

Workshops organized independently of the Main Programmes

ESI Anniversary - Two Decades at the Interface of Mathematics and Physics The [Un]reasonable Effectiveness of Mathematics in the Natural Sciences

Organizers: Goul'nara Arzhantseva, Piotr Chruściel, Adrian Constantin, Joachim Schwermer, Frank Verstraete, Jakob Yngvason (all Kollegium ESI, U Vienna), jointly with Wolfgang Reiter (Foundation ESI) and Klaus Schmidt (Foundation ESI)

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Report on the workshop

The years 1990–1993, preceding the official foundation of the Erwin Schrödinger International Institute for Mathematical Physics (ESI) in Vienna, had been a time of intense preparations involving many mathematicians and physicists. Walter Thirring, Peter Michor, and Heide Narnhofer, acting on behalf of the scientific community, played a decisive role in the foundational period of ESI and beyond. Their initiative was well taken up by the Ministry of Science and Research in Austria. One year before the official opening of ESI a Conference on “Interfaces between mathematics and physics” was held in Vienna in March 1992. From the outset, the ESI has put particular emphasis on fruitful interactions between mathematics and other disciplines, in particular, mathematical physics. With the 20th anniversary occurring in April 2013, the ESI celebrated this event with a Symposium, entitled “*ESI Anniversary – Two Decades at the Interface of Mathematics and Physics – The [Un]reasonable Effectiveness of Mathematics in the Natural Sciences*”. It was necessary to utilize this moment, as it were, not only to celebrate the rich heritage of the Institute but also to position the ESI for the future. The topics of the lectures given at this occasion ranged over a broad band of themes which relate different fields within mathematics and mathematical physics or themes manifesting the outreach of mathematics beyond its own borders.

During the opening ceremony, **Susanne Weigelin-Schwiedrzik, Vice Rector for Research and Career Development of the U of Vienna**, underlined the unique role the Erwin Schrödinger International Institute for Mathematical Physics fills in (post-) graduate education and scientific research in mathematics and physics at the U of Vienna, in Austria, and on an international level.

In his opening words, the **Director of the ESI, Joachim Schwermer**, gave a brief overview of the institutional structure of the ESI, now turned into a research centre at the U of Vienna, and the various programmatic pillars of its scientific activities. However, as he explicitly pointed out, in securing the future, one has to give careful consideration to ways in which the ESI arrangements might be made more stable as it goes into a period of various challenges regarding its financial resources.

Finally, **Marta Sanz-Solé, the President of the European Mathematical Society**, drew attention to the position the ESI has within the international scientific community of scholars as a research Institute with a specific unique character. The Institute is a place that is very conducive to research and, at the same time, integrates scientific education and research.

The following talks were given during the Symposium:

David Ruelle [IHES, Bures-sur-Yvette, France]

Post-human mathematics

Abstract: Present day mathematics is a human construct, where computers are used more and more but do not play a creative role. This situation may change however: computers may become creative, and since they function very differently from the human brain they may produce a very different sort of mathematics. We discuss what this post-human mathematics may look like, and the philosophical consequences that this may entail.

Elliott Lieb [Princeton U, Princeton, USA]

The mathematics and physics of the Bose Gas

Abstract: One of the ways in which mathematics and physics interact strongly is the quantum mechanical many-body problem, specifically the Bose gas. Some rigorous results will be reviewed, including those that were influenced by ESI. Since it is the 50th anniversary of the Lieb-Liniger model, which is now an active experimental area, this topic, too, will be briefly recalled.

Alain Connes [Collège de France, Paris, France]

Variability, time and the quantum

Abstract: I will explain how the mathematical notion of “real variable” is best encoded by the quantum mechanical formalism of self-adjoint operators in Hilbert space. After this mathematical preliminary I will explain how one can conceive of the flow of time as emerging, using the Kubo-Martin-Schwinger condition, from one fundamental aspect of the quantum world which is intrinsic and irreducible variability.

Michael Douglas [State U of New York, Stony Brook, USA]

String theory and the real world

Abstract: Superstring theory is the leading candidate for a theory unifying the fundamental interactions of nature. Yet, after almost 30 years of work, we have no clear experimental evidence for or against the claim that it describes our world. Why do we believe it? What are the prospects for testing it?

Jeremy Gray [The Open U, Milton Keynes, UK]

“The soul of the fact” – Poincaré and proof

Abstract: Throughout his working life Henri Poincaré was concerned to promote the understanding of mathematics and physics. This is as apparent in his views about geometry, his conventionalism, and his theory of knowledge, as it is in his work on electricity and optics, on number theory, and function theory. This talk will argue that this is one of the ways Poincaré discharged his responsibilities as a scientist, and that it accounts not only for a surprising degree of unity in his work but also gives it its distinctive character – at once profound and elusive.

Reinhard Werner [Leibniz U, Hannover, Germany]

How spectral properties may be irrelevant in the long run

Abstract: A simple model system is presented in which the spectral type depends very sensitively on a parameter: pure point, absolutely continuous and singular spectrum each hold for a dense set of values, although all finite time expectations depend continuously on the parameter. Therefore any spectral type is consistent with the dynamics in any long (but finite) run. Of course, this merely points out the lack of exchangeability of the infinite time limit and limits in the parameter, which is a fairly common occurrence. Related phenomena are the lack of direct relevance of ergodicity for establishing approach to equilibrium and the high (quantum-) computational complexity of finding the ground state of a large quantum system as opposed to the low complexity of simulating the dynamics. With regard to the last problem one may well

ask: If Nature does not find the ground state on any reasonable time scale, why should I care about it? This poses the challenge to come up with mathematical notions which have more to say about quantum dynamics on finite time scales.

Wolfgang Lück [Hausdorff Institute, Bonn, Germany]

An introduction to L^2 -Betti numbers and their applications

Abstract: Betti numbers of closed manifolds are classical invariants in topology. Atiyah proposed a generalization, called L^2 -Betti numbers, for the universal covering of closed manifolds taking the operation of the fundamental group into account. These invariants have analytic interpretations in terms of the heat kernel as well as topological interpretations in terms of simplicial homology. They can also be defined for more general spaces and for groups. They have striking applications to various prominent beautiful problems in differential geometry, topology and group theory, where on the first glance these invariants do not seem to appear. We will discuss a selection of such problems indicating where the L^2 -Betti numbers occur as an important tool in their solutions.

Yuri Tschinkel [Courant Institute, New York, USA]

Diophantine equations and their hidden symmetries

Abstract: The structure of solutions of diophantine equations is often governed by symmetries invisible from the shape of the equations. I will discuss representative examples of this phenomenon, involving linear algebraic groups and Galois groups.

Peter Goddard [Institute for Advanced Study, Princeton, USA]

Algebras, groups, and strings

Abstract: Aspects of the interweaving development of the study of infinite-dimensional algebras, such as the Virasoro algebra and Kac-Moody algebras, and of string theory will be reviewed to illustrate the symbiotic relationship between mathematics and theoretical physics.

External invited visitors/speakers:

Jean-Pierre Bourguignon, John Cardy, Alain Connes, Michael Douglas, Peter Goddard, Jeremy Gray, Nigel Hitchin, Helge Holden, Daniel Huybrechts, Horst Knörrer, Elliott Lieb, Wolfgang Lück, Herbert Spohn, Vincent Rivasseau, David Ruelle, Marta Sanz-Sole, Yuri Tschinkel, Reinhard Werner.