Spaces and models

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Alain Connes: I think we could start by talking about how in the past the notion of space appeared more and more precisely for physicists. So ... if you don't mind ...

Thibault Damour: Yes, indeed, it's a good question, a good way to approach the debate. It seems to me that we can start in New ton. Newton inherited a conception of space that dates back to the Greeks.
But, it was Newton the first who, as a physicist, really needed a space on which to base the physics he was creating. And what characterizes Newton's space is first of all the mixture of a mathematical structure, which Newton did not invent, which was that of the space of the Greeks, of the Euclidean space, with a simple structure, a set of points, a notion of distance and then, a certain philosophical position about this space which was the belief in the reality of this space, reality in the naive sense, that is to say that the space , really, is something that preexists outside of us.
Now this conception, which entered physics historically therefore in 1687, when Newton published his masterpiece, this conception was immediately questioned by other physicists-mathematicians, like Leibniz, let's say roughly for whom the space in fact was not a thing pre-existing to objects, but was, as he said, only an order of coexisting, say, the set of relations between objects, between things existing in themselves, the set of their relations reciprocal, their configuration

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Transcription of a viewable interview at the National Audiovisual Institute (INA). Video reference: https://
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geometric defined space, but space did not exist as an object itself. But in fact, these two conceptions, which one could believe to be extreme positions of the existence of space, either space exists, or there exist only objects, matter, it does not exist. space, were later reexamined by Kant, and although it was not the purpose of our interview to speak in every detail of philosophy, just remembering that Kant deeply perceived that in fact it was not was neither Newton nor Leibniz who were right, that one could not conceive of space as really existing, nor matter as only existing, and space being an illusion linked to the configurations of matter, but that in fact , necessarily, it was necessary to posit space mathematically, as a mathematical a priori, an ideal thing, which is not a thing of reality and that it is this space as a mathematical structure posited a priori which then makes it possible to make physics, and that objects acquire reality only if there is a background space in which we can ask them.

Alain Connes: Space is a word we use, it is the theater in which all physical phenomena occur. Obviously, we can say that there is no theater, that there are only the physical phenomena themselves, but I do not quite understand the opposition between Leibniz's point of view and the point from Newton's point of view, in the sense that, if you want, you would have to explain more in what sense for Newton, even empty space had a meaning, existed, unlike Leibniz. I could hardly see the nuance between the two.

Thibault Damour: Yes, in fact, obviously, you are familiar with the Newtonian approach because ...

Alain Connes: Absolutely.

Thibault Damour: ... for Newton, that's what you just said, space is a large empty theater, which preexists the existence of objects that can be put inside, even if there is no there is no matter, the space is there, first of all.
The space is there, really. So to try to ... well, Leibniz's idea, the problem is that it has never been mathematized. In modern terms, in fact, it is very close to an idea, to certain ideas for example from Bacry[1],

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[1] Henri Bacry, French physicist, 1928-2010.

with which you are familiar ...

Alain Connes: Yes.

Thibault Damour: ... which would be that what really exists is matter, good. In modern terms, say, matter is described as vectors, in, what must be called space, but which is not at all ordinary space, which is called Fock space, or Hilbert space. , more generally. So in fact, the matter itself is given, independently of the usual space in which it is placed, it is the matter which is given in a primordial way, and one could consider that Leibniz defined a program which, being given the mathematical description of matter in modern particle physics, that is to say this Hilbert space, can I somewhere find space with its usual three dimensions from the vectors of this space of Hilbert who defines matter.

Alain Connes: What I do not understand well in the point of view of Leibniz, precisely, is how, by rejecting the notion of space, even empty space, how we manage to make a physics which has a contact what a conch with the current models even.

Thibault Damour: Yes, well, in fact, you're absolutely right: Leibniz's program never worked. Leibniz sketched it out, it was an intuitive reaction; there is something which seemed unsatisfactory to him in the Newtonian synthesis and he has never succeeded in coming up with better. So it's still a program. We can try to look now at what Leibniz found unsatisfactory in the Newtonian attitude, since that, in fact, remains with us. So what bothered him was that space was something supposed to exist independently of objects, that it could never be touched, space, and yet, that it had some visible consequence, in particular, the notion of absolute rest, and the notion of displacement at constant speed, that is to say the principle of inertia, that we can distinguish if we are in a train which moves at uniform speed, or if we are in a merry go-round, in a merry-go-round that turns at high speed, there are effects, there, Newton said these are real effects, which distinguish an accelerated cue from an accelerated cue and therefore, there must be some cause outside of the two. And that bothered Leibniz.

Alain Connes: Yes, therefore despite the philosophical interest of the views of Leibniz, and of his critics, we can still say that it is Newton's point of view which prevailed for an extremely long time and in part, obviously, by the successes of the model he proposed for space, and the truly incredible successes of the predictions that can be made with Newtonian theory.

Thibault Damour: Yes, absolutely. For centuries after Newton, armies of mathematicians and physicists set in motion what was germinating in Newtonian synthesis and showed how, in an unprecedented way until then, all the phenomena around us, and especially this magnificent celestial clock , all the celestial mechanics, everything worked very well, so that for centuries, the Newtonian conception was imposed, and that it became completely evident; we no longer thought of the philosophical difficulties that it could include, really, the notion of space was identified with a real object existing around us, inside which the objects exist.

Alain Connes: We can say that it influenced considerably even the point of view of physicists, mathematicians like Lagrange or Laplace, who came to believe that the world was entirely deterministic, in the sense that, as they said to the At the time, if we knew all the positions and speeds of the particles present, we could predict what would happen.
There was an illusion, which lasted for quite a considerable time, and which made it appear that physics could not only be modeled, but that in fact one had caught reality, and this reality was bordering on deterministic.

Thibault Damour: Absolutely. And finally, it was not until the end of the 19th century and the beginning of the 20th century that a certain number of dark clouds accumulated, concerning the two aspects that you have just noted: the perfect character of determinism, and the absolute character of the space. The first doubts arrived at the end of the 19th century. Well, first of all, it's Planck and the mysteries of the black body: when you heat a body and keep it in thermal equilibrium, how much light energy radiates into the space around it? This simple problem posed a dreadful paradox to classical mechanics.

Alain Connes: We will see that this kind of paradox did not frighten the physicists of the 20th century, far from it. But anyway, let's say there was a rather serious crisis that had occurred, indeed from the moment when we were trying to understand the radiation of the black body, but we can say also the radiation which comes to us from the nearest star, the Sun.

Thibault Damour: Yes, well, the whole theory of radiation, it was clear that it was necessary to redo it to zero, at that time, the existence of the spectral lines, too, that is to say that indeed the stars do not emit any light whatsoever but that we see particular lines, in absorption or in emission, that the light emitted by the atoms is not a continuous frequencies but contains particular frequencies, it was all very mysterious and light began to come from 1900[2] when Planck, then whole generations of physicists after him tackled the problem of understanding the mechanics of the atom. And there, to understand that, they were not obliged to question the notion of space, since they could presuppose the same notion of Newtonian space, and even without any effect linked to high particle speeds for a long time. So they have modified the other aspect, which is the determinism of the mechanics of which you spoke, which is the fact that matter is not represented as points mass in a three-dimensional space but by more fuzzy things. He understood that the fact of introducing a new constant of nature, this Planck constant which measures this quantification of energy, by slices *hÿ* where *h* is Planck's constant, he understood, and it was for him a jump conceptual in the unknown because he did not have to question the notion of space at that time. But, purely mathematically, it was said “I introduced a new constant in physics (the constant of Planck), however, there is the speed of light, there is Newton's constant, and as early as 1905, at least in a book, and I checked that it was in a book, but I also believe in an article he said “playing with these constants, I can do other things, and in particular, I can do a length characteristic ”, and he found a length of 10ÿ33 cm.

Alain Connes: What is now called Planck length.

Thibault Damour: And he really found it. It's not because it is made with Planck's constant, it is because he saw it.

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[2] sic!

Alain Connes: Okay.

Thibault Damour: And there, perhaps, I do not know German, and there, without doubt, he did not immediately formalize that, and he said to himself "things are going to happen there" but since he was a very deep person, he must have said to himself “there must be something deep there and the appearance of a new constant in physics will change things in the notion of space and to the concept of distance. ” But that, things changed a lot before we manage to conceptualize in fact, which can happen at the length of Planck, since after this introduction by Planck of the discontinuous in physics, Einstein issued his first theory, which , from one point of view, modifies Newton's theory a lot, but which, from another point of view, does not modify it at all. That is to say, the big difference is that Newton thought of space as absolute, and that this notion of space absolutism disappeared in 1905 with the special theory of relativity, but it replaced the absolute space by a space-time which is just as absolute as Newton's space was; there is no longer any notion of absolute rest but ultimately, the preexistence of the theater you were talking about, of this great empty theater which preexists matter is again imposed by special relativity, so we are in the reign of absolute in 1905.

Alain Connes: Absolutely, yes. Besides, I wanted to say, all the same, it's true that on one side, there is that, and on the other side, as Einstein updated the notion of the particle as transporting light, there was a return to Newton in that way, in a certain form, since there had been this long debate after Newton, on the non-corpuscular character of light, but finally, at the same time, we came back completely to the same idea.

Thibault Damour: Absolutely, and in the usual popularization presentations, we tend too much to indicate that scientific truths arrive, are acquired. In fact, there is much more inertia, the old conceptions still live on, and we know that at any time the physicist does not know the truth, at all times the intimate nature of what we are talking about in physics can change completely, and that ultimately, we are not sure of anything in physics.

The Newtonian conception of space therefore, this theater preexisting the existence of objects, neutral, empty, flat theater, was completely modified in 1915, by Einstein, proposing a new physical theory of space, where space, in intuitive terms, becomes soft, is no longer a object pre-existing to matter, but a dynamic partner of matter, since space is a dynamic entity, now it contains degrees of freedom, it is a physical object, which evolves. Space for the first time becomes something that can evolve, that can have birth, that can mature, that can have death, and that interacts with what is in it.

Alain Connes: I still think that if we place ourselves, not at the level of space, but at the level of space-time, when you describe this dynamic and evolving character of space, when we see it at the level of space-time, it disappears since we can very well try to conceive of space-time as a whole, and to consider its evolution simply as the passage of time, a little bit like the idea that if we try to guess what an orange is for example in a 4-dimensional space, you have to imagine it as being nothing, then a small orange, which grows which grows more and more to reach a certain size then which then decreases, to finally disappear. So that is a slightly dynamic image that allows us to imagine what is happening in a larger dimension. And there is still always the same idea that, from the moment we consider space-time, there is still this framework, there is all the same this theater, which is nonetheless, you I was saying earlier “it is no longer frozen”, well, it is no longer frozen, in the sense that indeed, time is a parameter and the shape of the space-like slice will change over time, but it is nonetheless frozen in its entirety.

Thibault Damour: Absolutely, and here we should perhaps insist on the fact that, linked to this Newtonian conception of an absolute space, of an ab solu time, we still believe today that time is a absolute of physics, that there is a zero time, we speak of the Big-Bang in cosmological models.
However, in a relativistic cosmological model, the Big-Bang, it is not an event, it is not really a time that exists, and we cannot ask ourselves the problem of knowing what there is before, even without talking about the fact that classical physics stops at the Big-Bang, it is coherent within relativistic physics to say “time has a beginning” but we must not imagine that time is born nothing: we give ourselves a space-time block, which is the framework that will allow us to describe the material and this block has

a natural border, full stop. Because the inner reality, the way in which the human being perceives time, in the notion of reality of the external world and perceived by the human being has nothing to do with what physics says; physics ultimately has mathematical models of reality, which are completely separate from the real notion of intuitive perception of time.
So the problem of posing the Big-Bang as time is a false problem.

Alain Connes: Of course. And besides there is something extremely frustrating in the model as it is compared to the current intuition, which is that from the moment when one imagines that space-time exists of overall, and our universe-line, for example, as an individual is already drawn, one would become completely fatalistic, I mean. It is a vision of the world, of evolution, etc., which is entirely, how to say, which is written in advance. Finally from the moment we try to imagine a space-time globally, it's excellent as a model, but let's say that compared to intuitive ideas, as you said, compared to the intuitive perception that we have of time, it appears to be quite confusing.

Thibault Damour: Yes, but I believe that physics will never have anything better to offer, than in any case, this intuitive problem of time, this problem, let's say, of the now, the separation, for each of us, which 'There is a now, as opposed to past things, and as opposed to future events, this problem is completely unsolved and never will be.

Alain Connes: When you say “it never will be” you can't tell, I think ... You're a little pessimistic there.

Thibault Damour: Except that experience shows that ultimately, physics only gives models of reality, and that at any time, we know that these models will be replaced, can be replaced, and that in history of all these models, never finally, the true reality as it is lived appeared since the image that physics gives of reality is always neutral, factitious, let's say, it is a model, it is a representation of what is happening, but it is not what is.

Alain Connes: Yes. I think we can go a little further on this. I mean, before talking about space seen by mathematicians, I think that indeed, as we saw that after Newton, physicists had this idea that finally, physics could be deterministic, I think that as we have taken a considerable step backwards since then and as you said very well, we are at the point of giving certain models, I think we could try to define more precisely what is the link between a model and the physical reality that it is supposed to describe. So I would like to take an image which is the following: it is that ultimately, physical reality, the experiences that we have in physics, the experience that we have daily while living, I would characterize them as being a contribution. daily information. That is to say that every day brings a certain flow of information, and for me, the purpose, the goal of physics, is to eliminate information that is of no interest. So I will try to explain myself in very simple terms. What I mean is that, for example, it would never occur to a television presenter, in the evening when he speaks at 8 p.m., when he gives the news, to say “Tomorrow, the sun will rise. ”. Why ? Because we know that it is automatic, it is a fact that we noticed, we re marked that it was repetitive, that it happened, and finally I mean, it is information that is without interest because precisely, it is part of a model which is perfectly accepted, which is a model of physics, one could say, it is an elementary model which is perfectly accepted by everyone, and therefore as it is perfectly accepted by everyone, the quantity of information that it gives has been coded once and for all, it has been coded in this model and it reduces the information that we capture during the day to the information that is original , which are really part of the news flow. So the way in which I would like to present physics is precisely the possibility of capturing a quite considerable amount of information, such as for example that there will be a lunar eclipse on a given day, etc. , and to code them in such a way that they are coded by simple laws, which is also why we insist on simplicity, and when I speak of simplicity, I do not speak in aesthetic terms, I really speak at the quantity of information level, as we speak of quantity of information for a computer, etc., the number of bits of information that it is necessary to have to communicate the message. And so I think now we've come to roughly the following conception, tell me if you don't agree but ... to the conception of saying that we can never reduce the quantity of information which is provided to us by external reality, which is provided to us by Nature, which is provided to us by the Universe, we can never reduce it to zero, in the sense that, precisely, physics is not a deterministic system which means that, if we knew all the past, we ar would be trying to predict the future, but on the other hand, we manage to reduce this quantity of information, this new flow of information, to reduce it to a smaller and smaller quantity, and finally, this originality that the passage brings of time, it is of course due in part to quantum mechanics, it is partly due to the fact that we have indeterminism in quantum mechanics, but it is also due to the fact that the equations which govern even mechanics are - saying classical and deterministic are in general equations which are hyperbolic, and which do not allow us to really predict, in a precise way, what will happen.

Thibault Damour: This model, to make you feel what is new in quantum mechanics, this originality that time brings, it also makes you feel, even more violently, the fundamental incompatibility between quantum mechanics, of which not only the usual descriptions, but even the way of thinking about interpretation, needs a passing time, needs things which take place in a certain time, and the precisely fictitious and illusory nature of time, in theory of Einsteinian space-time, where space-time is given as a block, and where there is no now, where there is no present, there is not something that is past. So here, we can already see that at this conceptual level, there seems to be an incompatibility between quantum mechanics and general relativity, and all of this is further reinforced if we go back to what Planck had predicted a long time ago, c 'that is to say that if we combine purely, like a child combines legos, if we combine the little lego which is the constant of quantum mechanics with the little lego which is the constant of gravitation, we build a small fundamental length, 10ÿ33 cm, and here all of physics shows very clearly that if we managed to try to measure regions of space small enough to contain dimensions of the order of 10ÿ33 cm, then there , we could neither have general relativity, nor quantum theory, we cannot identify a point in space to better than 10ÿ33 cm, we must have something else.

Alain Connes: What that reminds me of, if you will, is that in fact, a problem that seems to me to be very interesting, is the somewhat preconceived idea that we have, that is, is an a priori, which is that the space, whatever it is, the model that we are going to make of it will always revolve around the theory of sets, that is to say ultimately, this idea that, a little bit by education finally, since this is how we are taught mathematics, a space is necessarily a set of points. So I think we're gonna try to think about it and let's say the question that we can decently ask ourselves, we can ask it on the one hand at a philosophical level: “is a point in space something good? defined ?". But one can on everything I believe to pose it at the level of the models, and I think that this is how I would prefer to approach the problem, not by trying to think about knowing if the space is formed of points or not, because basically that presupposes the existence of a well-defined notion of space outside of a model, but of a very real physical space. And on the other hand, I prefer to ask myself the question at the level of the models, therefore, and to know if we can manage to make interesting models, which are mathematical models, of space, and which no longer rely on l The idea that space is necessarily formed of points is necessarily a set. So I believe that if we want to try to understand this question, we have to start by analyzing, as the development of mathematics progresses, how we got there, with finally set theory, with Cantor, how we got there. trying to formalize everything in terms of sets. So I don't propose to throw the sets in the trash, of course, that's not my idea at all, but what I propose to do is to free the notion of space from its straitjacket. imposed, when it is imposed to be a whole. So what I would like to explain is first how this notion of sets, of set theory, has invaded mathematics to the point of preventing us from considering a space other than as a set, and then how in fact quantum physics liberated this notion, as far as classical mechanics is concerned.

So good, it's true that the notion of set, set theory has made it possible to formalize mathematics to such an extent that it even engendered the disease of formalism, I would say, that is to say to such a point that it ended up making believe that one could forget that mathematics had to do with a certain reality, to think that it was a game, like a little bit like an assembly game, therefore, c 'is what is called a formalism, which consisted of a certain number of propositions supposed to be true, which are called axioms, a certain number of grammatical rules, which are rules for assembling propositions, of a set of logical rules which make it possible to deduce from them other propositions, and what is therefore called formalism, is what leads to trying to deduce theorems only by a process which is practically automatic, in any case which is verifiable: we can know whether or not a demonstration is correct or not. So set theory contributed to this in particular, and it also contributed to making think in general the notion of space under this form it. We cannot say that set theory is a theory in the usual sense, in the sense that it has no general implication in current themes, we cannot say that the subtleties it contains has really important implications in current mathematics, but it is true that all mathematics is formulated in these terms. So, if we now want to analyze a space, we realize that the way in which space intervenes in the physical theory which is without most elaborate doubt, apart from general relativity, field theory, space does not intervene at all as far as we are interested in free particles. This is what I was saying earlier. There only intervenes the pulse space and say, even, the fields are parameterized by points in space, but these are just dumb parameters, they are like clues, we could give them any other name, that wouldn't matter. On the other hand, a space intervenes when we write the interactions, and it occurs when, finally, I believe that we can re-express the principle of locality of interactions in a more economic, and it seems to me that when we reach that point, we surrender count much more easily than there is this duality which has always existed, which is a bit like the duality between Leibniz and Newton, which is the duality between algebra and geometry. So geometrically, we manage to have a simple visual intuition of things, and once we translate them algebraically, that's a little bit what Descartes did, when he took the coordinates Cartesian, that he succeeded in replacing the points by coordinates, by a certain number of coordinates, when we manage to make things more algebraic, we realize that we are no longer limited by an intuition, even a set intuition of things and we manage to handle objects whose access is perhaps more difficult because it is more algebraic and more abstract, but which allows us to go further, in the sense that it just allows us to go beyond an intuitive notion of the notion of space, which is based on the concept of the whole. So it seems to me that there is a problem that is posed in a certain way, which consists in trying to understand which is the exact role that the algebra of space, that is to say the algebra of functions, coordinates on space plays, in physics, like the physics of elementary particles, or general relativity, and to arrive at this algebra greater flexibility, greater freedom, to possibly meet the need that we feel, which is to have this side

a little blurry of the notion of point, when we place ourselves at the scale which will appear at the level of quantum gravity, which is the Planck scale.
I think I can strongly defend the following idea, which is that finally, when people do, at Cern, the experimenters, do experiments on elementary particles, discover new particles, try to simplify their experimental data by creating models etc., what they give to mathematicians is a cryptogram, and a cryptogram that you have to be able to decipher, but to decipher it, I think it would be wrong to try at all costs to have a preconceived idea of what space time is, and to try to fit the data of this cryptogram with conventional data. And so the point of view that I would like to defend is that precisely we must give ourselves a certain freedom, both on what space is, and especially on what differential geometry is, therefore it is is why, in part, that I tried to develop this non-commutative differential geometry, to have more freedom to decipher the cryptogram that the experimenters give us. These models finally inform us about the structure of space, or space-time as we like, on a scale which is of the order of 10 millionth of a billionth of a centimeter.

So it's an incredibly small scale, and it's a scale that you have to realize that sight is useless. So our instruments of perception are no longer our visual organ, our instruments of perception are a model, and the dialogue between this model and experimentation. And we must realize that therefore our perception of space at this level is no longer done through images that we could show, we could of course show images of the accelerator where we see positrons or electrons moving in an electromagnetic field, that's not how to think, you just have to step back a little bit, and look at the model as it has been systematized, such as it was obtained by physicists, the current paradigm.

Thibault Damour: More generally, then, let's admit that physics has accumulated a terrifying, enormous amount of data, and that we are trying now, we have this cryptogram and we are trying to interpret it. Do you think that one day we will know the true nature of space? What is your philosophical position from the point of view of knowing what knowledge can be *obtained* from space?

Alain Connes: Absolutely. So, let's say that my philosophical point of view

Sophistically, first of all, I believe that the problem arises very early on because we are taught what real numbers are. So the mathematician is familiar with real numbers. And he realizes pretty quickly that when you give a real number, well, you have to give all of its decimals. And in physics, you never use, like you said about Planck length, I mean, the experimental precision is always limited and will be practically limited without further possibility to a number of decimal places that you could give as being about thirty or forty decimal places, and I think that one could manage to demonstrate, for relatively simple physical reasons, that certain quantities, one will never manage to know them with an infinite precision. The impression I have is that we have managed to catch a certain part of reality, but that part has little to do, indeed, with what you might call a physical reality.

Thibault Damour: I quite agree with that: the fact that physics in the end never catches reality, which will remain, whatever the succession of successive models more and more, open the quotes, “precise” that physics will give some clarity, open the quotes, “reality”, there will always be a distance, and a blur between the two. But I would like to insist on the fact that, me as a physicist I am supposed to speak more about the material reality, external to us than you, that there too, there is a ... as we say in Frenglish, a “ gap ”, a huge gap, and I want to stress this, between reality in the existential sense, that is to say the fact of being, of being here now, of speaking in the spotlight here , tonight, such and such a day in such a year, truly lived, the fact of being inserted into the being, and the fact of having a knowledge of a physical object. For me physics, even if it is refined more and more, and I think that it will never be refined until it has an infinite precision, there will always be this limit, but that, ultimately, is not very interesting. The fact that physics can have an asymptotic limit which is different from the maximum information that one can hope to have from reality interests me less than this enormous metaphysical hole between the fact that even if we have the physics of the twenty- fifth century as perfect as possible, it will leave as great a gap between what Newton knew of reality and what today we think we know about reality, which is that in any case, reality in what it has more real is completely unaffected by physics.