Between two, time and truth

Alain Connes and Daniel Sibony
An organized dialogue
by the Hugot Foundation
from Collège de France,
2019

ALAIN CONNES : Hello, Daniel Sibony.

DANIEL SIBONY : Hello.

ALAIN CONNES : So, it really makes me very, very happy to be able to discuss with you. So I'm going to start by introducing you because you have a quite atypical career really, that is to say that you are a mathematician, you have a mathematics thesis.

DANIEL SIBONY : At the start.

ALAIN CONNES : At the start, with Gustave Choquet. You are a philosopher, you have a philosophy thesis, I said with Desanti, but hey, more precisely. And then, you have a practice as a psychoanalyst. So the way I chose to be able to present yourself, it's actually to read a page of one of your works, that I found very touching, because ultimately, it says more about you that a lot of humming titles, etc.

DANIEL SIBONY : I'm curious.

ALAIN CONNES : So you allow me to read it, that's it. It's a book title *Between two, and the origin in share.* There, it will really be the origin sharing, and then it's going to be between two, that is to say that both of us will discuss between 2 extremes. You say : "I knew the dead end in the past, being student at Marakkech in a Jewish school which had the mission of westernizing the little Moroccans we were. My knowledge of Arabic, my mother

tongue, played tricks on me, teaching me a lot about this fear that the other may have of himself, of his awakened origin. I had the worst notes in French, often 0, when I liked to write, and my texts were quite picturesque. But the professor was repulsed to find strong traces of my native culture, culture and language which he was warned but that he seemed embarrassed to know and of which he resisted the blends with the French writing that he gave us as ideal, ideally flat. My essays were often read, when copies were delivered, and the whole class was laughing, including me. The correct expression hid badly and even complacently showed the grotesque gestures of our modes of being who seemed a little ashamed, compared to the small Larousse Classics who indicated true culture. Once read, my copies received their fair sanction : 0. After which the model copy was read where everyone, mine lying down, bored firm. Agreed phrases, lifeless twists. That's who is called "Well written" insisted the professor : "It is not farce, it is elegant."". I kept a strong disgust for well written and hollow texts. At the age of 12-13, I did not have the means to understand that I was wrinkling in his troubles unraveled with the origin, with what he repressed. I thought naively to write was to articulate blocks of meaning and memory, of feeling sations and reminders, with the words available, wherever they come provided that it is just, that is to say authenticated by the life of at least one being, who the occurrence was me. So I accepted, without believing too much, my label : null in French. And I was trembling when I arrived in France, at 14, I was placed in a boarding school where everyone spoke French naturally. However, the day the professor handed over the first copy, he declared, peremptorily : "Here, there is one who can write.". And he pointed at me. I turned around to see this guy whose handwriting smiled. Behind, there had no one. I learned quickly afterwards that this teacher loved above all else the original texts. It was his happy way of transmuting relationships with the origin. I mean, when I read that, I said, "It's really Daniel.". And it's true that...

DANIEL SIBONY : I am very touched that you came across this text, as it.

ALAIN CONNES : Absolutely. Because, what do you want, it means that in fact, in an individual, what interests me, precisely, is what comes out ordinary, which is original, which is not at all typical, etc., which is not at all like writing as you want it, etc. And it's like that's what I perceive you. And if you actually want to on your journey, there is something that is very striking is that, therefore, you were mathematical, this with Gustave Choquet, and

one day, you decided that... I don't know, you decided to change course a bit. How did it come to you?...

DANIEL SIBONY : There are many reasons, this is called an act overdetermined, that means a lot of vectors converge there, but it's true that, I realized that I was worked by all kinds of thoughts, who had to express themselves, and who did not find a mathematical expression tick.

ALAIN CONNES : Of course.

DANIEL SIBONY : And that mathematical work was not enough for me, to fulfill that desire. And so, I did philosophy, and then I did psychoanalysis.

ALAIN CONNES : You did a psychoanalysis before becoming a psychoanalyst.

DANIEL SIBONY : Ah yes, I did a long check, at the time, it was with Lacan. And since I also frequented him closely because he loved math and that he wanted me to explain things to him so it saved me from being Lacanian, and even among anti-Lacanians. So I'm very free, in relation to this thought. And so, that's how, little by little... But compared to this which we will surely discuss, i.e. time, physics, math,...

ALAIN CONNES : Yes, that's how you contacted me, yes, well sure...

DANIEL SIBONY : I realized that strangely, my work in math had focused, we'll come back to that later, had focused on an idea that you are familiar to, and that you have brilliantly applied in other areas, started from the idea of a package of functions that should have certain pro-intrinsic prayers so that it comes out of time. And that was the idea on time, the concern for time even marked my mathematical work and a few years later when I wrote one of my books like that, which is called "Psychoanalysis and writing", the title is more bizarre, it is "The other incastrable", I wrote a big text about time, where I really put all my questions and all that I felt, and so I was worked by time, all the time.

ALAIN CONNES : Okay, so in fact, if you want, what I think is that precisely, we are going to have a dialogue and that in fact, this dialogue, that is going to be an in-between, but not between the two of us, it's going to be between

two extremes. And these two extremes, if you want, precisely, you talked about Lacan, there was a period when you could say that there was a great danger in the post-modernism, which was to use scientific concepts that were not really digested, to finally try to acquire a psychological ascendancy over people who were unable to understand this language. So this is an extreme, it is an extreme which was highlighted in the book of Sokal, on intellectual impostures.

DANIEL SIBONY : I denounced this aspect all along...

ALAIN CONNES : You denounced it, of course, I trust you entirely. And there is another extreme, on which we must be very vigilant, it's the extreme of talking about technical notions, which to be correct, mathematically or physically, will still be difficult to understand by our audience. So we have to navigate between these two extremes and we have to stay understandable.

DANIEL SIBONY : I trust.

ALAIN CONNES : If you want, I think we could start, I had as an idea that we could start by approaching time, as you have proposed, and then, to go on another subject, which is very related, of a certain way, and that is the notion of truth. So these are really two basic topics core, two fundamental subjects on which I think we have both things to say and in fact I will stick to the only things I have to say about which I am certain but which I do not understand at all philosophically or at another significance, because I'm not a philosopher, because I'm just a scientist, if you like, but it will be the same on the notion of truth since there, really, there is a lot to say. Therefore over time, in fact, I had a mathematical trajectory on which it played an absolutely essential role, but, in fact, the main observation, the essential thing that I came up with is that in fact there is something which is much more fundamental than this one-parameter timeline...

DANIEL SIBONY : that the variation of time.

ALAIN CONNES : ...that the variation of time, that's it, exactly. I remember a time, I had a special math teacher, who interviewed me, and who do that (gesture of an index following a curve drawn on an imaginary blackboard in the air) and he said - Mr. Connes, what is the variable ? So we were doing

4

kinematics, etc., I had thought a lot about it, I had thought - is it x or y?, and then I ended up telling her "It's time". Indeed, what we find out here it would require technical explanations, and I will refrain to give them is that the real notion is less the passage of time than what I will call variability. And there, we notice a phenomenon which is absolutely fundamental, which is absolutely crucial, which is one of the most great discoveries of the XX$^{\text{th}}$ century so it didn't exist before, is that when we do experiments, in the microscopic domain, that is to say in what is called quantum, it is simply at the experimental level, there is no need to have a theory for that, so it turns out that there is certain experiments, for example, passing a photon through a very small opening, and then to land it on a target on arrival, so that can be done on a smartphone, well, the experience of to say that the photon will arrive at such a place is not reproducible. And it's because of this phenomenon that Swiss engineers manufactured a device, that you can put in a smartphone, which makes random numbers, but unlike computers that can make random numbers, even if we knew all the ins and outs of the system, we would be unable to reproduce the random numbers in question.

DANIEL SIBONY : So, if you allow, it's good because there are phenomena like that, and that touched me, that I was happy with the idea that we meet, and that we discuss, but really freely, and thoroughly. I resume the example of your teacher, who touched me, I thought about it, and imagine, of course, you answered "The variable is time.", because that's it...

ALAIN CONNES : That's what he expected.

DANIEL SIBONY : And then, it goes without saying, it seems to go without saying. In reality, you didn't have the quantum, spectral point of view that you got later, but you could have told him that the variable is the movements of his finger, and that, it is, not only is it logical, but it is non-commutative, that is to say that if he does that (sketching the end of the movement), he cannot do it if hasn't done this before (sketching the start of the movement). It was the dance of his body if you want.

ALAIN CONNES : I understand what you are saying.

DANIEL SIBONY : And that would join, I don't know, I'm thinking of a sentence of Heraclitus and who is "Time is a child who plays.". That is to

say a child which swaps gestures, movements, and it's not them who are the time, time is the ability to do it all. And that, by far, joins variability that is dear to you.

Alain Connes : Absolutely. Now if you want, what happened at the mathematical level, we realize if we really think about mathematics, cautiously, what a real variable is, then mathematicians believe they know what an actual variable is by saying that they have a set X and they have an application that goes from set X into $\mathbb{R}$, in the real. And in fact, you realize quite quickly, if you think about it, but in a naive way, but that's what to do, we realize that this notion, as standard as it is, real variable, in fact, it does not allow to live at the same time, the continuous and the discrete. Okay, so I'm not going to go into the technical details but then what is extraordinary is that quantum, by definition, almost by definition, at the outset was the appearance of the discrete in what should have been the continuous, that is to say, what that Planck did by hand : basically, I can tell the story a bit all the same, it is that Planck found, almost empirically, a mathematical formula, which described high temperatures very well, and low temperatures, he was in an institute where we were doing experiments of black body, at low and high temperature, and he found a mathematical formula. In this mathematical formula, he was forced to take a really non-trivial function of a quantity which had dimensions, the dimensions of an action. So it's impossible because in physics, normally, when we do calculations, we cannot add a length and the square of a length. So this function, it was forced... and this function, it was an exponential function... And the only thing he could do was to define an action constant, arbitrary, Planck's constant, and when he divided the amount he owed put by this action constant, it became dimensionless, and therefore we could apply any function to it. That's what he did. And good, afterwards, well we understood, he said it but he didn't really understand what was going on, that it meant that the energy was discrete, that is to say that they were whole multiples, that is to say that the energy of a photon, it's integer multiples of this constant times the frequency. So then in fact, what is absolutely extraordinary is that the mathematical impossibility to represent what are called variables, by something from simple, like applications of a set in the real line, it was completely resolved by quantum mechanics, and the formalism of quantum mechanics, which was found by Heisenberg and then by Von Neumann, made it possible to understand that in fact, a real variable, it is not necessary to milk not think of it as a function but you had

to think of it as an operator of a Hilbert space, and at this point, we see that the discrete variables coexist beautifully with continuous variables except that they cannot switch. That is to say, it is impossible, for a continuous variable, to switch with a discrete variable. And that's what makes the very beginning of the quantum miracle. So in my case, what happened is that what I understood in my thesis is that when we look at a quantum system, but if we only partially know this system, that means that we are interested in subsystems, the subsystem admits its own time, its own time emerges. The own time of the subsystem emerges from the fact that we does not know everything, because we have a limited knowledge of things, the time appears, then, that, I found it absolutely miraculous and during years and years, it was very interesting mathematically, because that it gave a whole series of invariants that I found in my thesis on what are called factors, Von Neumann algebras but during many, many years, I had tried, a little stupidly, to reconcile this with physics and I hadn't got there, it seemed obvious to me that this time which appeared naturally from the quantum, had to be linked to physics, until the day when, during a meeting that I told too often already, that of Carlo Rovelli, I met someone who was a philosopher, a physicist but who is in fact more a philosopher that physicist, and who by abstracting on gravitation, had found that at a thermodynamic state, therefore a thermo-equilibrium state dynamic, had to be associated with an evolution, a time, in that sense, and that's exactly what we had in the math. So looking at this, what does one feel who is not a philosopher, but a mathematician, or a physicist, etc., we say to ourselves "Well, there is something there". Well then we are not able to continue enough, but, we can not help to question yourself, you can't help but ask yourself all kinds of questions tions, and to really question... What is the question I have successful, what is the basic question is "do we not make a fundamental error in saying that all physics is based on evolution in time, that is to say by saying that in fact, the equations of physics are equations of the form $\frac{d}{dt}$ equals something else", that's it.

Daniel Sibony : So you want me to tell you my feeling : my feeling is that first of all I love quantum phenomena precisely because that they are of a completely different order, notably this variability of a simple electron going through a hole, which gives you something absolutely not repeatable, and at the same time universal. I used to build a concept which I call singularly universal, that means that it is absolutely singular...

ALAIN CONNES : It remains singular, that's what's extraordinary, it's not a cloud.

DANIEL SIBONY : It's singular and it's universal.

ALAIN CONNES : Absolutely.

DANIEL SIBONY : So already, it subverts a false opposition. Other thing that I liked a lot is that finally, I explained that to someone who knew nothing about physics who asked me "the quantum, what's this ?". I said to him : "Here, you take this glass, it contains a countless number of particles, billions of particles,

ALAIN CONNES : Billions of billions, yes.

DANIEL SIBONY : Billions of particles, and it's happening inside. But if we go into it, we won't understand anything. To understand something, to approach the truth of this phenomenon, we must place ourselves in a space of infinite dimension, in a Hilbert space, with observables, which are operators, and there we understand, we have a clear vision ; in other words, and that is very beautiful, and it is very beautiful both philosophically and poetically also, and I would even say, from a therapeutic point of view, which is mine, psychoanalytic, because often we want to attack directly, at the level behavior : there, this person has this twisted behavior, we will tell him regulate, we're going to straighten it, like the other wanted to straighten my style, and we will straighten it directly. And it's not true, you have to go through a space totally abstract, which can be the unconscious dimension, or even more concretely the dimension of the symptom and there, we see things, and I tell you will give examples, which allow to unblock this behavior. Then since we are talking about time, I think of an example : once, there is a person who came to see me, who was not even my patient, and who exposed me her drama : it was that she had a fever all the time, but not just any what a fever, she still had 38.

ALAIN CONNES : It's not ordinary yes.

DANIEL SIBONY : All the time. I was a little confused. Naturally, here, what I have better to do, to make her tell a little her story, not that the story

is completely reliable, but it gives elements. And she tells, she tells, and suddenly, this question comes out of my mouth : "But you still don't have 39 ?". And she said to me "Well, in 39, my mother denounced my father to the Nazis", she was a German, and therefore, I no longer had a father. And the father's disgrace started in 38, because in 38, she started threatening him. And so, it crashed, it was hiding, from 38. And there, I said to myself, time, by temperature, did a small passage and registered as a kind of burning stone, in the body of this woman ; obviously, when we deployed this, that is to say when we took this marking 38 in a story, in 1938, the woman lost his fever.

ALAIN CONNES : (flabbergasted) But wait, wait, then there, because we has an example, I'm going to play devil's advocate, because I'm rational, but why didn't she go to a doctor before going to see you ?

DANIEL SIBONY : She went to see doctors...

ALAIN CONNES : And they did nothing.

DANIEL SIBONY : Well, yes. They treated her with her fever and when she stopped anti-proud treatments, she still had this fever.

ALAIN CONNES : But they managed to cure her by giving her, I don't know, aspirin.

DANIEL SIBONY : But herself. Today someone who is 38, she takes 3 dolipranes, she's not 38. And one day, she stops, and then hey, she was 38.

ALAIN CONNES : But then here, we also touch on another singular point, if you want, which is the numerical coincidence : it's true that 38 and 39 had such a signification for it, it still never fell below 36.

DANIEL SIBONY : But if you want, the problem is not there, it could have been another sign. It was just to tell you that sometimes you have to pass by things that have nothing to do, for... And from there, the point that me interested me a lot, fascinated me in your conferences, it's precisely that you start from this data, this variability. You say "we commit a error in associating this variability with time". I would plead for a some indulgence for the common man who uses the right as time tracking. But nobody saw time as

a point that moves to the right. So, in reality, I realized that the time that is identified as a point of $\mathbb{R}$, this is I would say the vital minimum which we have needed, to note things that relate to time, knowing that in life is something entirely different. Look there, we're living a moment, this moment is present.

ALAIN CONNES : Yes.

DANIEL SIBONY : It involves our presence and we don't have the impression that it is an instant t which has already passed, so there is a certain stability of the present. And there are also other phenomena, that is to say that if you take two moments, you tell yourself t_1 is before t_2, and when we are in t_2, it's screwed up, it's over, we don't talk about t_1 anymore, it's over and people have written poems on Nevermore, etc. And in reality, that's not it, because the points t_1 and t_2, it's like they're wearing fibers, or fiber spaces, these dimensions moreover, the fibers become entangled, and the subsequent instant t_2 can meet before t_1 in... it's not the time that is reversed, it's the relationship at the time.

ALAIN CONNES : I'm going to bounce back on what you said because it reminds me it happened to me to have to make a presentation on a seminar by Antoine Compagnon, who Proust's expert, and at that time, indeed, I gave an image which is very close to the one you give and which is basically the following : we are actually used to see time like this straight line and very rightly, you say that it is an image which does not correspond in fact really to the best description and I think... the description I had given, roughly, was this : what I was saying is that in fact in the everyday life, in the writings of Proust, etc., what happens is not at all a straight line like that, undefined, but it's a straight line that rolled. It rolls up on itself, and finally I had given a picture geometric, I gave the torus, and it rolls up like a leaf, not like a bundle, that is to say that in fact, what happens is that in our daily experience we have a number of periodicities, the daytime, year-round, a bunch of habits, and those are the habits which give marks on this torus and not on this indefinite line, and which make a kind of eternal return, and which is perfectly described in Proust, in fact. I had spotted a number of statements in Proust, who showed precisely that the structure of his time, he had succeeded in structure by writing *In search of lost time* and that we came to see this global object on which the naive time was wound, which is

this indefinite line finished, etc., but that in fact, the real mental construction he came to, and which can happen to people who look back on their past, it was a structure much more interesting geometrically than the straight line defined. I think it fits what you said.

DANIEL SIBONY : The interesting thing in the example you give for me, by compared to Proust, the time at which Proust arrives, this global time...

ALAIN CONNES : multidimensional, yes, of course...

DANIEL SIBONY : It's the 3 volumes of *In search of lost time.*

ALAIN CONNES : It's his book.

DANIEL SIBONY : It's his book and the word leafing is very welcome...

ALAIN CONNES : Very appropriate.

DANIEL SIBONY : Because with the flipping of the torus that you describe, that is to say with a straight line which wraps up in an irrational way and which therefore goes all cover, the sheets, you can have previous sheets which come synchronize on subsequent sheets. That is to say, in fact, you give, with this example of winding, you give like fibers because the leaves are like fibers.

ALAIN CONNES : This is what you mean by fibers, locally, it's a fibration.

DANIEL SIBONY : It's a local fibration but it gives you, it allows you to approach this phenomenon that turns a lot of heads, that I called, that calls synchronicity which is that, it doesn't matter, it's not extraordinary, that subsequent paths seek to synchronize on a point given, knowing that in addition, they carry meaning, imagine them as small vehicles which carry a package of meaning and which bring meaning where it was not. It's beautiful and it really helps to demystify and it keeps this straight, in this case curled...

ALAIN CONNES : She stays there ?

DANIEL SIBONY : covered with bundles.

Alain Connes : It stays there, yes.

Daniel Sibony : I just understand you when you say, for the quantic, it's not the real variable that will give the real variation, it's the operators with their spectrum, that is to say, precisely, moreover the spectrum is not unwelcome either as a word, because ultimately, the spectrum of an operator. However, it is something that, except for a factor, gives you a variation of identity.

Alain Connes : Especially if you want, that I insist a lot on the fact that precisely in quantum, what happens, therefore, a real variable is replaced by a self-supporting operator, and the values of the real variable which can be either discrete or continuous are replaced by the operator spectrum. So in fact, the operator spectrum has its own variability means that the space in which it operates can vary itself.

Daniel Sibony : So what I really liked was that, with this montage, this device, you manage to extract from the simple fact that the operators algebra is non-commutative, you manage to extract...

Alain Connes : time,

Daniel Sibony : ... to extract a group with one parameter, i.e. time, that is to say you manage to show that algebra evolves...

Alain Connes : Okay, so it generates its own time, if you want.

Daniel Sibony : And that, a phenomenon that generates the time in which it unfolds itself, that, I find it great.

Alain Connes : Absolutely. I totally agree with you, actually...

Daniel Sibony : Wait, I find it very beautiful because, precisely, this that I worked on a lot over the years, that's what I called already, from this book of 78 *The Other Incastrable*, where I introduce the notion of object-time. Do you mind, can I say a word ?

Alain Connes : Of course, of course, please.

Daniel Sibony : And what I call object-time is an object carrying time and from which you can extract some time seams, exactly, what we just said is that the device of a quantum situation, with the algebra of operators and the spectrum is a time object. My device, I had previously studied the theory of potential, where I started from a cone of functions and that's what I liked and what made me tick. You start from a set of functions which you require to verify certain properties that have nothing to do with time, and you extract time from it.

Alain Connes : There you see, I think we hit a point if you want crucial, I'll tell you why. Because precisely, I believe that... I had read for example in Desanti, he spoke of object-time.

Daniel Sibony : He talked about a time object ?

Alain Connes : He talked about a time object, but not at all in your opinion. And what I want to emphasize is this kind of razor-sharp edge on which we must if you want to walk around and stay with the greatest care possible that makes us stay in the real scientist but that at the same time, we get to touch, if you want, concepts that are acceptable, understandable, but we remain in the truth, you understand what I want to say.

Daniel Sibony : This is my major concern. I don't know these texts from Desanti. Desanti was a friend but his thoughts on mathematics, they weren't quite mine, it wasn't my cup of tea. No I try to have an intrinsic reflection, that is to say that when there is a situation, be it philosophical, analytical, physical, the situations you have learned worn, I find it wonderful, not so much to illustrate my concept We don't care about a object-time, but to show that, ultimately, each thing in its time, in its time. What is important, we put away programs, is, in a living situation, to see how it produces the time in which it can take place. And this is great, and there is another phenomenon that I found very beautiful, which you also talked about, it was quantum entanglement.

Alain Connes : So we're going to come there. Why are we going to come ? Because if you want, one of the ideas that emerges from this relativization of time is precisely that we should perhaps be interested in the variability

before to be interested in time, so in fact, we found in our first book with Danye Chéreau and Jacques Dixmier, we found a formula, well, it's always nice to have formulas. So we said "The quantum alea is the ticking of the divine clock.".

Daniel Sibony : Well, I will tell you what I think about it.

Alain Connes : But then wait. Let me bounce on it to say the following, in relation to entanglement. When we discuss the quantum intrication, first of all what is it. So I can tell in two words what it is. There was at the beginning of quantum mechanics, it is not wrong, countless philosophical discussions. The philosophical discussions, for example, were more important, between Einstein and Heisenberg, etc., as the equations themselves. Philosophical discussions were absolutely fundamental. So there was this well-known episode of Bohr, Einstein thought he had found a refutation of the principle of uncertainty...

Daniel Sibony : ...which was false...

Alain Connes : And Bohr found, thanks to Einstein, thanks to his theory of general relativity, that... But Einstein was not discouraged. That was in the early 1930s. But Einstein was not discouraged. And some years later, with Podelski and Rosen, he produced a paradox, to which at first nobody paid attention. Bohr had refuted it using a completely muddled method but when now we look at the curve of the number of citations of this article, of Einstein, it always grows exponentially. So it was an absolutely major contribution. And what was their idea, what was the idea that they put forward, the idea that they have put forward is that... Well then, it's true that...

Daniel Sibony : As if by chance, it's 2 particles.

Alain Connes : Yes, it's 2 particles. What is possible is to create two particles, whose moments are exactly opposite. And, like them are created in the same place, their position must also be linked. Well so what do we do now ? This is called quantum entanglement, what does the description of the experiment say ? What we know, finally, of what Einstein, Podelski and Rosen said, is that after all, we will be able to measure the position of one and the moment of the other, and if we do so how they are causally

separated, well, at that time, we will have, as we know the equality between the moments, etc., we will have the 2 informations. So, in fact, the situation is much more interesting than that, and we did the experiment and we realized that there was indeed this quantum entanglement, that it exists, and in particular, there is Alain Aspect and all the experiences that he has made have shown that there was indeed quantum entanglement but it seemed extremely odd then and it always seems extremely bizarre, that when we experience one, it means that something will happen to the other, while they are causal separated, that's what Einstein called Spooky action at a distance.

Daniel Sibony : Of course there is no remote action. It's very clear, because if the distance is huge, it's too much, but, what I like in this is that there is the idea of unity. It is as if it forms a unit.

Alain Connes : This is exactly the case.

Daniel Sibony : And if it forms a unit, what you say about this point is already passed on to the other.

Alain Connes : This is my philosophical interpretation of this kind of situation. So it turns out that the man always tries to write a story, the past. And in this case, when we try to write a story involving the time, we get confused, it doesn't work. Why? Because the colon are spatially separated, causally separated, so in fact we cannot write a real story. What is my interpretation? My interpretation is that in this situation the alea of quantum at the point that is here and the alea of quantum at this other point are not independent. They form a unit.

Daniel Sibony : Well, they are connected, that means that even if the variability is absolute at each point, it is surmounted by a connection.

Alain Connes : And what should that mean? This should mean that exactly as Einstein had done the analysis of time, from experience on the train, you'd have to be smart enough to hear at the analysis of variability and understand that instead of the alea of the quantic is completely random, in fact, because of quantum entanglement, it has a structure, and it is from this structure that must emerge, not time, because that, I know how to make it emerge from equations, and from noncommutativity, but that the

general structure must emerge. So this is a problem, I don't pretend that this problem is solved, but I pretend the question is getting hot, because you can't write a story consistent especially since the past as you know is not determined, because of Wheeler's experiments.

Daniel Sibony : But precisely, perhaps to raise, to highlight this aspect of quantum entanglement, we should talk about the experience with two holes and two photons that surround a galaxy.

Alain Connes : Yes, of course, especially the experience of Wheeler. In fact, we can do it either with the 2 holes, or with mirrors and photons which surround a galaxy. Well, we're not going to go into technical details, but basically, we will still explain what is the paradox. The paradox, is that at some point, when we have such an experience where we have a photon which divides into 2, a priori, when we think of it as a wave, etc., and it can interfere on arrival, well by putting on arrival either what it takes for there to be interference, which is what it takes to know which path the photon took, well, that will actually determine what happened in the past. And so Wheeler imagined an experiment with an intermediate galaxy, which means that the 2 photon trajectories will join at one point much later maybe a billion years more late, well, what would that mean ? It would mean that depending on what we do now determines what the photon did there is a billion years. So what does that mean ? It means that in fact the past is not written once and for all, which is absolutely fascinating.

Daniel Sibony : And that's what is... Well, needless to say, that excites me because, if you want, first of all, there is a common phenomenon, which one finds in psychoanalysis but which each one knows and which one calls after the fact. It means after the fact, there was an event for you, at 4-5 years, which was a bit striking but not too much, and then 30-40 years later, very long after, another event takes place, which is not identical, which has a small variability with the first, but close enough, to make a kind of resonance and there, the subject notices after the second event, that the first was traumatic. That is to say that the nature of the past event only appears when it questions the past, with a large gap, and also with a certain interest in the truth, that is to say what happened, really ? We may not know but what we do know is that something else happened than we thought. In other words, if we resume now in physical terms, I think it's great that the past is unstable,

ALAIN CONNES : not fixed,

DANIEL SIBONY : and depends on overlays that we don't know depends in our way of questioning it.

ALAIN CONNES : Absolutely, at the present time ?

DANIEL SIBONY : At the present time. If we think about it, maybe it's the montage even writing. You were talking about Proust. Someone who writes a novel, who writes a somewhat creative article, etc., he writes what appears but he lets approach with things that will otherwise reconvene the past, other said that will transform the past. There is a caricature of that, today, but who should have existed, it is the people who want to rewrite history.

ALAIN CONNES : Obviously, it's obvious.

DANIEL SIBONY : But the idea by itself is very beautiful, it means also say that the truth, it can only be a dispersion, a distribution discreet, always partial, and that it is in the recurrence of re-questionnement that we get closer to the effect of truth, than by writing, ultimately what the truth should be.

ALAIN CONNES : Yes, that's very interesting, because that leads to what the truth is and there, if you want, I think it takes that we are fairly organized, for the following reason : when you take work of the mathematician, at first glance, the job of the mathematician is that he is trying to demonstrate, he's trying to find out if something is true. Ultimately, he can test with his computer if something is true etc., and at first glance, even the mathematician will feel that either something is true, or something is false, and then it goes and navigate in a universe that is quite simple, that is to say that he will just navigate from one to the other. In fact, we realize that this idea, even in mathematics, I will come back to this for the rest afterwards, even in mathematics, is a misconception and the reason is that what I have just said applies perfectly to decidable propositions ; for example if I want to know if a number for example 31 is a prime number or not, this is called something decidable, that is to say either it's true or it's false, and we can do it in a finite time. So, now when we discuss truth in mathematics, you have to be

very, very careful, because you have to be able to qualify the statements, and you have to see that there are statements that are called existential or universal statements. So what I call for example a universal statement, for example, whatever x, I don't know, whatever an even number, there are... okay. But what's going to happen is that therefore, what I call a universal statement, it's whatever x, but whatever x, we're going state a decidable property, for example, we will ask that x if it is even be the sum of two prime numbers. It's a statement that we can decide. So what's absolutely amazing is that if we only take the integers, everyone knows what the integers are, which is absolutely incredible, is that we know, if we work with integers, etc., we know that if a statement is demonstrable, it is true. We have all kinds of nuances between this which is demonstrable and what is true and in fact what we also know is that in fact, if we look at most of the statements that are true on integers, most of the true statements are not demonstrable. So this is something absolutely incredible, if we look at the proportion, among the statements true, of those that are demonstrable, we know that there is an incredible amount true statements that are not demonstrable. And then a typical example of a statement which is true but which is not demonstrable in the axioms from Peano, this is an example that I like to give, I'm not going to give it technically but I'm going to say it like that, and say it, I hope, so coorect : it is that we take a number, like for example, the number 5, we write it in base 2, we write that 5 is equal to 4 plus 1 but we write that 4 is 2 power 2, we write everything in base 2, okay, and then we do an operation which I call the hare, because it will increase the size considerably of the number. We replace all 2 by 3, okay ?! Next, the turtle arrives and the turtle subtracts 1. We take the result, we write it again in base 3, etc. and we replace all 3 by 4, that's the hare, then the turtle arrives and it subtracts 1, we rewrite the result in base 4, and we replace all 4 by 5, etc. The statement which is incredible but which is true, and I will tell you why but which is not demonstrable in the arithmetic of Peano is that it is the turtle that wins. This is the story of the hare and the turtle of course that I want to illustrate is that although the hare takes steps absolutely huge, the turtle that does almost nothing. What's going to happen produce ? There are going to be situations in which when we subtract 1, you can no longer write it in the same way, you must have changed the script, etc. and then, what is incredible, is that this statement, we know that we cannot demonstrate it in Peano's arithmetic. How do we know it ? We know that because we know that the function that gives the number of steps that it takes for the turtle to win is a function that

grows faster than any function you can write. If we could demonstrate it, we would have a terminal for this function. So we know it's not demonstrable in Peano's arithmetic. Why do we know it's true ? You will understand since you were a student of Choquet so you will understand immediately : what do we do ? We do something that is beautiful, we replace the base, which was 2 then 3 then 4 by the smallest ordinal infinite, by ω and we realize that when we put ω, the hare does not change anything, and the turtle decreases by 1. Since we have an ordinal, it will eventually happen towards 0. The demonstration is incredibly simple, but it escapes to Peano's arithmetic, so here is an example that shows how the notion of truth in mathematics is an incredibly subtle notion and it is a notion which, well, in practice, the mathematician who works has not to do these things. But in fact, it may very well be that there are pretty common situations, in which this stuff has a role. In any case, the image that emerges has been wonderfully described in a little book which is Jean-Yves Girard's book, on Gödel's theorem, he explains what was above, the image that emerges, if you want, is that you have to see the mathematician as someone who is in a court and who is going to try analyze the truth. He's going to have some means, but certainly not that to know if something is true or not, if you want, except in very simple cases, of course.

Daniel Sibony : This problem arises, of course, in mathematics, but in many other areas. Since you are talking about court, there are cases who are really fair, irrefutable, etc., and who are lost in court, because the language in which the formulas would be needed for them to be even understandable, is not there.

Alain Connes : Does not exist.

Daniel Sibony : So the interesting thing means that the truth is not an entity, it is not even an emergence or an effect, it is a correlation between two languages, that is to say between the language that carries the object, and then the language which must receive, which must authenticate.

Alain Connes : Yes, but then wait, then there, we have to be very careful because if you want, for example in this book, there, from Bricmont and Sokal, which I read carefully why, because in fact, I have things to say on that, so what was their idea ? Their idea, the idea of Intellectual impostures was that, and now we're both in agreement on this, there was abuse, at the

19

time of post-modernism, that is to say that there was abuse consisting of using mathematical language, which was in fact, not supported by rigor, and which is especially not knowledge.

Daniel Sibony : If you allow, even today, it's not just post-modern. I sometimes listen to popular science lectures and to hear amazing things, for example, to hear that because of relativity, time, there is no more, time no longer exists. And that, I find than...

Alain Connes : ...that it's difficult to swallow, yes.

Daniel Sibony : People who don't know relativity, happily resist and know that there is time, that time here, is not the same as there, on the other galaxy, it confuses time and time measurement, and we confuse time with relation to time, so there is a lot of confusion, even among scientists.

Alain Connes : Of course. Being a scientist does not absolutely exclude that kind of confusion. But then, what struck me a lot, reading this book, precisely, of Sokal, their goal was laudable in a certain way, but what really struck me was that the people they criticize, there is Bruno Latour, there is Lacan, Lacan is on the front line...

Daniel Sibony : I hope.

Alain Connes : but, which struck me enormously and there, I am sure that I'm going to get attacked but it doesn't matter, there is something that these people felt it but they didn't have the language to say it, and I think it's going to match what you said earlier.

Daniel Sibony : Tell me.

Alain Connes : People say that I have a fad, it's the topos of Grothendieck. But I'm going to talk about it. I'm going to talk about why, because this fad, in fact, Grothendieck considered it his greatest discovery, and when you really understand it, making it, what amused me also, it is to see that the people who criticize this thing, in general, don't know what it is, and criticize it saying "yes, it didn't matter, it didn't have impact on mathematics" but precisely, if it had no impact on math it is because people just didn't

understand the sense that it has. And it has an extraordinary meaning. I'll explain what it is, and explain to you in what sense precisely, that would allow these people, who have tried to express themselves, if they had had this concept, they could have expressed themselves.

Daniel Sibony : These people, that is to say ?

Alain Connes : Bruno Latour, etc.

Daniel Sibony : Ah yes, people who tinkered with things, like they were able to.

Alain Connes : With common language.

Daniel Sibony : Yes good.

Alain Connes : With common language but they didn't have the words to say it. So I'm going to try to explain, first what is the concept, and in what sense it completely changes the notion of truth because that's it which is fundamental. The interest of this concept is that it changes the concept of truth, which is absolutely fabulous. So if you want, well, it's an abstract mathematical concept. What was the great discovery that made Grothendieck ? He first noticed the following. I will employ technical words, but if I don't use technical words, we'll accuse me of... good.

Daniel Sibony : You have to.

Alain Connes : What Grothendieck found was that at one point, he had to write an article for, it was an article a bit of... everyone would have said it was easy, these were things he had to compile etc., it was on homological algebra, and basically he wrote the axioms of the abelian categories, but people knew them more or less at the time, it was not what was important, and by developing these ideas, taking examples, very interesting examples, which he called the categories of diagrams if you want, he noticed the following thing, he noticed that in fact, when we do the usual beam theory, we took bundles of abelian groups, and then we looked at the cohomology, etc., and he had the idea of no longer looking at bundles of abelian groups, but bundles of sets, and at that time, he made two observations that are great,

the first observation he made was that, if we give the category of bundles of sets on a topological space, we can reconstruct the topological space, with its topology, so it's enough extraordinary, because you give an abstract observation... Second observation, when working in the category of bundles of sets on a topological space, it's as if we were working in the sets category. All the properties that we usually use are true, except the principle of the excluded third party. In other words, we no longer have the right to reason by the absurd, otherwise, everything works flawlessly. That is to say if you work in the category of bundles of sets, in a topological space, you can talk about a group, you can talk about a ring. And it will come back to talk of a bundle of groups, a bundle of rings, etc. But third observation : the really great thing is that it's not just the beam categories of sets on a topological space which verify these properties, there are others. And that means there are new spaces. And among these new spaces, which are no longer topological spaces, these are basically spaces with inner movement, okay, with relationships if you want. And then what is happening now and which is really wonderful, I think. I met that, if you want, what convinced me was that before, I bleated like the herd, that is to say, "Oh, the topos, it's not interesting, it's a generalization of the concept of space, we don't care, etc.", that's what I was doing until a number of years ago, maybe 5 or 6 years, and 5 or 6 years ago, I realized in my work with Katia Consani that in fact there was an underlying topos to the space that we had found, and whose point space was the non-commutative space we had found, at that time, I was amazed, I was completely wrapped by this notion, and afterwards, I realized the depth that it has, and in fact, what's really deep in this notion is that when you work in a topos, where you can no longer use the third-excluded, the principle of contradiction well, we have a replacement for the true and false, I'll explain what it is, a little technically, we have something that replaces the true and the false, but which is more subtle. And I have recently gave a lecture last year at the teacher training college, in which I wanted to give an example of that. And so I wanted to give an example, and this example was to say that we are 3 steps from the truth, 4 steps from the truth, 10 steps from the truth. So I'm going to give you a topos that I'm going to name, you see, okay, so technically this will be correct, and that will say that we are 10 steps from the truth. What is this topos ? Well, instead of talking about a whole, as we would normally talk about, well, we're going to talk about a set with a transformation, without special property, it is a transformation. You have a whole and a transformation. Theorem : that is a topos, that is to

say that if you work with these things, you can work exactly as if you were working with sets. Everything works well, except that you no longer have the third-party excluded. So where did it come from that you no longer have the excluded third party ? Well, it comes from the fact that normally, when we work in the topos of the sets, when we has a subset, we can say, we can speak about the subset by the function which is worth 1 if we are in the subset, let's say true if we are in the subset and false if we are not in it. Okay ? Well ! Because obviously you have the set, and you have its complement, so it works good. So now, let's take this topos I told you about, which is made up of a set and a transformation. So we will try to classify the subject together. What is a subset, a subset is a set which is stable through transformation, of course, it's a sub-object, if you will. So is it possible to take the function which is worth 1 on this subset, and 0 on the complementary ? No why ? Because you can get to be in the complementary, but after a certain number of steps, you get in the starting set, so in fact, it's not this 0-1 that will classify the sub-objects, no, it's the number of steps it takes to arrive in the truth.

Daniel Sibony : It's proximity...

Alain Connes : It's proximity to the real. And at that time, what do you have ? You have a theory that works exactly like the theory of sets but in which the true and the false, which were simply the true and false in the ordinary sense are replaced by the nuances on the true, that is to say you have the true, you have the 1 step to the true, you have the 2 steps to the true, 3 steps to the true *(laughs)* and you have the false. And this is wonderful I mean, from there, we can do variations, we can do all kinds of variations and my idea, now, my idea is this : in carefully reading the writings of these people, I realized that what they were actually trying to say was that, not in the area of mathematics, but in the social sciences, where for example, when attend a discussion on television, where we will say "This one is right...", "Is Martine Aubry right to say that the 35 hours were a success ?". If you want, things like that, as soon as we are in a situation like that, to say so-and-so is right, so-and-so is wrong, is heresy, because it's an incredibly simplistic view, compared to the complexity of the problem we are tackling, and as a rule, the only tool that people have to try to make up for their lack of concepts, that is to say : this one is 50 % right, etc. But it's ridiculous. So what I'm saying, simply, my conclusion, if you will, is that in all these cases, I think there is a topos difficult to determine, but which would allow, if you want, to...

precisely, name all these nuances on the notion of true and false, and allows trait of being infinitely more effective precisely in this kind of situation. And I think these people understood it intuitively.

Daniel Sibony : So I think it's a very good thing, obviously, that it is developed in mathematics, and in mathematical language, and on mathematical objects, in a precise and radical way. At the same time, everything that is developed, in art, in a science, in a practical field, is that it exists in life. And in life, me, before a debate, I realized that my question was not "who is right ?" and "this, is it wrong ?", my question is that everyone is between two, that is to say between the truth he believes in and approaches a little the other truth, which is in question, it is "in between". And it's very fun to observe that some see well the approximation of the common truth or of the truth that is in question but do not find the strength to risk their place, which they obviously identify with the truth, because that's what that in psychiatric terms, we call a narcissistic investment, that is to say, there is a minimum of self-love *(He laughs)*.

Alain Connes : Let me bounce on what you say, let me interfere to break up. Is that the topos have points, and when you are in a point of topos, there, you have the true or the false, that is to say that you have no nuances. And that is very important.

Daniel Sibony : It is obvious that it is a theory of nuance, but which, punctually is not nuanced.

Alain Connes : Occasionally by definition of the point, the point is not shade. That is to say that the point is the theory of ordinary sets, by definition.

Daniel Sibony : And so, to come back to that, and these days, with all kinds of tensions and even underlying violence, we feel that the dimension of truth has taken a hit.

Alain Connes : Ah, terribly, terribly, of course.

Daniel Sibony : Because someone you respect, who is someone good, there is no arguing about it, to see him not seeing the truth who could threaten

the position he is responsible for, and who often identifies himself with his position, that is to say that if he takes risks with this truth be told, he's on an ejection seat.

ALAIN CONNES : He jumps, yes.

DANIEL SIBONY : See that, I assure you that for someone... if there is something to which I am sensitive more than all, it is the truth, justice, which is a little truth in sharing, and desire, which is also a little truth in our existence. Well, I had moments of suffering, when I had to take this new set of truth, which is a kind of parade, and who doesn't make me judge or despise people at all, but who says that places are expensive, to defend... So now, I would like, before we finish, which has always fascinated me in this area of quantum, through what you say. You said at one point, "The emergence time, it is due to the fact that we do not know everything".

ALAIN CONNES : Exactly, that's exactly the truth.

DANIEL SIBONY : There is one thing, this thing, and I would like to bounce back on the other, which is "The alea of quantum, it is the ticking of the divine clock.". First, the alea of quantum is the ticking of the clock, even if there is no clock, and the Divine, we don't really know where it is, but this sentence reminds me I like it a lot, and I deeply agree with it, in the following sense : in the sense that it means that, and we agree that, there is no time universal, but this sentence, it says that time is taken in what I call "the infinity of possibilities".

ALAIN CONNES : Absolutely, that's exactly it, of course.

DANIEL SIBONY : And for me, the infinity of possibilities, that's the Divine, and the Divine, both in the religious sense and in the non-religious sense. Religious do prayers to the Divine so that He gives them a little more, and non-religious seek the possible, and say to themselves - if I could have a shot luck ! -, etc. So this sentence, it says very exactly that time may not be the same here and elsewhere, it may emerge, and not be already given, etc., between us, moreover, if it emerges from a situation, it is because it was already there.

Alain Connes : Yes, in a way, of course.

Daniel Sibony : Time takes in the infinity of possibilities, that means to say, what I call being, that is to say that there is a time of being, which is insituable, which is unaffordable, but which one takes, where one can. And it's that the role of time-objects, the meaning I give to them, objects carrying time, works of art, a theory, whether of potential or quantum, or something else is that the important thing are objects where we can take time, and the important thing is how to take it. So now, that leads us to the question of the unconscious, of non-knowledge. You say "The time emerges from the fact that we don't know everything.". And I ask myself a question, and maybe we will finish on this, I ask myself a question, in a meaning, I wrote in this text over 50 years ago - time rises with the repression -, that means when the repression is lifted, there is a source of time that appears.

Alain Connes : And there, you identify time and freedom, in a certain way last, yes.

Daniel Sibony : Maybe that means that in reality, we take a scene of your second novel, where the heroine is totally unconscious, and there she has total knowledge, but she is not aware of having that total knowledge.

Alain Connes : No, not at all.

Daniel Sibony : So she's totally unconscious.

Alain Connes : Since consciousness is linked to time, by definition.

Daniel Sibony : And it is by leaving this state of unconsciousness towards the aware...

Alain Connes : that let her realize what she went through.

Daniel Sibony : That there is a flow of time that appears. And that makes me think of a small Talmudic story that I once knew that says that the infant, the fetus, in the womb until its time of birth, he knew everything.

Alain Connes : We agree *(He laughs)*.

26

Daniel Sibony : He had total knowledge, and then an angel passes, he makes a cut, and, by the way, when it enters time, it loses everything, and we notice, coming back this time to the quantum model or your novel, we realize that he loses all knowledge and that the equivalent of these knowledge that was unconscious, that's the time it will take to acquire.

Alain Connes : Absolutely.

Daniel Sibony : In other words, there is a balancing...

Alain Connes : Between time and knowledge.

Daniel Sibony : Between time and unconscious, or time and knowledge, that is to say that in a certain way, time enters the world crossing the threshold...

Alain Connes : The maternal threshold.

Daniel Sibony : The threshold between the unconscious and the conscious, or between the real and the practicable.

Alain Connes : Well listen, I think it's a very nice conclusion.