIN CONNES : Were you taken first? Okay.

JEAN-PIERRE SERRE : In 49. So there, I can answer all these questions there... because... the way I knew Bourbaki : then, well, sure, I had looked at Bourbaki's books ; there weren't many. There was the General Topology, chapters 1 and 2, I believe, and there was Algebra 1, I believe. Algebra 2 hadn't even come out.

ALAIN CONNES : There were the foundations too? (*laughs at JD*)

JEAN-PIERRE SERRE : No, no, no, there was none of that.

JACQUES DIXMIER : There was the Bundle Results booklet.

JEAN-PIERRE SERRE : There was the Results booklet. That was very useful by the way. And then, personally, what happened was that I passed the aggregation and there was an analysis test, well, that I had more or less understood, not much, suddenly, and then I go back to Ecole Normale Supérieure and there, Bourbaki had a congress which was in a room and I returned by any chance in this room...

ALAIN CONNES : Ah, okay, you came into this room by chance, okay.

JEAN-PIERRE SERRE : And then I saw that they were discussing issues that were very close, you see, to those who were in my problem of approval. And suddenly, it intrigued me. And I learned like that by accident that the next Bourbaki congress three months later, this was in June or July 48, so in October or November, there would be a Bourbaki meeting, in Nancy.

ALAIN CONNES : In Nancy.

JEAN-PIERRE SERRE : Rue de la Craffe. So at the time in question, I was with my wife in Auxerre, because she had been professor in Auxerre, and I didn't ask anyone, I took a train ticket and left for Nancy...

ALAIN CONNES : Of course (*laughs*).

JEAN-PIERRE SERRE : Oh, well for me, it was normal because maths are open, you see, I had no idea of politeness, that it was necessary to ask, no, no, I went to Nancy, and there, there was an amphitheater and I sat in the amphitheater ; they were discussing body theory. I remember very well, there was a proposal from Chevalley a little bit extraordinary which was to suppress the Galois theory (*laughs*), which he transformed in a choice "or else you put it, but in this case, you also put Lie algebras..."

ALAIN CONNES : First appearance of Lie algebras by Chevalley.

JEAN-PIERRE SERRE : Suddenly, they had the good sense to say yes. And I think I intervened, because I could speak you see, and I believe that I intervened to say something stupid that I will remember until the end of my life which was that the Galois theory was useless, you see (*laughs of all*). And that struck me, because for the rest of my life, I would always use it. There, the good Lord did not miss me. So they were happy with the comments I made about topological vector spaces, stuff easy, like that, they invited me this time for the next congress. So it must have been in January, or in February-March.

PIERRE CARTIER : It must have been in 49.

JEAN-PIERRE SERRE : In 50, uh, in 49. At the next congress, they told me "We take you". There, that's how I got home.

ALAIN CONNES : And who was there at the time? So there was Cartan, Dieudonné, Chevalley...?

JEAN-PIERRE SERRE : No, but I forced the Bourbaki door in some way so.

ALAIN CONNES : Oh, very well, you opened it, and it remained open! (*addressing Jean-Pierre Serre*)

JACQUES DIXMIER : So let's see, so, then, I think it was you we took? What year did they take you?

JACQUES DIXMIER : Wait, before you, there was Godement and Schwartz, who were recruited just before.

JEAN-PIERRE SERRE : Yes, but (*designating AC*) he said among us three.

JACQUES DIXMIER : Ah, among the three of us. So yes, after, it's me, yes.

JEAN-PIERRE SERRE : What year is it, you?

JACQUES DIXMIER : In 49. You were even... you were in charge, Samuel and you, to recruit me.

JEAN-PIERRE SERRE : To find out if you accepted.

JACQUES DIXMIER : And you had no effort to do because, I was (*laughs*)... I must... Let's face it, I was flattered, and then interested.

JEAN-PIERRE SERRE : Yes, but it changed your ideas, anyway also a little bit.

JACQUES DIXMIER : How did it change my ideas?...

JEAN-PIERRE SERRE : Compared to Hilbert spaces.

JACQUES DIXMIER : And then, oh yes, it greatly changed my ideas, yes, of course! And even immediately, I realized that it interested me more than my

own work, there, then, immediately.

ALAIN CONNES : And the first congress then ? Go ahead telling me...

JACQUES DIXMIER : The first congress, then, in 49, but where it was exactly, I don't remember. Wait, I come back to something else, we are a little free ? On your introduction, because there is a point on which I do not agree, finally, I want to dissociate myself, it's because you call this period the golden age of Bourbaki. So maybe it is, I don't know, because precisely I completely lost contact with Bourbaki, it bothers me to say "this is the golden age of Bourbaki", when I don't know anything about what's going on or almost nothing that has happened since. You understand that it is to take a position of superiority which is a little unpleasant anyway. For me anyway.

PIERRE CARTIER : I think that's the general opinion.

JEAN-PIERRE SERRE : (*designating Alain Connes*) Let's say it's his opinion, because for example,

JACQUES DIXMIER : (*addressing Alain Connes*) Okay, that's your opinion, yours, okay.

ALAIN CONNES : Not only that, it is what is written in the books that talk about Bourbaki, that's all. We're not going to drag on this.

JEAN-PIERRE SERRE : It will not be written, anyway (*addressing Alain Connes*), you said it anyway, we can't delete it.

ALAIN CONNES : I said, I take responsibility for it.

JEAN-PIERRE SERRE : But actually, I was a little annoyed when Cartier made speeches, and in writing, him, on the death of Bourbaki, that it was over because he was no longer in Bourbaki, suddenly, Bourbaki was dead.

ALAIN CONNES : Well, Bourbaki's death has been announced, like you know, in 1968.

ALL TOGETHER : It was a hoax.

JEAN-PIERRE SERRE : It was a stupid hoax.

JACQUES DIXMIER : Oh, stupid, he was no more silly than a lot of other hoaxes.

JEAN-PIERRE SERRE : No, but that is saying a lot, already (*laughs*).

JEAN-PIERRE SERRE : Okay so this is your entry into Bourbaki. (*Turning himself towards Pierre Cartier*) And you, Cartier, it's a little more complicated, because you sort of came in twice?

PIERRE CARTIER : Yes, yes, well then, how did I know Bourbaki? Well, obviously, Bourbaki's books, I knew them. I discovered them when I was a schoolboy around 16 because my grandfather who was a teacher at Henri IV wanted to give me a gift for... I don't know for what occasion, he took me to Hermann, he left me there, all afternoon in the shop, and when he came back, he took out the check he had written in advance and the bookseller had calculated exactly, he had told him in advance... Good, and I remember, I think I chose the *General topology* of Bourbaki among other books there were also the books of Lichnerowicz. And my grandfather told me "You know who Bourbaki is?", "Ah no, not at all", "You know who is Simone Weil?", I said "You talk a lot about her!". Because they had been normaliens together. "She has a brother, André Weil, who founded a group which is called Bourbaki and maybe one day you will know them.". Then I rushed in, spent my whole summer trying to read it. The difficulty, I did not have the booklet of set theory summaries. And so all these notations, I did not know them. And so I had a hard time to reconstruct them.

ALAIN CONNES : And what were the topology chapters, 1 and 2?

PIERRE CARTIER : 1 and 2, 1 and 2.

ALAIN CONNES : 1 and 2 only, there were no real numbers.

JEAN-PIERRE SERRE : But there were quite a few things anyway.

JEAN-PIERRE SERRE : There were compacts, for example.

ALAIN CONNES : There were uniform structures.

PIERRE CARTIER : There were things, but the terminology of the sets was not re-explained. This is what caused me difficulty. So good, therefore, this is how I heard about it. And then, how it happened. Well, my first year at school, there was a seminar Cartan where Eilenberg was, Samuel Eilenberg was there.

JEAN-PIERRE SERRE : What year was that ?

PIERRE CARTIER : It's 50. I came back in 50. There is a seminar 50-51, one of the major seminars Cartan, Eilenberg had spoken of the cohomology, then there were the beams, finally there were a lot of things. And Eilenberg had done a parallel course on it which I attended well and I admit that I was very good, we got along very well Eilenberg and I and I think he was the one who said to Cartan "You should invite him, we should invite him", so I was a guinea pig in June 51. And I arrived, so Cartan had a little reserve because I had to pass I don't know which licensing exam and I dried it off promising Cartan that I would pass it right in September and I would have it (*laughs*). And I got it, finally good. Then I arrived and it was at Pelvoux-le-Pouët, historic site, I arrived there, and I saw everyone. There was Cartan of course, there was Schwartz, I remember Schwartz very well, I remember Dieudonné, I remember Godement, well, of all this, it was all very, very hectic. This place, Pelvoux-le-Pouët, we went there a number of times, there was a nice little inn and legend wants, I can't guarantee it anymore, legend has it that one day Weil and Dieudonné got so bad at each other that the owner of the restaurant, was afraid and that she told her son "Go get the gendarmes." and someone reassured her "No, but no, nothing will happen, rest assured, they're not drunk.". So that, it was my first congress, and I remember we discussed the Lie groups, well, there was a Schwartz report on Lie groups.

ALAIN CONNES : Of Schwartz, not Chevalley then ?

JEAN-PIERRE SERRE : There was also Weil afterwards but finally, Weil, it was not Lie groups, it was the varieties, it was not Lie groups, if it's Schwartz.

PIERRE CARTIER : Schwartz had made Lie groups and I remember that

there was the abbreviation Pass.Ad. I no longer know why. (*Addressing himself to JPS*) Pass. and Ad., I don't know what that meant anymore, it had made everyone laugh, obviously, passing. This is where I learned by example, there was one thing that inspired me a lot, it was a theorem of Schwartz that the enveloping algebra of the Lie algebra of a group was the algebra of distributions supported by the origin for the convolution. And that, it inspired me enormously, so that I started my thesis on this, on these ideas. Okay what happened. We did an expedition to the white glacier.

JACQUES DIXMIER : Not me.

PIERRE CARTIER : But there are still photos lying around a bit everything on this expedition, we are all photographed, and then Weil had persuaded us, or the guide had persuaded us that we had to wear a alpine hunter beret. You couldn't do a mountain expedition without putting on an alpine hunter beret. And Weil was very proud of his alpine hunter beret and (*speaking to JPS*) it seems to me that you had made an unscrewing elsewhere there?!...

JEAN-PIERRE SERRE : And that I didn't have a beret either ! But by against, I went down a slope, they were a little bit worried, in particular Cartan, to see me tumble. And then I stopped at the bottom of the slope so there.

PIERRE CARTIER : You still unscrewed 50 or 100 meters, really ?

JEAN-PIERRE SERRE : Yes, but it was a slope that was not vertical, it was tilted enough for it to continue, you see (*imprint mines of AC and JD*) yes but it ended with the glacier you see, it was not...

PIERRE CARTIER : And then, I also remember that this is where I did the knowledge of Chevalley for example. And Chevalley, he was there with his daughter, because he had just remarried and he was there with his daughter Catherine who was 2 years old. All these are my memories of this first congress.

JEAN-PIERRE SERRE : Is this the second or is it the first congress ? Because you weren't caught at that time ?

PIERRE CARTIER : I was taken several years later, in 54.

ALAIN CONNES : Ah, you were taken several years later, okay.

JACQUES DIXMIER : (*speaking to PC*) Unless I'm mistaken, but I don't trust too much in my memory, I was in charge of recruiting you, as well as Bruhat. But does that fit with your own memories ?

PIERRE CARTIER : Yes, yes, we were recruited, Bruhat and I, roughly at the same time, I know.

JACQUES DIXMIER : Ah no, no, no, we weren't recruited at the same moment. Me, it was in 49 and you say in 54.

ALAIN CONNES : No, no, he talks about Bruhat and him.

JACQUES DIXMIER : Yes, Bruhat and you, finally, in my memory, Bourbaki said to me "recruit them", finally, "tell them we are recruiting them", it was not me who recruited, it was Bourbaki of course.

PIERRE CARTIER : Do you remember the recruitment of Koszul ?

JEAN-PIERRE SERRE : Yes, I was going to talk about it too because it had been pretty funny.

JACQUES DIXMIER : No, but then, we come out of the three of us there.

ALAIN CONNES : No, no but of course, go ahead, go ahead.

JEAN-PIERRE SERRE : So Koszul, we decided to take it, and we had instructed Cartan to "contact" him. And then at a next congress, we have asked Cartan "So, did you contact Koszul?" "Yes, well no, it's nothing said, finally...". At the end of 6 months, (*addressing Jacques Dixmier*) finally, you probably remember.

JACQUES DIXMIER : The dates, the durations, no, I don't remember the durations, no, morals, I remember.

JEAN-PIERRE SERRE : After a year, finally, after a certain time anyway we

said to ourselves “Someone else...” (*to JD*) you, maybe?

JACQUES DIXMIER : No, it’s not me.

JEAN-PIERRE SERRE : It’s not me. I don’t know who among us said “Go and ask Koszul”. We asked Koszul who said instantly yes and we asked him “But didn’t Cartan ask you?” “No, I had not understood...”. Cartan had been so discreet, you see, that Koszul hadn’t understood that he was invited (*laughs*).

PIERRE CARTIER : That was the thing.

JACQUES DIXMIER : We must say that Cartan was the thesis boss of Koszul, maybe it made relationships a little bit more formal.

PIERRE CARTIER : Cartan was the thesis boss of practically everyone, at least officially.

JEAN-PIERRE SERRE : He was not formal but he was...

JACQUES DIXMIER : He was not a good thesis director.

JEAN-PIERRE SERRE : He was very polite, however, he could...

ALAIN CONNES : Sure, he was very polite, so maybe effectively...

JEAN-PIERRE SERRE : Where is it, I think it’s in Proust, that there is like that, right at the beginning, there is indeed a story of the aunt, who asks...

ALAIN CONNES : But of course! No, but who wants to thank Swan for having given bottles of wine.

JEAN-PIERRE SERRE : To thank Swan, that’s it, they do it in such a way discreet...

ALAIN CONNES : They do it so discreetly that ultimately...

JEAN-PIERRE SERRE : ...that nobody realizes it.

ALAIN CONNES : Of course, of course, I wanted to talk about this passage from Proust, because it reminded me of that, exactly.

JEAN-PIERRE SERRE : Cartan behaved like that.

ALAIN CONNES : Okay, so I see that the round table is working well. So we are now going to tackle a slightly more delicate question.

JACQUES DIXMIER : Wait, if you're talking about recruiting then, we're talking recruitments or not? We three talked about our recruitments.

ALAIN CONNES : You can talk about recruitments.

JACQUES DIXMIER : We just talked about that of Koszul. There are others recruitments, for example Grothendieck, recruiting Grothendieck.

ALAIN CONNES : It was done in what year, the recruitment of Grothendieck?

JACQUES DIXMIER : It was in Nancy, Grothendieck went as young student.

JEAN-PIERRE SERRE : Ah no no, he was not at all in Bourbaki when he was in Nancy, he was taken clearly after you.

JACQUES DIXMIER : Finally it's linked anyway, listen, he went to Nancy immediately after graduating from college, he...

JEAN-PIERRE SERRE : No, he was a student of Bourbaki members but this is not the same.

ALAIN CONNES : Of Schwartz and Dieudonné.

JACQUES DIXMIER : He was a pupil of Schwartz and Dieudonné, that's it, but Schwartz and Dieudonné said right away "he's great" so... (*Jean-Pierre Serre, Alain Connes, together*) Ah, but it is not the same!

JEAN-PIERRE SERRE : (*continuing*) It's completely different : we haven't thought of taking him as a member of the Bourbaki group right away, that,

I'm almost sure.

JACQUES DIXMIER : So there, my memories are not precise enough, I can't say anything, it surprises me a little but well, well... I would say that he was recruited in 55, there.

JEAN-PIERRE SERRE : Yes, me too.

JACQUES DIXMIER : (*designating Pierre Cartier*) That is to say right after you, at least, your official recruitment.

PIERRE CARTIER : Around the same time, around the same time approximately.

JEAN-PIERRE SERRE : No but we are not going to review all the members of Bourbaki.

JACQUES DIXMIER : Well, I don't know...

PIERRE CARTIER : No, there are archives that allow us to know.

JEAN-PIERRE SERRE : We will come back, let's come back to us, rather.

JACQUES DIXMIER : Recruitment of members after us is part of our personal memories.

JEAN-PIERRE SERRE : There were a lot of weird ones, you see, by example, Atiyah was invited once, and he did not return.

ALAIN CONNES : Mac Lane came once too.

JEAN-PIERRE SERRE : So he wasn't even member, it's not the same, he has come as a visitor you see, it's not the same.

JACQUES DIXMIER : No, but the recruitment of Borel, for example, there, you must have memories. And it's interesting because it must be one of the first non-normaliens that we recruited.

ALAIN CONNES : Foreigner you mean.

PIERRE CARTIER : And one of the first foreigners as well.

ALAIN CONNES : And Eilenberg, right ? Eilenberg had been recruited before.

PIERRE CARTIER : Eilenberg it was in 50.

JEAN-PIERRE SERRE : I don't know when it was, Eilenberg, it was in 50 ?

PIERRE CARTIER : Yes I'm sure, it was in 50.

ALAIN CONNES : So, Eilenberg was the first foreigner to be recruited.

PIERRE CARTIER : It was when he came to Paris, the year he spent in Paris.

ALAIN CONNES : And Tate then, when was he recruited ?

JEAN-PIERRE SERRE : It is clearly later, but indeed, he came several times. It was not at all illusory, like Atiyah, you see, no.

ALAIN CONNES : Atiyah came once, then ?

JEAN-PIERRE SERRE : Once, and then he didn't like it, it didn't match...

PIERRE CARTIER : In his way of doing things, that's for sure.

ALAIN CONNES : So I'm going to address another question, the following question boasts. This is a question that has to do with Bourbaki's math, but for being much more specific than that, the question I would like to ask you is "how did you reconcile your own research work with the work you had to do for Bourbaki, the editorial offices, etc., and were there relationships in one way or the other, that is, did the work for Bourbaki influence you for your own work and vice versa. That, for me, it is a crucial question.

JEAN-PIERRE SERRE : You would have to choose an order, because...

ALAIN CONNES : Well, we can respect the same order as earlier since it allows...

JEAN-PIERRE SERRE : I'll start then.

ALAIN CONNES : Yes, you start.

JEAN-PIERRE SERRE : It's hard to say. I always had a great pleasure to work for Bourbaki.

ALAIN CONNES : Oh good?

JEAN-PIERRE SERRE : For me, it was a great pleasure.

PIERRE CARTIER : A great... pleasure?

JEAN-PIERRE SERRE : Pleasure because at the same time, I was trying to write it as good as possible but at the same time I knew someone else would resume it afterwards, and correct it.

ALAIN CONNES : So you were quiet, yes, okay, I understand.

JEAN-PIERRE SERRE : So I could let myself go, you see.

ALAIN CONNES : You had security, you mean.

JEAN-PIERRE SERRE : And then Bourbaki's style obviously suits me perfectly. To write my own articles, if possible, I followed the Bourbaki style, I pretended to be writing for Bourbaki, explaining just a little bit.

ALAIN CONNES : But which way did it go, I mean, when you writego articles after having, since you started very early with Bourbaki, so in fact, you followed this style as soon as you started writing.

JEAN-PIERRE SERRE : It was my spontaneous style in fact, curiously. As soon as I started, I naturally wrote in this style.

ALAIN CONNES : Okay, and you were typing, how were you doing ?

JEAN-PIERRE SERRE : I was typing, yes, I had a typewriter, and then at the time, we typed very easily because when it didn't go well, we typed Xs on what we wanted to erase, we made letters by hand if necessary, it was amazing. So, from the influence point of view...

ALAIN CONNES : Yes, the influence point of view, in both directions, that's it, it's what interests me.

JEAN-PIERRE SERRE : Oh, hummf, I may have influenced Bourbaki a little, it's difficult, a determined person, apart from Weil, did not influence really Bourbaki, anyway.

PIERRE CARTIER : Except for commutative algebra, anyway.

JACQUES DIXMIER : He's the one speaking, but we can correct it afterwards.

ALAIN CONNES : You will correct after, of course. No because...

JEAN-PIERRE SERRE : No, I don't think I pushed, well, then commutative algebra, I pushed a little bit, I helped. I had the feeling to help, and above all, I had this feeling on Bourbaki, that it is a work, something of public safety, of public space.

ALAIN CONNES : That's what I was trying to say at the start.

JEAN-PIERRE SERRE : That it's done to be of service. People don't have absolutely understood that, the people who comment on Bourbaki... And also, that I understood well, because in the discussion, it was clear, that what is done in Bourbaki, it was not maths specially interesting, it was useful math to do interesting things.

ALAIN CONNES : To do interesting math, of course.

JEAN-PIERRE SERRE : And that is different. I mean we had to take, a bit as if in a kitchen, where you have very pure salt, where you have this and that's not cooking. Cooking is mixing and making things...

ALAIN CONNES : It's the tools that are perfect, and then after, we use them.

JEAN-PIERRE SERRE : To clean, to prepare the tools well, so that after, we can use it. And that, unfortunately, the people who commented Bourbaki did not understand anything. They saw it as a business for seeing, for example, in the style of Bourdieu's ideas, what is good... (*evasive gesture*)

ALAIN CONNES : And the other way around? Did what you were doing for Bourbaki influence you for your own research?

JEAN-PIERRE SERRE : The style necessarily influenced me, you see. But I didn't have the idea to work on Bourbaki subjects, it doesn't make sense, but to use it.

ALAIN CONNES : Ah, anyway, anyway, let me ask you a more specific question, for example, when the Lie groups were treated, in Bourbaki, etc., could that have influenced you to make things about Lie algebras or Lie groups...?

JEAN-PIERRE SERRE : No, it's the opposite, I needed it for what I was doing, the l -adic, stuff like that, and instead it is rather than it gave me more enthusiasm, in Bourbaki, to insist so that the Lie networks are not on \mathbb{R} or \mathbb{C} .

ALAIN CONNES : Okay, so I have another more specific question, this is, when we see for example, the proof of Dwork on... we see well that you have to use Banach p -adic spaces, etc., is that you who insisted that the EVT* also be done in the p -adic setting.

JEAN-PIERRE SERRE : No, that was already done I believe. Besides, they have not been so well done from the p -adic point of view, it's not usable...

ALAIN CONNES : No, it's too much superficial.

JEAN-PIERRE SERRE : No, no, it was already... Dieudonné, of himself, said "a body with a standard" and then here's what.

*. Topological vectorial spaces

ALAIN CONNES : And then we do everything on that, okay.

JEAN-PIERRE SERRE : We do everything but we do nothing interesting. The interesting things is Hahn-Banach, and all.

ALAIN CONNES : It's on \mathbb{R} too.

JEAN-PIERRE SERRE : Except for Banach's theorem, it is also true for p -adic, on closed graphs.

ALAIN CONNES : Still, yes, there was something.

JACQUES DIXMIER : On spaces of finite dimension, locally compact.

ALAIN CONNES : So Jacques, go ahead.

JACQUES DIXMIER : On the one hand, Bourbaki influenced me enormously in my personal works.

ALAIN CONNES : So that is very important.

JACQUES DIXMIER : And even 3/4 of what I did was influenced by Bourbaki. If I had not been in Bourbaki, I would have worked on the Hilbert spaces for all my life.

JEAN-PIERRE SERRE : Maybe but maybe not anyway.

JACQUES DIXMIER : I still did other things. And it's thanks to Bourbaki.

ALAIN CONNES : It is thanks to Bourbaki, ah yes ?! It's strong when-even.

PIERRE CARTIER : (*teasing, designating AC*) There are other people who take care to Hilbert spaces.

ALAIN CONNES : No, but not only that, no, no.

JEAN-PIERRE SERRE : You know we said that about Dieudonné too, that if he hadn't been in Bourbaki, he would have continued on polynomials in

one variable because that was his only subject.

JACQUES DIXMIER : I am surely not the only one, I am a sort of Dieudonné with the little foot, if you want : I wrote a lot. So there, my role in Bourbaki was not a role... (*pausing*). So, first, when even, I repeat what he says to him (*designating JPS*), when he says that he did not had an influence on Bourbaki, it's a huge misinterpretation.

JEAN-PIERRE SERRE : You make me happy but...

JACQUES DIXMIER : No, but wait, you have to distinguish two parts :from the moment if you want where the founding members left, they have started leaving with the law of 50 years in 54, Cartan was 50 years old he left. At that time, well, it was you and Borel who influenced Bourbaki and its branches.

JEAN-PIERRE SERRE : It is not as precise as that, all the same, it was true collective, anyway, but it helped.

JACQUES DIXMIER : If you compare the influence you had on Bourbaki and the influence I had on Bourbaki, I'm not going to put that on the same level. On the other hand, I rendered great service on Bourbaki because, like Dieudonné, I wrote a lot. So 10 times less than Dieudonné, because that Dieudonné, it is out of competition (*laughs*) but finally, I... Well then, again once, this point being settled, from my point of view, Bourbaki influenced me by teaching me a lot.

ALAIN CONNES : New things that you didn't know, okay.

JACQUES DIXMIER : Well, at the same time, I talked about my influence on Bourbaki, as editor.

ALAIN CONNES : Not only, all the same, you don't only...

JACQUES DIXMIER : No, I did not influence Bourbaki other than making work easier, writing a lot.

ALAIN CONNES : But wait, for a little more precise question, it is that, if you like, there were still, in Bourbaki, new contributions, I mean, Serre is

right to say that we cannot situate this at all on the same level since we were developing tools, but for example, before 45, there were filters and ultrafilters, which had nevertheless been developed by Bourbaki, and which did not exist before. So the question I'm asking is...

JEAN-PIERRE SERRE : But it's really not one of the most important things bearing, then that, nobody actually uses it, I think.

PIERRE CARTIER : It's not true.

ALAIN CONNES : Yes, ultrafilters...

PIERRE CARTIER AND ALAIN CONNES : In logic, they use them.

JEAN-PIERRE SERRE : (*accompanying his word with a gesture*) Oh yes good...

ALAIN CONNES : But whatever, but what I mean is in what sense, the question was a little bit more specific, in what sense was there a creativity of concepts, etc.

JACQUES DIXMIER : Yes, you already asked me the question outside of this meeting, we must distinguish.

ALAIN CONNES : It was not the goal, of course.

JACQUES DIXMIER : Yes, during the congresses themselves, I don't think it there are a lot of... For example, an example that came to my mind, associated ideals, to make the primary decomposition, it is a creation of Chevalley, he did it outside of a congress : he arrived, he wrote it on, it appeared in a Bourbaki editorial office.

ALAIN CONNES : Ah anyway, yes, so that means...

JACQUES DIXMIER : For the outside world, it's an invention of Bourbaki.

JEAN-PIERRE SERRE : Ah, do you think it didn't exist before?

JACQUES DIXMIER : It didn't exist before. In any case, Samuel and I have it asserted and Samuel knows the question perfectly, and he was also a little angry, Samuel, because when there was the review of this book Maths reviews, no hint that it was a new point of view but that doesn't matter.

JEAN-PIERRE SERRE : It doesn't matter.

JACQUES DIXMIER : The reviews are often confusing.

JEAN-PIERRE SERRE : No, that's not it, when there are new things in a text, this is precisely what the reviewer does not understand, you see, so he doesn't talk about it, that's for sure.

JACQUES DIXMIER : Ah yes, but that was the essence of the chapter, finally well, let's move on.

JEAN-PIERRE SERRE : No, it's not the main thing.

JACQUES DIXMIER : Another example, then. So there, it has happened during a convention, so I'll tell you about it, we were discussing chapter 5 of integration and we read, if I remember correctly, an essay by Dieudonné there are a lot of things in chapter 5 there were the measurement products, image measurements, induced measurements, density measurements, and suddenly, Schwartz had an idea that unified all of this : instead of that either 4 or 5 (*gestures with both hands as if it were turning a crank*), where we repeated every time, to demonstrate the theorems, he had an idea that united everything.

JEAN-PIERRE SERRE : And what was it, I forgot ?

PIERRE CARTIER : Families of measures.

JEAN-PIERRE SERRE : Families, right ? Pierre Samuel.

JACQUES DIXMIER : What Choquet called diffusions after...

JEAN-PIERRE SERRE : No, no, it was not called diffusions in Bourbaki.

PIERRE CARTIER : It corresponds to probabilities diffusion.

JACQUES DIXMIER : Anyway, you're taking a measure that is not a parameter, and this parameter is itself in a measured space, and you have to integrate.

JEAN-PIERRE SERRE : And that gives the different cases, that ?

JACQUES DIXMIER : It contains all the different cases. So I remember very well that it excited everyone, I think we can say it, me in particular and I was in charge of the drafting.

ALAIN CONNES : The writing according to you, you mean ?

JACQUES DIXMIER : The following wording, I wrote that, it took me more than six months. I was thrilled there, (*then laughing frankly*) when it was read in congress, it was seriously skimmed.

JEAN-PIERRE SERRE : We were used to that.

JACQUES DIXMIER : It happened in the exercises...

ALAIN CONNES : It's okay, it's a very important point, I think, because if you want...

JACQUES DIXMIER : But it's not new math.

ALAIN CONNES : Oh no but still, which is interesting I think, what we would actually like to know is when there were aloud common readings, like it happened all the time...

JACQUES DIXMIER : Well, yes, since that was the principle.

JEAN-PIERRE SERRE : Do you want to talk about that, maybe, about aloud readings aloud, because it's very important in Bourbaki.

ALAIN CONNES : Absolutely!! Okay, I'll put it off a little later, then, okay, this will be the next topic. But actually, this is a very important point. Fi-

nally, this is the only time when there was a common reflection that was happening. So Pierre now. So Pierre, the influence of your math on Bourbaki and of Bourbaki on your math.

PIERRE CARTIER : I can say that for almost 30 years, I have devoted 1/3 of my scientific activity to Bourbaki.

ALAIN CONNES : 1/3, so yes, okay.

PIERRE CARTIER : roughly 1/3, anyway, in volume : the editorial offices to do, the tribes eventually I did when it was no longer you (*at the address of JD*) who did them, the proof reading, well, well, I have really a lot, a lot... I would say about 1/3 of my activity. It was time consuming.

ALAIN CONNES : 1/3 of your activity is enormous.

JACQUES DIXMIER : And the trial?

ALAIN CONNES : No, no, we don't talk about it yet.

JEAN-PIERRE SERRE : Oh, my God, no.

JACQUES DIXMIER : Yes, but time consuming.

PIERRE CARTIER : No, the trial, I'm not talking about it. On the other hand, my own scientific interests did not always coincide with those of Bourbaki ; for example, I was very interested in probabilities, awhen Bourbaki was not very interested in it.

JEAN-PIERRE SERRE : Schwartz anyway, Schwartz was interested in it.

ALAIN CONNES : (*laughing*) But not Dieudonné.

JACQUES DIXMIER : I cut, Cartier is interested in everything, so necessarily he overflowed Bourbaki.

ALAIN CONNES : No, no, no, but there he has something very particular in mind, yes, yes, of course.

PIERRE CARTIER : Good, but it is true that gymnastics to write in an imposed style, because it was a very imposed style, it shapes you, that, it's clear that it trains you. And then on the other hand, if you want for my scientific interests, I said that I had worked a lot on probabilities, which was not in Bourbaki's sights, but I also worked a lot on Lie groups, on things like that, and then there, well I mean that it is certain, the writing of books on Lie groups, that was a crucial moment for me. And besides I told everything at the time that I have just finished a demonstration started there is 60 years on this subject.

JEAN-PIERRE SERRE : (*nodding very interested*) Ah?!

PIERRE CARTIER : to build the diagram on the integers associated with a semi-simple Lie group... Well, I have a new demonstration which took me 60 years to develop.

ALAIN CONNES : We were talking about Chevalley's article of 55.

JEAN-PIERRE SERRE : But it must be complete, this demonstration. Because there are lots of people (Kostant, etc.) who have made them but who are not, you too, you had done, and where there were not all the details that are needed, so it was useless.

PIERRE CARTIER : There will be all the details there, I guarantee it.

JEAN-PIERRE SERRE : There really has to be all the details, huh.

PIERRE CARTIER : No, but that I agree, I know the subject. Well. Anyway. Let's say I've become really excited about a lot of Bourbaki did. For example, we were talking about probabilities, the only place where we are getting close to probabilities, this is the last book on integration.

JACQUES DIXMIER : Last "chapter", sir.

PIERRE CARTIER : Ah yes, last chapter. Chapter 9... which was done under the impetus of Schwartz, and essentially written by Paul-André Meyer, who was not from Bourbaki, but who served as, say, an advisor.

JEAN-PIERRE SERRE : But he was not in Bourbaki, Paul-André Meyer ?

PIERRE CARTIER : No, no, no, no, no.

JACQUES DIXMIER : He came as a guinea pig, I think, once or twice. But he was not a member of Bourbaki.

JEAN-PIERRE SERRE : Ah, he was not a member.

PIERRE CARTIER : No, he was not a member. But he participated, indeed, to this writing of the book on probabilities.

JACQUES DIXMIER : This is a very special case, a non-member who wrote, finally, 50 % because you have written 50 % too.

PIERRE CARTIER : There was Schwartz, there was Meyer and me. And we succeeded to do it. Well, it was... let's say that for me it was... If there weren't had been this Bourbaki impulse, I probably wouldn't have been interested in what had been done by the Russian school at the time, Prokhorov, Kolmogorov and all that, and that, it served me enormously afterwards. So, I mean my mathematical interests...

JEAN-PIERRE SERRE : They overflowed a lot, they overflowed a lot of Bourbaki.

PIERRE CARTIER : They overflowed a lot of Bourbaki : I got involved in mathematical physics and that was not a subject that interested a lot Bourbaki.

ALAIN CONNES : No, for sure.

JACQUES DIXMIER : It interested Bourbaki a lot but we had other things to do, that's it !

JEAN-PIERRE SERRE : But that's not math, anyway.

PIERRE CARTIER : No.

JEAN-PIERRE SERRE : We couldn't write it with corollaries and demonstration.

JACQUES DIXMIER : Finally, there is a continuous spectrum, from one to the other.

JEAN-PIERRE SERRE : Aaahh, (*making the gesture of a loophole in the mountain*) there is a discreet rupture (*PC laughs while showing AC, which makes as if he said "Too bad"*), there is a rupture, no, no, no.

PIERRE CARTIER : Let's say that it was not a motivation to make math is different.

JEAN-PIERRE SERRE : There, that's different. It can even inspire.

ALAIN CONNES : But besides, it touches on a question which is however very important is "how was it designed and how has the architecture of Bourbaki's books changed?"

JEAN-PIERRE SERRE : When I arrived, it was already launched if you want, they started this very early you see, the general plan...

PIERRE CARTIER : ... dates from 1935.

JEAN-PIERRE SERRE : Dieudonné had it in his pocket for a long time, they must have done that in the thirties you see, roughly.

ALAIN CONNES : You mean, he had put EVT before measure theory (*laughs*)?! before integration?!

PIERRE CARTIER : No, that's a detail.

ALAIN CONNES : No, I'm kidding, we'll talk about it again.

JEAN-PIERRE SERRE : Lie's book was not originally planned. It happens kept on topological vector spaces. Integration, that was inside, I believe it ended there (*turning to PC*).

PIERRE CARTIER : There were two parts : first part, the first 6 books, and the second part... There was a big debate.

JEAN-PIERRE SERRE : Second part, it was a bit of a myth. It was there... Second part, we'll take care of it afterwards", that was it, you know.

PIERRE CARTIER : No, but I mean that what was however planned, it's a little more than that, it was that the first 6 books were the essential foundation for all that followed, and that they followed it in very precise order.

JEAN-PIERRE SERRE : They followed it in a specific order, while that afterwards, we could mix the...

PIERRE CARTIER : After that diverged (*indicated with a gesture of separation of branches, up*)

ALAIN CONNES : But wait, what I don't understand, I'm a little taken aback by something that comes to my mind, beam theory, where in these 6...

JEAN-PIERRE SERRE : It is not there, it is not in Bourbaki.

ALAIN CONNES : Because I found it in...

JEAN-PIERRE SERRE : (*interrupting*) Well, no, it's not worth it, since there was a Bourbaki member who wrote it, we weren't going to copy, there was that you see too.

ALAIN CONNES : But then the question that we can ask is in what sense precisely, Godement was influenced or influenced Bourbaki, precisely, for something like that, because we were talking about it with Pierre, precisely.

PIERRE CARTIER : Godement, Godement, hey, he wrote "the" book on bundles at a time.

JEAN-PIERRE SERRE : And it was he who wrote the integration, on the measures of Radon, but that was on Bourbaki's order, it was not only his taste.

ALAIN CONNES : It was a request from Bourbaki, okay.

JEAN-PIERRE SERRE : There, I attended this session.

PIERRE CARTIER : The beams, the beams, that was part of the current discussions, obviously of course, we could not ignore the beams, we were all...

JEAN-PIERRE SERRE : We didn't want to talk about it in Bourbaki.

PIERRE CARTIER : No, no but beware, we have all been trained by Cartan, and the beams, it was, it was...

ALAIN CONNES : But what was the reason why you didn't want not to talk about it, precisely then?

JEAN-PIERRE SERRE : We should have made a book on algebraic topology, it was not worth it, we were building it, you cannot do a Bourbaki on something that is being done you see, it's not reasonable, that.

PIERRE CARTIER : It was the way of seeing.

JEAN-PIERRE SERRE : No, you do a Bourbaki on things that are already quite well established, that you rearrange.

ALAIN CONNES : Okay, but the general beam theory, however, it was perfectly well established, I mean...

JEAN-PIERRE SERRE : Well, listen, look, experience has shown that it had to take coatings other than stupid coatings, it was necessary to take some...

ALAIN CONNES : Yes you mean to have Scholze's theory with pro-etale coatings and all that, yes, of course.

PIERRE CARTIER : Rather Grothendieck, the etale topology out.

JEAN-PIERRE SERRE : And then, we are not going to redo what is well done in Literature.

JACQUES DIXMIER : For architecture, there, I have a very clear opinion, it is that, by the way you said, it was settled when we entered, it was done between 35 and 40, by the founding members, it was not done at all in 35 nor in 36, since they wanted to write a treatise on analysis, but in the 2 or 3 first years, they understood, finally, they imagined something else, and besides, it resulted in 4 or 5 participants of the first congresses got the hell out, because that was not what they wanted to do. Finally, I don't know (*contacting JPS to confirm it*).

JEAN-PIERRE SERRE : Yes, yes, yes, Dieudonné had this plan. So there was a sequel, a second part, as Cartier said, but it was completely vague.

JACQUES DIXMIER : It was clarified a little... First, I specify, Dieudonné, when he talked about that, it was implied “with the approval of Cartan, Weil and Chevalley”, Dieudonné, he would not have allowed himself to... Good. Second, they still structured this second part a little tie since it had become “Algebraic structures, functional structures, geometric structures”.

JEAN-PIERRE SERRE : No, they didn't do that, no.

JACQUES DIXMIER : Well, I heard that once. The second part was divided into 3.

JEAN-PIERRE SERRE : No, no.

JACQUES DIXMIER : Ah well, I can tell you that it was.

JEAN-PIERRE SERRE : No, I assure you that it did not continue, no.

JACQUES DIXMIER : It was not done, no, it was not done at all, it was in their minds.

JEAN-PIERRE SERRE : In the second part, what was there : there was Spectral theory...

PIERRE CARTIER : Commutative algebra... Spectral theory, Commutative algebra and Lie Groups.

JEAN-PIERRE SERRE : And the theory of varieties of which only the fascicule remained... Thanks to Dixmier, by the way.

ALAIN CONNES : What is terrible, when you look from a far, is that there has this book which is very very good on the theory of functions of a real variable.

JEAN-PIERRE SERRE : Oh yes, it is magnificent, no one knows him. The big O's are splendidly defined.

ALAIN CONNES : But on the other hand there is not a book of the functions of a complex variable so anyway, that's...

PIERRE CARTIER : Yes, but that is historic, if you will.

ALAIN CONNES : It's sad, that...

JEAN-PIERRE SERRE : But that, they could do it now, you see.

PIERRE CARTIER : At the time, at the Sorbonne, we only talked about that. The infinitesimal calculus courses in my generation, or a little before, the courses, there were only the functions of a complex variable.

JEAN-PIERRE SERRE : Yes, but there were no useful things for the number theory. There weren't the big O's, the estimates, the things like that.

PIERRE CARTIER : Yes, but then I mean that there was a weariness before these functions of a complex variable.

JEAN-PIERRE SERRE : Yes, that's it, they had seen enough, anyway effectively.

JACQUES DIXMIER : But I wonder if the variety book was not part, in the spirit of the founders, but then there, I dare not advance...But in my opinion, that was part of the fundamental structures, given the importance they brought to the work of Elie Cartan...

JEAN-PIERRE SERRE : No, I don't think so.

JACQUES DIXMIER : Ah, I would like them to be there to ask them the question.

JEAN-PIERRE SERRE : But in any case, we can thank you because it is thanks to you that the results booklet exists.

ALAIN CONNES : For varieties you mean.

JACQUES DIXMIER : Ah no, then there.

JEAN-PIERRE SERRE : And I'll tell you why...

JACQUES DIXMIER : Go ahead (*laughter from AC*).

JEAN-PIERRE SERRE : We had the book on Lie groups, which we wanted start. And so in particular chapter 2, chapter 3. Chapter 3, all the basics. And Dixmier vetoed, he could veto, as long as he there would not be a text on varieties. Because you see, in chapter 3, we always use theorems on integrable stuff, things like that, the sub-varieties, and you vetoed.

JACQUES DIXMIER : I forgot.

JEAN-PIERRE SERRE : And it was I who wrote most of the booklet of results, at full speed (*holding the head in the hands*) : I think it there were days when I wrote 10 pages of this results booklet, and it was horribly difficult to write because without demonstration, not to say stupidity... Ouh! (*sighing*)

ALAIN CONNES : It's not obvious, that's for sure.

PIERRE CARTIER : In addition to the preconceptions that we had made that we had to process all kinds of varieties at the same time, finite dimensions, infinite dimensions.

JEAN-PIERRE SERRE : the Banach p -adic, but (*detaching the words*) WE DID IT! And you forced your hand, you see, and it's great to have the forced hand, there is nothing more useful, to write books.

JACQUES DIXMIER : It reminds me, then, of Godement's comment on chapter 3, once Godement, after he abandoned Bourbaki.

ALAIN CONNES : On Lie groups ?

JACQUES DIXMIER : He believes that it is a monster (*laughs*) .

JEAN-PIERRE SERRE : It is very very good, this book, it is big indeed.

JACQUES DIXMIER : He's big and heavy, but I don't see how we can do better.

JEAN-PIERRE SERRE : Do you still remember that you vetoed, you remember it ?

JACQUES DIXMIER : No, I don't remember.

JEAN-PIERRE SERRE : No kidding, you don't remember (*flabbergasted*).

JACQUES DIXMIER : But that doesn't surprise me (*laughs*). I forget things...

JEAN-PIERRE SERRE : What surprises me about you is that you vetoed, this is the only time in your life, I believe, moreover, there was very little veto in Bourbaki.

JACQUES DIXMIER : I trust you, in fact.

JEAN-PIERRE SERRE : I do not remember another, (*speaking to PC*) do you remember ?

PIERRE CARTIER : Chevalley, on algebra 2. He only wanted spaces of finite dimension.

JEAN-PIERRE SERRE : But it was not a veto, it was a proposal and then Weil jumped up. He wanted to delete all the modules, you see, Chevalley. Only the vector spaces were useful.

ALAIN CONNES : (*flabbergasted facial expression and sigh*) Ouh!

PIERRE CARTIER : And of finite dimension.

JACQUES DIXMIER : It is all the more extraordinary, what you say, that a little later, when we studied separable algebras, Chevalley sold us a huge kit with separable algebras of infinite dimension. And then we got confused in there at a convention, “Look, we could let that aside...”. Chevalley says “Ah no now it’s too late, we read it often enough, you have to keep it” (*laughs*). Fortunately, he has not...

ALAIN CONNES : He did not insist too much.

JEAN-PIERRE SERRE : We never followed what Chevalley wanted, he had a very bad taste for elementary writing, you see; he was writing fine the difficult theorems but then when he tried himself to make elementary books, his book on tensor products, powers symmetrical, etc.

JACQUES DIXMIER : It was undrinkable, but on the other hand Theory of Lie Groups, it’s a masterpiece...!

PIERRE CARTIER : All his books that contain something are wonderful.

JEAN-PIERRE SERRE : Ah well, absolutely, all those containing something. But the exhibition was awful.

PIERRE CARTIER : But the one he published in the United States, on algebras, relatively basic.

JACQUES DIXMIER : And his book on spinors, it is very pleasant too!

JEAN-PIERRE SERRE : Yes, except for its systematic side, but he didn’t want not mix different views and so.

JACQUES DIXMIER : And then I think of something else, we come back to foundations. Finally, let’s come back, when we ended up writing chapters 1 and 2, of set theory...

ALAIN CONNES : When was that done ?

PIERRE CARTIER AND JEAN-PIERRE SERRE : In about 50.

ALAIN CONNES : When was it finished ?

JACQUES DIXMIER : When I got home in 49, I had in my hands an editorial by Dieudonné which was really not engaging. And after that, there was a Chevalley writing...

JEAN-PIERRE SERRE : And that was good ?

JACQUES DIXMIER : Ah, I thought it was great.

JEAN-PIERRE SERRE : Oh well, okay.

JACQUES DIXMIER : So I was in charge of my usual job, of good smooth, I wrote after Chevalley.

JEAN-PIERRE SERRE : No, but because it is good, the book of theory of together, for lots of uses.

JACQUES DIXMIER : Oh well, it has lots of flaws, again, but finally, in any case, Chevalley did a very good job of writing on it, it was what I meant.

ALAIN CONNES : So he was able anyway...

PIERRE CARTIER : Chevalley, it was said that he entered a tunnel, all black, and that the exit was 4 km away, with a little glow.

JACQUES DIXMIER : Yes, well listen, and André Weil's book on number theory, when I tried to read it, that's kind of it...

JEAN-PIERRE SERRE : Ah, the Basic Number Theory ?

ALAIN CONNES : Oh the Basic Number Theory, oh, it's superb, it's heavy, because it is locally compact, so... (*laughs*)

JEAN-PIERRE SERRE : No, but there are complete demonstrations.

JACQUES DIXMIER : No but I'm not saying, he surely has enormous qualities, etc., but... for a non-expert.

JEAN-PIERRE SERRE : Does it demonstrate the body of classes in it, I believe, yes ?

ALAIN CONNES : No, but if you want, he pushes a point of view, good, which is the locally compact view of adèles, etc., and it pushes it very well until end.

PIERRE CARTIER : But it's an important point of view anyway.

ALAIN CONNES : It's a very important point of view, but hey, it does not it lightly, that's what we can say.

JEAN-PIERRE SERRE : Okay so what did you have for questions ?

ALAIN CONNES : No, we should now talk about the meetings around the table, and the kind of discussions that were going on, because that's very important, I think. No, what I mean is not easy to understand : finally therefore, there was an editor, who made an edition work, who spent time there, who tried to write the best possible, and then this writing was read aloud.

JACQUES DIXMIER : Not always.

ALAIN CONNES : Not always? Were there cases where it was not read aloud ?...

JEAN-PIERRE SERRE : 100 pages, 100 pages, we were reading aloud, it happened...

ALAIN CONNES : 100 pages that are read, of course.

JEAN-PIERRE SERRE : ...I remember the writing of Dieudonné on integration, and they read the first 3 pages, and they said, I attended to that, and they said "oh no, that is not possible, it is undrinkable, it must be done otherwise, Godly, you will do with Radon measurements. Here". And hop on

the hatch.

ALAIN CONNES : Hop, trapped? (*hissing*)

JEAN-PIERRE SERRE : Ah yes yes yes.

JACQUES DIXMIER : And unless I am mistaken, my wording of Chapter 1 of the varieties, because I insisted on detailed writing... (*hand sign that the writing has been forgotten*). I was not there, I was in the United States.

ALAIN CONNES : Ah, but it's normal, you didn't attend. If you were not there, what do you want?!...

JACQUES DIXMIER : No, I didn't suffer from it, for me, the important thing, was that I had learned what a variety was.

JEAN-PIERRE SERRE : I also did Bourbaki writing even hopeless that they'll be taken, I can't remember what I had drafted, whatever, I knew they wouldn't be taken, but like that, I had clarified for myself, it was written and then here it was.

JACQUES DIXMIER : We didn't say that earlier.

ALAIN CONNES : No, we didn't say it, we didn't mention it enough.

JEAN-PIERRE SERRE : Let's say what?

ALAIN CONNES : Let's say if you want that in fact, there was one of the positive roles of Bourbaki which was to force someone to go to the end of a subject and it's really important that someone learns it in depth.

JEAN-PIERRE SERRE : Yes, I wanted to say that for me, I learned a lot of things because, to write them for Bourbaki, it forced me to give full demonstrations.

ALAIN CONNES : And until we have done that, I mean, we cannot say that we understood.

JEAN-PIERRE SERRE : It was perfect : I learned semi-simple Lie algebras, roots, etc., when writing it.

ALAIN CONNES : I brought this book because I think it's a book...

JEAN-PIERRE SERRE : Ah well, yes, this is the most famous of Bourbaki, basically, the most read.

ALAIN CONNES : We can't talk about Bourbaki...

JACQUES DIXMIER : Chapter 4, 5 and 6, oh yes, he's the one with the most success, no one has dared to say anything bad about it, to my knowledge so far.

JEAN-PIERRE SERRE : In the citations index, I'm sure it must be at all the others.

ALAIN CONNES : I mean, there are people who have criticized Bourbaki.

JEAN-PIERRE SERRE : And this one, I think we owe it to Cartier in large part.

JACQUES DIXMIER : I would like to know what Arnold would have said.

JEAN-PIERRE SERRE : You insisted, you insisted but we were all enthusiastic.

PIERRE CARTIER : I insisted, but everyone participated.

JEAN-PIERRE SERRE : We were enthusiastic when we saw that it was... (*gesturing to something that is developing well*). And then, in a way so, the byproducts are very interesting, the invariants of these groups, things like that.

JEAN-PIERRE SERRE : I learned like that Coxeter hyperbolic groups, for example, by writing exercises for Bourbaki. And they are very fun, Coxeter hyperbolic.

ALAIN CONNES : There is a clear back and forth between personal interest

and the role in Bourbaki.

JEAN-PIERRE SERRE : Bourbaki is really extremely useful for his editors, maybe even more than for its readers, you see.

ALAIN CONNES : Yes, but that's kind of what I had in mind when I was talking about the back and forth between research activity...

JEAN-PIERRE SERRE : Me too, I believe that I benefited in good part like that, writing.

ALAIN CONNES : By writing and getting to the bottom of things exactly.

PIERRE CARTIER : Let's empty an old account. You (*designating Jacques Dixmier*), you wrote the commutative algebra until chapter 5 or 6.

JACQUES DIXMIER : Finally, it was almost over.

PIERRE CARTIER : In a determined plan. Then Serre arrived and everything turned upside down, introducing flatness.

JEAN-PIERRE SERRE : No, you wrote after platitude.

JACQUES DIXMIER : Finally, you completely changed the beginning by putting the flat modules.

JEAN-PIERRE SERRE : Yes, but you, you had to...

JACQUES DIXMIER : I was not able to do that, me.

JEAN-PIERRE SERRE : Yes, but you had to write specializations, stuff like that, which have disappeared.

PIERRE CARTIER : That's it, specializations, that's it.

JACQUES DIXMIER : What ?

JEAN-PIERRE SERRE : Specializations.

JACQUES DIXMIER : No.

JEAN-PIERRE SERRE : No ? Because Weil...

JACQUES DIXMIER : No, I wrote the valuations, which were very threatening created at one point by Grothendieck, but they stayed.

PIERRE CARTIER : But finally, the point of view that we suggested to you elsewhere to write, he was, when Serre intervened, he was completely up, this point of view on general algebra.

JEAN-PIERRE SERRE : Yes, indeed, I had an influence at that time...

PIERRE CARTIER : You talk, you talk !

JEAN-PIERRE SERRE : On commutative algebra you see, well because I could see what was useful.

PIERRE CARTIER : It was you who introduced platitude anyway, finally who insisted on platitude.

JEAN-PIERRE SERRE : Yes and some other things of the same kind.

ALAIN CONNES : So there is a question, and we come to questions a little more delicate, if you want, why is that in the chapter of algebra, no commutative algebra, on the homological algebra, why is it that it was done not in the context of the abelian categories but in the framework of the modules ?

JEAN-PIERRE SERRE : Because there was no language of categories, simply.

ALAIN CONNES : But we couldn't add it a little bit there ?

JEAN-PIERRE SERRE : The categories, you can't add "a little bit" , no no no ! *(laughing)*

ALAIN CONNES : There could not be a fascicle of results on the categories, anyway, oh listen...

JACQUES DIXMIER : It might have been a solution, yes. But I don't think we thought about it.

JEAN-PIERRE SERRE : No, but then, is it a set or a kit?...

ALAIN CONNES : Yeah, well...

JACQUES DIXMIER : That is the question.

JEAN-PIERRE SERRE : When you talk, you can say "oh, yeah, etc.", but you can't say "wow" in writing!!

ALAIN CONNES : But I agree.

JACQUES DIXMIER : There are two questions : on the one hand, talking about categories, that's one thing, and second, the question of the foundations of this theory, these are two disjointed problems, both very annoying, but...

JEAN-PIERRE SERRE : But not disjointed anyway, you can't talk if you don't know what it is anyway.

JACQUES DIXMIER : Yes, yes, very well, very easily!

JEAN-PIERRE SERRE : You can talk about it, but write it, no, anyway, no, not in Bourbaki.

PIERRE CARTIER : Not in Bourbaki style.

JEAN-PIERRE SERRE : You can't say "either C a category" without saying what a category is, that is not...

JACQUES DIXMIER : In any case, in the book of Algebraic Topology including we are meditating profusely (*laughter from AC*), there are some numbers

on categories...

PIERRE CARTIER : Small categories, small categories.

ALAIN CONNES : Yes, small categories, of course they are small categories?

PIERRE CARTIER : That's the problem. Small categories do not pose no problem. The big categories...

ALAIN CONNES : It's true that Demazure and Gabriel's book on algebraic groups is cannulated from the start by this discussion, huh, the universe, etc..., and they are forced to discuss all that...

PIERRE CARTIER : No, but Dixmier, everyone knows that talking about the whole of all the sets, it's complete nonsense, everyone know that, good.

JACQUES DIXMIER : Yes, but we also know that if we don't orient ourselves in a certain direction (*making the gesture of taking a path of cross*), we will not fall on glitches.

JEAN-PIERRE SERRE : No, but that is scum, that, you can't put this screed in writing. No, however, what we can do, and Bourbaki the fact, and I too have done it often, that is to say "the functor thing" to just say "the construction I just made"... has such properties-tees, that, it makes sense, we allow ourselves that.

PIERRE CARTIER : Bourbaki allows that.

JEAN-PIERRE SERRE : We allow ourselves "that functor" being simple function rather, or construction.

ALAIN CONNES : But I mean you can't speak, you don't speak of a category, you never name a category. The category, I don't know, me, pointed finite sets for example.

JEAN-PIERRE SERRE : But I don't know where they are?...

ALAIN CONNES : Well, you take a skeleton, which is the whole $\{0, \dots, n\}$

with the point T_0 .

JEAN-PIERRE SERRE : (*laughing with AC*) Okay, look, this is not the place... to do that, we're not going to do a Bourbaki congress and yell at the categories because we will inevitably yell, you see.

ALAIN CONNES : Yes, except that I think, if you want, that it would be good, because that it would show the kind of discussions, that were happening around the table...

JEAN-PIERRE SERRE : Oh well already, it started, we are already in doing it, you see (*all laughing, happy*), this bad example that we give, there, but that is not such a bad example, moreover.

ALAIN CONNES : Well no, it's not such a bad example.

JEAN-PIERRE SERRE : No, we couldn't get out. It should have been all redo, you see, all the foundations.

PIERRE CARTIER : There were two reasons : first, everything would have needed to be rebuilt, and second, the foundations were not solid.

JEAN-PIERRE SERRE : So in addition, yes. You know there are topologies of Grothendieck which even Deligne does not dare to use...

ALAIN CONNES : Because precisely, they are not sets.

JEAN-PIERRE SERRE : Flat topologies, for example, because you can't increase, you know, a flat thing (*turning to PC*), while a spread stuff, when you have a kit, you increase it, you have models.

PIERRE CARTIER : (*nodding*) Yes, it's easy.

JEAN-PIERRE SERRE : But flat, you never know if by changing of universe, you see, that could change cohomology, it's silly what, one must not...

ALAIN CONNES : No, it's true that we are a bit in the unknown, but.

PIERRE CARTIER : There are Gabriel and Demazure who were compelled to this kind of verification, pfffou! (*two expirations to show the difficulty*)

ALAIN CONNES : But that's what I said earlier. And their book ochange, algebraic groups precisely, it starts with that, so we say "Well, we stop reading".

JEAN-PIERRE SERRE : They have...

ALAIN CONNES : They exaggerated, good.

JEAN-PIERRE SERRE : They exaggerated for the typography too, they made their book impossible because of that (*new PC review laughing, JPS cutting it*). We will not criticize, we have enough to do for criticize ourselves.

ALAIN CONNES : Absolutely.

PIERRE CARTIER : No, but you are talking about typography.

JEAN-PIERRE SERRE : The typography of Gabriel and something, you see, they have invented a complicated system of Gothic, Italic, etc. letters

ALAIN CONNES : That you can't even read.

JEAN-PIERRE SERRE : Didn't you notice that?

PIERRE CARTIER : Yes, yes, yes.

ALAIN CONNES : You can't even read the letters that are written.

JACQUES DIXMIER : In the first Bourbaki, there were letters of so ubiquitous themes that people could not read, it was Bourbaki who...

JEAN-PIERRE SERRE : No, no, but you did not look at the Gabriel. It is not the same, no.

ALAIN CONNES : Okay now, a slightly lighter subject, but anyway I think I would absolutely like to hear it is that there are... Each of you has, I think, anecdotes, which have happened at Bourbaki. I only know one but... In fact,

Jacques told me about it another but I dare not tell it. But the kind that one day, knowing very good that Dieudonné became furious when, well,...

JEAN-PIERRE SERRE : Yes, oh well, this one is known... The resignation of Dieudonné by Godement, that's it.

ALAIN CONNES : No, but wait (*pointing to PC*), and the editorial staff was made by Dieudonné. That was Cartier who told me about it.

JEAN-PIERRE SERRE : Really?

PIERRE CARTIER : I no longer know, there was a discussion...

JEAN-PIERRE SERRE : But he had a double anyway (*we hear the laughter from AC*).

PIERRE CARTIER : It must have been on the sets, so hey, we didn't disagree.

JEAN-PIERRE SERRE : There weren't many duplicates at the time.

PIERRE CARTIER : We did not agree and there was a complicated discussion one evening. So Dieudonné, furious, grabs the manuscript, which was all finished, annotated, to send it for printing, tears it apart (*PC does the gesture of tearing a book in 2*), and then he had a fist that allowed him to tear it in half, and drop everything. So Cartan and Eilenberg rush with tape to try to glue the pieces. And the next morning Dieudonné, who had calmed down, quietly descends breakfast by bringing another copy (*laughs*), which he had reconstituted during the night, or that he had made in advance, I don't know.

JACQUES DIXMIER : It was a little bit of set theory. It was perhaps on the lemmas of Serre... (*handing the pole to the others, who laugh*).

JEAN-PIERRE SERRE : Yes, so you don't know that. So there was a period when, in Bourbaki, there were Weil's theorems : the theorems of Weil, then, that were chosen to be particularly obvious, you see, that was that that was funny. (*All laugh*). What if $f \circ g = 1$ and $g \circ f = 1$ then f and g are bijections you see. And then that, we laughed, it's convenient you see, right now, when we want to show that the thing is bijective, we manufactures its inverse. It's

a lot cleaner first of all. And the lemme of Serre, it was something like this, so watch out, what this is that Serre's lemma : we have $a \rightarrow b$ and $b \rightarrow c$ then if $a \rightarrow c$ is injective then...

ALAIN CONNES : $a \rightarrow b$ is also injective.

JACQUES DIXMIER : This is part of the lemmas of Serre.

JEAN-PIERRE SERRE : ... then $a \rightarrow b$ is also injective. It's a part, yes. So me, you see, I needed it in C-theory, when you neglect a class of groups, I needed to know if it was also true in this theory is slightly less obvious, you see. But then to make fun from me, they called it Serre's lemmas.

ALAIN CONNES : Ah, okay, very good (*laughs*) .

JACQUES DIXMIER : It served to demonstrate Weil's theorem (*all are claffent*).

JEAN-PIERRE SERRE : Exactly, Weil's theorems are lemmas of Serre, I don't know how, but surely.

JACQUES DIXMIER : But Weil took it very well.

JEAN-PIERRE SERRE : But me too, finally they are lemmas, they are true, what. As an anecdote, it's not really an anecdote but it's an expression of Bourbaki that I liked very much and which is worth being kept for posterity, that is, we always said on the editorial staff "yes, and then we will have to be careful to replace false theorems with others."

ALAIN CONNES : ssss! Yep!

JEAN-PIERRE SERRE : With the ambiguity that is...

ALAIN CONNES : So wait, I have a question, which comes back a little to the question of discussions around tables, etc. That is to say, if you want. Usually when we actually think alone, we have every now and then (*gesture of an index which turns near the temple*) the recall of the brain which says "That can be wrong" something like that, and we have time. That is to say that it can come, maybe, an hour later, or something like that. So how it was, when

there were discussions around the table, that is to say were the discussions going slowly enough, did that people went out, left to go and think.

JACQUES DIXMIER : Very often.

JEAN-PIERRE SERRE : But at the same time, they were not going slowly, the discussions were quick. They were like we are talking there.

ALAIN CONNES : Yes, but that doesn't really leave time to think, so uh...

JEAN-PIERRE SERRE : It excites a little bit and after you can, after you have the time.

JACQUES DIXMIER : Ah well, I'll give you an example of a discussion super-fast : Schwartz, all alone, saying "Yes yes yes, no no no, there is no doubt!" (*laughs*) .

JEAN-PIERRE SERRE : Yes, that's in Bourbaki, we asked him a question and he answered that "Yes yes yes, no no no, there is no doubt!" (*laughing*).

JACQUES DIXMIER : This is an extreme case.

JEAN-PIERRE SERRE : Nice case of quick reflection, you see. But maybe that you could say a word about this tradition of reading texts aloud, because in Bourbaki, there was this rule that when we were brought an essay, we were supposed to have read it before.

ALAIN CONNES : Of course. So someone was reading it.

JEAN-PIERRE SERRE : Someone wrote it, we didn't read it : anyone of us was reading it out loud.

ALAIN CONNES : One of us was not a different person...

JEAN-PIERRE SERRE AND PIERRE CARTIER : It could turn out. It was not spinning much, I read quite often. There are people who don't like to read aloud, me, it did not bother me.

JACQUES DIXMIER : There were good readers and bad readers, for example, I remember, at my first congress, I was asked to read, and immediately someone said “he’s a bad reader”, I shouldn’t have to read a lot.

PIERRE CARTIER : You wrote enough.

JEAN-PIERRE SERRE : It was really important, you see, because otherwise, the traditional method, people sent texts, you’re supposed to read at your place, and then you come back, and in discussion, you say “oh yes, I found such error”. No, that’s a joke, for math, it’s not going at all.

PIERRE CARTIER : You have to go everywhere.

JEAN-PIERRE SERRE : While there, out loud. And I did a few times with people who wrote a course for me, I said to them, “We will check, we’re going to do your course, you come to my house, and we’re going to read it out loud”, they were “Baba”[†]; of course, it takes the time it takes, it takes hours.

ALAIN CONNES : Of course, it takes time.

JEAN-PIERRE SERRE : Every sentence goes there and there.

PIERRE CARTIER : There is the filter, there is the filter.

JEAN-PIERRE SERRE : We instantly see that things are not going well.

ALAIN CONNES : Okay, but the question is, don’t we read too much quickly so people can think.

JEAN-PIERRE SERRE : No, we read reasonably.

JACQUES DIXMIER : And then I have something to add to what he said, it was not at all prohibited to have read the drafting in advance, it was rare that we do it because we didn’t have time, we had too much work, but not only was it not prohibited, but I even believe it was encouraged, we knew it

[†]. Baba : familiar word that means very astonished.

was very little done but finally, well listen

JEAN-PIERRE SERRE : It was very little done, that was not the main thing. The essential was reading aloud.

JACQUES DIXMIER : I remember reading aloud, redaction. Maybe I'll mix the names, let's put a draft of Chevalley on Lie algebras, no, an essay by Godement, and Chevalley read it before we discuss it in congress, he sent a comment that this writing was pissing and worn out.

ALAIN CONNES : (*whistling*) Ouh la la!

JEAN-PIERRE SERRE : That's it : this collection of the most pissing methods and worn out. That was style.

ALAIN CONNES : It's hard!

JACQUES DIXMIER : Well tough, no! It's a bit of a Bourbaki discussion style.

JEAN-PIERRE SERRE : And we were not angry!

JACQUES DIXMIER : It may have been a bit limited, but not so much...

ALAIN CONNES : You told me, Jacques, I don't know if my memories are correct.

JACQUES DIXMIER : It's a fact that Godement had not a very good character.

JEAN-PIERRE SERRE : But he still hadn't taken it too badly.

PIERRE CARTIER : He was provocative, let's say.

JEAN-PIERRE SERRE : No, but it was Chevalley who said that, it was not Godement. No but its drafting was not particularly bad, I don't remember one... But he had taken standard methods and that, Chevalley indeed had better things and we have improved them.

JACQUES DIXMIER : It was on the first essays of Lie algebras, maybe even “the” first.

ALAIN CONNES : Ah, on enveloping algebras, and all that.

JACQUES DIXMIER : No, not on the enveloping algebras...

PIERRE CARTIER : Replicas, Chevalley was fond of replicas.

JEAN-PIERRE SERRE : Oh, it’s not bad, the aftershocks, but still it had a sense, of algebraic envelope.

JACQUES DIXMIER : No, but that’s to say that sometimes the newsrooms were read before we got to the convention.

JEAN-PIERRE SERRE : Yes, of course, but still, the realwork, it was at the time of reading.

JACQUES DIXMIER : Yes, it was much more frequent, indeed, we discovered them, editors, at congresses.

JEAN-PIERRE SERRE : The real criticism. Real work was probably afterwards, the next editor who, then, also had the right not to remain pecter.

JACQUES DIXMIER : So there, I would say 50 % because there were the critics during the congress and then, as you say, after that, the guy who read was digesting, he had his job.

JEAN-PIERRE SERRE : He decided, he decided, because if he found that it was silly...

JACQUES DIXMIER : If he found that the critics were silly, that they shouldn’t be taken into account, that often happened.

ALAIN CONNES : So there is another question which is somewhat related to this is that for a long time which is at least until Dieudonné’s retirement, Dieudonné was there, so he was doing the final work of..

JEAN-PIERRE SERRE : Yes, he did all the final work for the publisher.

ALAIN CONNES : What happened when he retired, just-is lying?

JACQUES DIXMIER : I don't remember there, who did the last essays and sent them to the printer.

JEAN-PIERRE SERRE : (*addressing JD*) You, you had to do it yourself.

JACQUES DIXMIER : But I don't remember.

PIERRE CARTIER : No, it was shared, there was no replacement.

JEAN-PIERRE SERRE : I did not do it, I never gave a feedback action to the printer, there has always been someone intermediary, for me.

PIERRE CARTIER : I don't think I gave any either.

JEAN-PIERRE SERRE : For example, with Bruhat, we completely mined in that book (*designating the book brought by AC that was discussed previously and addressing PC*), there were little bits you see, we wrote...

JACQUES DIXMIER : I wonder, listen, do we happen to write to Dieudonné by saying to him "here, the job is finished...", no but I'm not kidding, he was very devoted Dieudonné, he would have been ready to do...

JEAN-PIERRE SERRE : Maybe it was just the secretary of Bourbaki who gave it to the printer, you see, maybe we prepared it work...

JACQUES DIXMIER : I can't remember, it's a shame because... it's not very important, but still.

ALAIN CONNES : Ah anyway, yes, we would like to know, yes indeed.

JACQUES DIXMIER : I cannot answer you.

JEAN-PIERRE SERRE : I think it was she who had something to do with the editor because I don't see one of us going to Hermann's.

JACQUES DIXMIER : Ah, I went to Hermann quite often.

JEAN-PIERRE SERRE : Ah you went there?

JACQUES DIXMIER : Yes, it was for example for... I don't know why anymore I was going there, I had met the two vendors, who were good guys, it was not even Berès that I saw... Well, well no, we don't know how to answer.

ALAIN CONNES : Indeed, there is another question that arose, that is, you want, when people had to retire, in fact, I have a particular example since we had discussed it with Jacques. I mean, this rule that had been laid down in advance, there were bound to be cases where people were not very happy, I think of André Weil, to leave on the date given, they were forced.

PIERRE CARTIER : He was the one who reminded us the rule. In 54. In 54 or 55, I remember we read a letter on the birthday of Dieudonné, which was July 4, American National Day. We were celebrating it, well, we had watered it, and at the end, Cartan took out a letter from Weil. At the end of the pot.

JEAN-PIERRE SERRE : As a birthday present, it was well chosen.

ALAIN CONNES : Yes, it's nice, yes.

JEAN-PIERRE SERRE : This is Bourbaki!

ALAIN CONNES : Weil was there?

PIERRE CARTIER : No, he was not there. I received a letter from Weil and to this opportunity, blah blah blah, I will read it to you. And this is where Weil put the feet in the dish "we had promised to leave at 50...".

JACQUES DIXMIER : We promised ourselves, so the rule already existed before.

JEAN-PIERRE SERRE : Yes I think it existed before. They must have said that before, the founding members.

PIERRE CARTIER : We promised ourselves, well... And there was rather the reserve, indeed.

JEAN-PIERRE SERRE : Okay so let's see, you two left at 50 years, both?

PIERRE CARTIER : No.

JEAN-PIERRE SERRE : You, at what age?

PIERRE CARTIER : A little later.

JEAN-PIERRE SERRE : Oh well, how much?

PIERRE CARTIER : 53 or something like that.

JEAN-PIERRE SERRE : Ouh la la, and why you did that, it's not good, is it?! Okay so luckily, I made up for it, because I left 2 or 3 years before.

PIERRE CARTIER : Borel and you left before.

JEAN-PIERRE SERRE : No, Borel left at 50, he would gladly stayed, while I had a little bit enough, I came home at 21 or 22, you see, at 50 it was a lot, and so I left. As I have left the College 2 or 3 years before. I like to leave before.

JACQUES DIXMIER : It must be said and it is still linked to your question that Bourbaki immediately got into the habit of inviting retired people to a congress from time to time. And I have been invited twice to congresses later.

JEAN-PIERRE SERRE : I don't think I was invited, I was punished because I had gone before.

PIERRE CARTIER : Yes, I was invited also.

JACQUES DIXMIER : You have never been invited, that surprises me.

ALAIN CONNES : But Jacques told me the story, when Weil had been invited a few years later, at some point, the meeting in which the editorial offices were assigned for the following timebrag, and apparently you had intervened

with Borel, to tell him that he could not attend this meeting. It's correct ?

JEAN-PIERRE SERRE : Commitments of the congress, it was called, the commitments-Congress, and indeed, we were a little bit merciless, you see. The concept of pity was not in Bourbaki.

JACQUES DIXMIER : Then there, I remember this scene well enough, nobody did... I talked about it afterwards with Koszul and we had exactly the same point of view, that is to say that we, we would have accepted Weil, we didn't think it would have biased the discussion but that if there were members of Bourbaki, for example Borel and Serre who did not want to, so that imposed the refusal... But that was not very human if you want.

JEAN-PIERRE SERRE : No, you see, on the commutative algebra, he had a very different point of view. And yes, that's it...

ALAIN CONNES : Ah yes, on the commutative algebra, he had a very different point of view.

JACQUES DIXMIER : Yes, but it was a discussion where we made the plans for the future, we distributed the newsrooms, if Weil started to intervene, we could cut him off there, okay... But forbid him to attend the meeting there is harder, anyway.

JEAN-PIERRE SERRE : I think it's quite the opposite, you see...

ALAIN CONNES : I agree with Serre.

JEAN-PIERRE SERRE : To cut him off, it's really not possible.

ALAIN CONNES : It's really rude.

JEAN-PIERRE SERRE : From the moment you come to a discussion, you are on an equal footing with others.

ALAIN CONNES : I vaguely remember a story you had told me one day, which was that Weil had gone one day saying "it smells like Chevalley, I'm going".

JEAN-PIERRE SERRE : Saying what ?

ALAIN CONNES : He said that ?

JACQUES DIXMIER : “It smells of Chevalley.”. Yes, yes, he said it, I see the scene again.

JEAN-PIERRE SERRE : In what sense was he saying it ?

JACQUES DIXMIER : Oh well, he was furious about the discussions. Chevalley annoyed him particularly and he left being a little bit insulting for Chevalley, here.

ALAIN CONNES : That, it is a trait of Bourbaki which is that...

PIERRE CARTIER : It was the first generation, anyway, more than the following.

JEAN-PIERRE SERRE : You mean that we were more polite to each other. No, with Borel, listen : “Borel, you are kidding, anyway,...”.

PIERRE CARTIER : Yes, but hey, it didn't matter !

JACQUES DIXMIER : It's true that it's not the same thing as “it smells Chevalley”.

JEAN-PIERRE SERRE : In other meetings, you see, even that.

PIERRE CARTIER : Weil could be really biting, very biting, more than each of us.

ALAIN CONNES : But if you want, I think that freedom precisely yells at people, in mathematical discussions, I think, that's something that went hand in hand with the fraternity that was installed.

PIERRE CARTIER : In principle, there was no...

ALAIN CONNES : Animosity?

PIERRE CARTIER : No, there was no hierarchy...

JEAN-PIERRE SERRE : The Normal School undoubtedly has a lot to do with it. Because we had fun once, guessing what a Bourbaki congress would be made by Japanese you see. At that time, they would all have done that, simply (*making a greeting by bowing to the Japanese*) and the next day, we would have found one of them who would have made harakiri and there, we would have understood that the writing was not good (*bursts of laughter*). And that's why there has not been Japanese Bourbaki.

PIERRE CARTIER : There was the Normalian spirit, that's for sure.

ALAIN CONNES : And it's true that there was a direct, good atmosphere.

JACQUES DIXMIER : Well then, there is the sentence "there was no hierarchy"; then it's true and it's false. It's true that we could..., that I had no qualms about throwing valves at Dieudonné who was at the Academy, etc.

JEAN-PIERRE SERRE : Rather the Academy, everyone didn't care then. The Academy was rather negative, if you will, as a quality.

JACQUES DIXMIER : But the hierarchy, it existed, it is true that there, I have not asked others. It's a bit as if you want to like in a pack, there are some who drop their ears in front of the pack leader.

JEAN-PIERRE SERRE : Weil was the founder, anyway. It was clear, it was him...

ALAIN CONNES : He was the founder, but when there was a discussion on mathematics...

JACQUES DIXMIER : I cannot say that I discussed basic math-mental with Weil, here... Well, maybe I was wrong. In any case, I think I was right. And that's the situation for me. But here I am perhaps exaggerating by saying that there was a hierarchy.

JEAN-PIERRE SERRE : It's not a hierarchy, but there were people more influential than others.

PIERRE CARTIER : "Primus inter pares" as we say in Latin.

JEAN-PIERRE SERRE : That's normal.

PIERRE CARTIER : A relationship of order.

JACQUES DIXMIER : It is better an order relationship, because it is not necessarily a total order.

ALAIN CONNES : Yes, go ahead and say, Serre.

JEAN-PIERRE SERRE : If you want Delsarte, for example, when he attended at the discussions, at the start, he quickly retired, but, he said nothing, you see.

ALAIN CONNES : He did not intervene.

JEAN-PIERRE SERRE : His specialty was analytical number theory, and we absolutely didn't do it, we didn't even do anything that was useful, we could have but...

JACQUES DIXMIER : I heard that he had written the end of book 4.

ALAIN CONNES : Of topology ?

JACQUES DIXMIER : FRV.

ALAIN CONNES : Ah, Functions of a Real Variable.

PIERRE CARTIER : It was in his style.

JEAN-PIERRE SERRE : Yes, but that was before I entered Bourbaki, you see, I knew him, he never said a word. And then we have had Pisot and Roger who were lost, too. They were simply not in their place.

JACQUES DIXMIER : Yes Bourbaki kicked them out.

JEAN-PIERRE SERRE : Ah yes, and then it was tricky.

ALAIN CONNES : Oh well, more?!

JEAN-PIERRE SERRE : Ah well, you know how to kick the door, it's not... We still tried to do it properly, but I don't know how the founding members did.

PIERRE CARTIER : In any case, in the archives, there is no trace.

JACQUES DIXMIER : Except that their name no longer appears.

PIERRE CARTIER : Yes, they appear until a certain date, and after they no longer appear but there is no record of their exclusion.

JEAN-PIERRE SERRE : No but note, there is no trace either of the fact that Leray was in discussions at the start, and that he was not there after.

JACQUES DIXMIER : Well, we did well not to put that in La Tribu.

JEAN-PIERRE SERRE : But I know that we had discussions between us to find out how we would get rid of it.

JACQUES DIXMIER : I was not there at the time, I had to come a little after.

ALAIN CONNES : But how did you do it?

JEAN-PIERRE SERRE : I don't remember, I think we invented a system where they didn't have to come and they understood.

JACQUES DIXMIER : Chaboty was fired too.

JEAN-PIERRE SERRE : It is true that he was a member of Bourbaki too.

JACQUES DIXMIER : And Ehresmann, he is the one who left.

JACQUES DIXMIER : Which means that at a certain time, I think Bourbaki found himself in front of a huge job and in my opinion, it's for that they started to recruit all the time. Anyone could come to Bourbaki (*laughs*) Finally but in your case, as you say...

JEAN-PIERRE SERRE : No, I forced the door, not me, it's not the same.

JACQUES DIXMIER : But you are a unique case.

JEAN-PIERRE SERRE : I believe, yes yes yes. What's it called in America, when there is a party, you know, the people who hear in a building that there is a party and who go to the door and slip into it, what is it called "to crash", in any case, I crashed Bourbaki.

PIERRE CARTIER : I don't know the slang expression.

JEAN-PIERRE SERRE : So do you still have things in your mind ? to say now ?

ALAIN CONNES : Not really no, not really. I think otherwise, after, these are generalities on Bourbaki, and hey...

JEAN-PIERRE SERRE : Perhaps we could say, a little about, what we think from what we read on Bourbaki, because...

PIERRE CARTIER : You said that we could talk about the seminar.

ALAIN CONNES : Oh, well, it's a different subject, no.

JEAN-PIERRE SERRE : Ah, I would like to say good things about this seminar, that it's an extraordinary thing, it justifies to itself the maintenance of Bourbaki, after our departure, which obviously (*by carting a little PC*) was a disaster for Bourbaki, but...

PIERRE CARTIER : Of course, of course.

ALAIN CONNES : It is indeed important to express ourselves on what we bed.

PIERRE CARTIER : I was actually looking at volumes 40-50 and on 6 exposed, there are 4 by members of Bourbaki.

JEAN-PIERRE SERRE : And also they represent a perfect complement to the text, because it's interesting math, on the contrary. They are not bourbachiennes at all.

JACQUES DIXMIER : With a huge difference which is that often there are no demonstrations.

JEAN-PIERRE SERRE : Yes but now, be careful, there are many texts which are 40 to 50 pages long and which contain practically demonstrations.

ALAIN CONNES : But Serre, you mentioned a subject which is actually very important, which is that we read reviews of Bourbaki, I contacted before our meeting, there, precisely, someone I know well, who was professor Mole and who had interviewed a number of his colleagues on Bourbaki and he realized that the people who criticized him had not read no Bourbaki book.

PIERRE CARTIER : We live on reputation, in other words.

ALAIN CONNES : Simply, like that, by hearsay, etc., who criticize unfounded. But I think it's important to express yourself in relation to some critics, you talked about Arnold, for example...

PIERRE CARTIER : Laurent Lafforgue too.

ALAIN CONNES : How ?

PIERRE CARTIER : He claims that he has never read a Bourbaki book.

JEAN-PIERRE SERRE : Well, that's not the same.

ALAIN CONNES : No, but it's not the same, because Arnold...

JEAN-PIERRE SERRE : Arnold, it's worse than that. Let's take for example in literature, you have people who in the same sentence, will say "Bourbaki is too abstract!" and "Why didn't Bourbaki do category theory?". From an

abstraction point of view, category theory is much more difficult but they don't realize what they say. They are happy to criticize and that's it.

ALAIN CONNES : That is perfectly true.

JEAN-PIERRE SERRE : But there is still something of Bourbaki that should be mentioned is that many people accuse Bourbaki to have tried to acquire a certain university power. And that is se-laughing, anyway.

ALAIN CONNES : It's serious, yes, you can talk about it, but I think the reason is simple, it's in fact that the members of Bourbaki were the best mathematicians of the moment, and so... I mean, it's not surprising, they ended up at the Sorbonne in such a place.

JEAN-PIERRE SERRE : But finally, that's one of the things we see on Bourbaki and which are unpleasant to me because it was not the case when I was a beginner, because they were in Nancy, they weren't...

ALAIN CONNES : Yes of course.

JEAN-PIERRE SERRE : But after they all came to Paris.

ALAIN CONNES : Indeed if you want, the question we can ask ourselves is "was there a time when Bourbaki played the role of a coterie, of a sect, finally?". I mean, ultimately, people meeting when-even a month a year, had time to talk together, to take decisions...

JEAN-PIERRE SERRE : Extremely rarely. I do not remember table discussions, for example, on posts, things like that.

ALAIN CONNES : On the posts, no...

PIERRE CARTIER : No, that was prohibited.

JEAN-PIERRE SERRE : Yes, it was morally prohibited, yes.

ALAIN CONNES : It's all the more outrageous, this kind of accusations, if you want, that me, if I have a memory of coterie which is not at all Bourbaki,

is that I was the rapporteur for Grothendieck in 1984, for the CNRS.

PIERRE CARTIER : Ah, it was you.

JACQUES DIXMIER : When, on what date ?

ALAIN CONNES : In 1984. Grothendieck was a candidate for the CNRS, okay, and I was his rapporteur. I asked Jacques to write a letter, I had to still write letters of recommendation, he wanted to do math.

JEAN-PIERRE SERRE : And at the time, did he really want to do math ? Because he asked for jobs at a time when he was leaving math.

ALAIN CONNES : He had his Sketch of a program , he had a program which was quite extraordinary.

PIERRE CARTIER : Magnificent work, by the way.

ALAIN CONNES : Magnificent, and then if you want, what happened is that I arrived at the CNRS commission therefore, I was part of the CNRS commission and I realized, before we started, that the order of the day had been established so that the Grothendieck case would be discussed when there are no more vacancies. So I made a scandal, I put... and that was done by the unions, that is to say that what was behind, upstream, was not Bourbaki, it was the unions.

JEAN-PIERRE SERRE : Ah no, but that, the power of the unions, in the commissions.

JACQUES DIXMIER : It has nothing to do with it.

ALAIN CONNES : No, but what I mean is that we accuse Bourbaki, while there were maneuvers of coteries which were much worse.

PIERRE CARTIER : Oh, well, there was Malliavin too.

ALAIN CONNES : There may have been Malliavin, and then Grothendieck had a position, he had a temporary position, it worked, but I mean, it was

an absolutely untenable situation precisely because of...

JEAN-PIERRE SERRE : Yes, I may have been wrong to mention this problem there, but me, it's unpleasant for me to put this on Bourbaki's back, which was not like that at all.

JACQUES DIXMIER : Like something else that we put on Bourbaki's back, there's modern math, the boat accusation for years, now however, there is no more modern math, so we don't talk about it anymore.

ALAIN CONNES : No, but you have to defend yourself, and...

JACQUES DIXMIER : For 10 or 20 years, it was the cream pie.

ALAIN CONNES : You have to defend yourself, and Pierre, you said...

PIERRE CARTIER : Well, the most active in there was, for example Lichnerowicz.

JEAN-PIERRE SERRE : It was essentially Lichné and Choquet.

PIERRE CARTIER : And who were not from Bourbaki.

JEAN-PIERRE SERRE : But who were supporters of Bourbaki, note, both of you.

ALAIN CONNES : Oh good? Lichnerowicz was sympathetic to Bourbaki.

JEAN-PIERRE SERRE : Ah yes, he even hinted more or less, finally, people said that he was a member of Bourbaki, and he did not deny.

ALAIN CONNES : He let it happen.

JACQUES DIXMIER : I wonder if there was not a dispute between Weil and Lichné, but hey, I never tried to deepen...

JEAN-PIERRE SERRE : Weil simply had a bad opinion of Lichne, that's all. Ah well, I have a precise memory, the precise memory is that Lichné was

supposed to have demonstrated I don't know what anymore on symmetrical Riemannian spaces and then Weil arrived at the congress and we told him that, and he said "Ah no, that surely not, he surely didn't demonstrate that". And effective, a few days later, we found out that the demonstration was wrong, you know ?

PIERRE CARTIER : Ah good!

JEAN-PIERRE SERRE : No, he knew in advance that, he felt that... (*grimace*)

JACQUES DIXMIER : Finally personal fights, it exists everywhere, in all circles, and in mathematics, and in Bourbaki, and inoutside Bourbaki, oh!

JEAN-PIERRE SERRE : One thing that was nice about the dinners, if there were things that were morally prohibited which were to talk about posts, but there was also talk about our own work. I remember Dieudonné saying something, and Weil saying "listen, your scholarly works, etc., your scholarly works..."

ALAIN CONNES : We don't care! Ah well, that's very good, that.

JACQUES DIXMIER : Finally, there weren't many who wanted to talk about their own work, finally, the slightest modesty.

JEAN-PIERRE SERRE : Ah no, but in discussion at the table, you see... with a neighbor, anyway we could like that, no I think Dieudonné had used that as an argument for something, he said "I got it served in that, of...". And then Weil said to him "your scholarly works" (*gesture of signal the other to be quiet*).

ALAIN CONNES : Ah well, that's perfect, it's perfect. Precisely it is an essential element in what I was trying to say at the beginning is precisely that there was modesty and people put their ego aside, and they contributed to that, and that is fundamental, because it created this fellowship, and it created that spirit of dedication in a way.

JEAN-PIERRE SERRE : If you want, that's how I got in Bourbaki without asking because it never occurred to me that I had to ask permission, it was

math, and I felt that math, I always had the right to be there, it was spontaneous for me.

JACQUES DIXMIER : It reminds me that I never asked myself the financial question to Bourbaki, things were going well, I never asked myself the question of how it worked.

JEAN-PIERRE SERRE : Bourbaki's finances, from where they received the nice?

PIERRE CARTIER : The sale of books, the sale of books.

JACQUES DIXMIER : Yes, but finally, we had to organize, we had to pay taxes or not to pay them, well...

PIERRE CARTIER : Well, there was an association for that.

JACQUES DIXMIER : It was created quite late, the association.

ALAIN CONNES : From 52.

PIERRE CARTIER : In 52, yes before it was Delsarte, before.

JEAN-PIERRE SERRE : Imagine that they took me as Treasurer, then that I had no idea..., but it was Delsarte who did everything, it was on paper, I was Treasurer and then maybe there was a President who may have been Delsarte, but I've never seen a single account of my life.

PIERRE CARTIER : In the first 15 years, it was Delsarte who did all that.

JACQUES DIXMIER : As an anecdote, does the presence of cinema at conferences interest you?

ALAIN CONNES : Oh yes, of course, all the anecdotes interest me, I had planned...

PIERRE CARTIER : Piccoli came to see us.

ALAIN CONNES : Oh good ?

JACQUES DIXMIER : Yes, Piccoli, we were in Saint-Rémy-de-Provence, it was a friend, that was Douady's distant cousin, that's it. He came as attendee one or more of our discussions.

JEAN-PIERRE SERRE : No, not several, no.

PIERRE CARTIER : He was perfect, he was perfectly modest.

JACQUES DIXMIER : He didn't say a word, he sat in a corner.

ALAIN CONNES : Oh good ? Anyway, that is not bad then, as an anecdote.

JEAN-PIERRE SERRE : Sitting in a corner... I have an anecdote, you see. It was at Royaumont Abbey and we were talking about Topological Vector Spaces so it's really a very long time ago. And we were on a terrace with sun, and there were lounge chairs, and we had settled in the lounge chairs, and Weil was there, and the Topological Vector Spaces, that bothered him, you see, so Weil was asleep. So when we saw that he was asleep, we all left without making the slightest noise, and we went to look out the windows and we saw Weil at one point returned and he could not yell at anyone, there was nobody! (*laughs of all*).

PIERRE CARTIER : That, I didn't know that.

JEAN-PIERRE SERRE : That's in 49, I think. I think, because it's my first year, surely that was it. Oh, it was the EVT in finished dimension, you see.

ALAIN CONNES : Finite dimension EVT's, no...

JEAN-PIERRE SERRE : Look, when the body is not complete, it's not completely trivial and for example, there are things that are not necessarily closed one inside the other, and on the other hand, if the body is complete, it does nothing is happening. Yes, you see, it's not that great. But I still see the head of Weil, he was a little sheepish, you see.

ALAIN CONNES : Surely (*laughs*) .

JACQUES DIXMIER : And to tell you that the discussions were sometimes not formal, I remember a session where, it was summer, we were in a courtyard, a garden, and Douady left with a shovel and returned a few minutes later showing us what he had done : he had killed a viper and he proudly showed us the viper.

JEAN-PIERRE SERRE : There was once, he threw me a snake, it seemed a snake, but it's quite unpleasant when you receive a snake in your arms, alive, then !

PIERRE CARTIER : Douady, hunting vipers, that was one of his obsessions.

JEAN-PIERRE SERRE : He was killing them.

JACQUES DIXMIER : Douady and the counterexamples.

JEAN-PIERRE SERRE : At the time, we killed vipers, now they are protected.

ALAIN CONNES : Douady and the counterexamples is precisely to send vipers to someone.

JACQUES DIXMIER : We were discussing what Godement calls the monster, the chapter 3 of the Lie groups.

JEAN-PIERRE SERRE : Oh, it's not a monster, I tell you again, it's a very nice chapter.

JACQUES DIXMIER : And Douady was there and I was there and so one evening, we had the habit to go to bed quite early, around 10 pm, but then, I don't know why, I stayed working until around midnight and Douady didn't either, hadn't gone to bed, he went back to the hotel, he saw me, he was excited, he said to me "I found the counterexample on Lie groups, guess what it is!". So I try to suggest to him "did you find this?", "No it's not that!"... "This?", "no, that's not it!". And then all the same, at some point, I said to myself, "Isn't that right?", it seemed almost impossible to me, by chance, he would have found a Lie algebra which does not correspond to any Lie group?...

JEAN-PIERRE SERRE : That's it, Banachique or something like that, but no finished dimension anyway...

JACQUES DIXMIER : So I tell him "is that it?"... "Yes, that's it!" And then he started screaming, it was midnight, he had to wake up half from the hotel (*laughter*). I had a hard time calming him down.

PIERRE CARTIER : It was a real problem, that.

JEAN-PIERRE SERRE : I hope it's in exercise in Bourbaki, that.

JACQUES DIXMIER : It went into exer. of course. You talk about the importance of the counterexample, all the same.

JEAN-PIERRE SERRE : Ah yes, and then there was also, Dixmier, you had, it seems to me, funny counterexamples, didn't you make an example where a family has a parameter of Lie groups and the universal coverings cannot be extended (*gesturing with the arms to illustrate his point*).

ALAIN CONNES : There is no increase in homotopy, you mean.

JEAN-PIERRE SERRE : I think.

JACQUES DIXMIER : Not me, no, I have no memory of that.

JEAN-PIERRE SERRE : It is not you?... But whoever you say, I believe that it was an unclosed subspace in a Banach, there was something like that.

JACQUES DIXMIER : Ah, it was of infinite dimension, of course.

ALAIN CONNES : So the creative aspect is often in the exercises, effectively.

JACQUES DIXMIER : Oh no, you're exaggerating.

JEAN-PIERRE SERRE : There are a lot of things in the exercises.

JACQUES DIXMIER : Things that Dieudonné went to seek.

JEAN-PIERRE SERRE AND ALAIN CONNES : No no no, not at all.

JEAN-PIERRE SERRE : I made a lot of things that I put in exercises, you see, that weren't in the literature, no, that were sometimes not known.

JACQUES DIXMIER : I'm sure you put a lot of things, but well, it wasn't the majority of exer.

JEAN-PIERRE SERRE : (*getting up and going to the book table*) From of these books, you see, Dieudonné did not contribute to any exercise there inside : these are exercises entirely ours... You see, the Coxeter hyperbolic for example, which are there, moreover with imprint errors. Well it was badly printed, well. No, lots of things, I had set group invariants, generated by characteristic p reflections.

JACQUES DIXMIER : Characteristic p , Dieudonné knew that, when even.

JEAN-PIERRE SERRE : No, he knew absolutely nothing. The invariants perhaps.

JACQUES DIXMIER : No, not the invariants. Finally, I don't know, its about classical bands, I hardly read it.

JEAN-PIERRE SERRE : Look, there are exercises there, they are so pretty. It's not at all the same as the first Bourbaki books where effectively Dieudonné took articles and things and put them while that there, me, when I made it, it was different, I wondered, or else, then, something that I knew to be true, I could find a way of writing it as an exercise. No, there are a lot of things.

ALAIN CONNES : I think it's a perfect time to finish.

JACQUES DIXMIER AND JEAN-PIERRE SERRE : Let's finish.

ALAIN CONNES : We could chat for hours.

JEAN-PIERRE SERRE : We could “get angry” for hours, to give an idea of what it was! (*laughs*)

ALAIN CONNES : I think it still gives a good idea of what that was.